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REINFORCED CONCRETE

[A TEXT-BOOK FOR ENGINEERING STUDENTS]

by

H. J. SHAH

M. E. (STRUCTURE)

Lecturer in Applied Mechanics
Faculty of Technology and Engineering
M. S. University of Baroda

also

Consulting Structural Engineer

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[FIRST EDITION]

CHAROTAR PUBLISHING HOUSE
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PREFACE

There are a number of books on this subject which have been written by many specialists, both on the continent and also in U.S.A. These books are more or less like reference books for the use of practising engineers in the profession as these books are based on standard specifications of design of respective countries. As text-books for young engineering students, they are, of course, very valuable, when the course calls for a study of few subjects and the syllabus demands study in the subject of specialisation, as in foreign universities. They are also prohibitively costly for our Indian students preparing for various examinations of the Universities in India. For such students, this book is a boon.

This book presents the basic principles involved in the design of reinforced concrete structures. It is written completely in SI units and based on latest revision of IS : 456-1978. The material presented in the book covers the syllabus of most of the Indian Universities. The principles of design are explained with numerous examples and neat sketches in a manner of drawings for work at site. Supplementary details are included to provide the practical knowledge of design. The use of design aids is also emphasized. It also includes exercise problems at the end of each chapter and objective type questions with answers at the end of the book.

Generally, now-a-days, the elastic theory of design is not followed in practice except for deflection and crack width calculations, and where the design methods of limit state are not available e.g. liquid retaining structures. However, the elastic theory of design is a basic one and the beginners must learn it; because of this, in most of the Universities, elastic theory is taught first. It is believed by the author that after understanding the basic principles of analysis and design by an elastic theory, the student should learn the complicated elements like different types of footings, frames, retaining walls, etc., directly using the limit state method. This book is written as a first course in reinforced concrete analysis and design. Therefore, simple concrete elements are considered with practical details.

Suggestions to improve the usefulness of the book are most welcome.

Baroda **H. J. Shah**

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Baroda **Hiten J. Shah**

May 10, 1985

CONTENTS

NOTATIONS

Standard Symbols from Clause 3.1, IS : 456

A	area
b	breadth of beam, shorter dimension of a rectangular column
b_{ef}	effective width of slab
b_f	effective width of flange
b_w	breadth of web or rib
d	effective depth of beam or slab
d'	depth of compression reinforcement from the highly compressed face
E_c	modulus of elasticity of concrete
E_s	modulus of elasticity of steel
EL	earthquake load
e	eccentricity
f_{ck}	characteristic compressive strength of concrete
f_{cr}	modulus of rupture of concrete (flexural tensile strength)
f_d	design strength
f_y	characteristic strength of steel
k	constant or coefficient or factor
L_d	development length
LL	live load or imposed load
l	length of a column or beam between adequate lateral restraints or the unsupported length of a column
l_{ef}	effective span of beam or slab or effective length of column
l_{ex}	effective length about x-x axis
l_{ey}	effective length about y-y axis
l_x	breadth or shorter side of slab
l_y	length or longer side of slab
l_o	distance between points of zero moments in a beam
M	bending moment
m	modular ratio
P	axial load on a compression member or a pull
q_o	calculated maximum bearing pressure at the base of a pedestal
r	radius
s	spacing of stirrups
T	torsional moment
V	shear force
W	total load
WL	wind load
w	distributed load per unit area
x	depth of neutral axis (various subscripts)
Z	modulus of section
z	lever arm
α, β	angle or ratio (various subscripts)
γ_f	partial safety factor for load
γ_m	partial safety factor for material
σ_{cbc}	permissible stress in concrete in bending compression

σ_{cc} permissible stress in concrete in direct compression
σ_{sc} permissible stress in steel in compression
σ_{st} permissible stress in steel in tension
σ_{sv} permissible tensile stress in shear reinforcement
τ shear stress
τ_{bd} design bond stress
τ_c shear stress in concrete
$\tau_{c\,max}$ maximum shear stress in concrete with shear reinforcement
τ_v nominal shear stress
ϕ diameter of bar

Other symbols used are:

A_c area of concrete
A_s area of minimum reinforcement or longitudinal tension reinforcement which continues at least one effective depth beyond the section
A_{sc} area of compression reinforcement or area of longitudinal reinforcement for columns
A_{st} area of tension reinforcement
A_{sv} total cross-sectional area of stirrup legs or bent-up bars within a distance equal to spacing of stirrup or bent-up bar
A_T area of transformed section
a centre to centre distance between bars or group of bars perpendicular to the bend or width of contact area of concentrated load measured parallel to the supported edge
a_1 distance of the concentrated load from the face of the cantilever support
BS bottom straight bars
BU bent-up bars
b actual width of an isolated flanged beam or width of slab or footing
b_1 centre to centre distance between corner bars in the direction of width
C, C_c force in concrete in compression
C_r reduction coefficient
C_s force in steel in compression
D overall depth or depth in respect of the major axis in column
D_f thickness of slab in flanged beam
DL dead load
d_1 centre to centre distance between corner bars in the direction of the depth
E_{ce} long term modulus of elasticity of concrete
EB extra bottom bars
ET extra top bars
F characteristic load
F_{bt} tensile force due to design loads in a bar or group of bars
F_{tc} total flange compression in a flanged beam
F_{ts} total tension of the steel in a flanged beam
f characteristic strength of material
f_c stress in concrete due to axial compression
f_{cb} stress in concrete in compression due to bending
f_{sc} stress in steel in compression

f_{st}	stress in steel in tension
G	going of stairs
h_{agg}	maximum size of aggregate
I	moment of inertia (various subscripts)
IS	Indian Standard
i	radius of gyration (various subscripts)
j	lever arm constant
K_t	coefficient for moment of inertia of a flanged beam
k	neutral axis constant
kN	kilo-newton
kNm	kilo-newton-metre
L_0	anchorage of bars beyond the centre of the support
l	span of beam or slab
M_{bal}	moment of resistance of a balanced rectangular section
M_1	moment of resistance of the section assuming all reinforcement at the section to be stressed to f_d
M_{e1}, M_{e2}	equivalent bending moments
M_{tc}	moment of flange area of flanged beam about bottom of a flange
M_{ts}	moment of transformed area of steel about bottom of a flange in a flanged beam
m, mm	metre, millimetre
N	newton
p	steel percentage, soil pressure (various subscripts)
Q	moment of resistance factor for a balanced rectangular section
R	reaction at support
s_v	spacing of stirrups or bent-up bars along the length of the member
T	tension or total tensile force in steel
TA	top anchor bars
V_e	equivalent shear
V_s	strength of shear reinforcement
w	distributed load per unit length
y_1	long dimension of stirrup
α_x, α_y	bending moment coefficients for two-way slabs
ε_c	strain in the concrete
ε_{sc}	strain in the steel in compression
ε_{st}	strain in the steel in tension
σ	tensile stress
σ_1	major principal stress
σ_2	minor principal stress
σ_{ct}	maximum tensile stress in concrete
σ_s	stress in bar at the section considered at design load
τ_{ve}	equivalent nominal shear stress
θ	angle
Φ	diameter of a tor steel bar

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Introduction

1-1. Structural design — Role of a structural engineer: A building or any structure in general can be built in a way one likes. However, when the question of building a systematically planned structure arises:

(a) It should satisfy the functional requirements of the client and should be aesthetic which needs an *Architect.*

(b) It should be structurally safe so as to withstand the loads it has to bear which needs a *Structural engineer.*

To achieve the best possible results, both have to work together.

The general structural frame work with which a structural engineer is concerned, usually consists of load bearing masonry structure, reinforced cement concrete frame or a steel structure. Many other types of frame work are also used. The choice of the type of frame work depends on site conditions, architectural planning of the building, economy and safety requirements. Building regulations and client's desire also play an important role in framing the building. In any case the safety of the building is the ruling factor for the structural design.

The working drawings of a building are prepared by the architect with the help of the structural engineer.

The general working of the structural engineer is as follow:

(1) Finalising the frame work and tentative sizes of beams, columns, etc., considering the site conditions and structural requirements.

(2) Estimating the loads on structure or a part of the structure using available loading standards and his experience. For normal structural work the loading standard is IS : 875.

(3) Analysing the structure for shear, moments, axial loads, deflection, etc.

(4) Designing the structural members using results from (3).

(5) Preparing the detailed structural drawings for the work at site.

(6) Checking the work done by the contractor on site when the work is in progress. In the interest of smooth and proper execution of work, it is essential for the contractor and client to get the work checked by the structural engineer frequently.

The job of structural engineer is challenging and carries a great deal of responsibility. Thorough knowledge of analysis and design of structure with current codes of practice, creative ability, confidence and practical experience are required to get a safe and economical design of the structure.

The different types of analysis treated in this book are based on Code of Practice for plain and reinforced concrete IS : 456-1978 and other relevant codes wherever necessary. Use of design aids is also explained wherever necessary and should be read in conjunction with SP : 16-Design Aids to IS : 456-1978.

1-2. Concrete: Concrete is a mixture of cement, fine aggregates (sand), coarse aggregates (kapchi) and water mixed in a definite proportion. The chemical reaction between cement and water causes hardening of the concrete. Because of the chemical action, cement crystallizes and releases the water which may evaporate. If this water is retained by some means or in other words if the water is continuously available, the chemical action can be continued as long as all the cement is hydrated. The process by which the loss of water from concrete is prevented is known as *curing*. To achieve this, concrete is kept moist and at about 10°C. temperature by sprinkling water on it or covering wet gunny bags on concrete. This also can be achieved by covering the concrete surface with polythene sheets or by applying a thin layer of liquid sealing compound. For further details one may refer to standard text books on properties of concrete.

Different strengths of concrete can be achieved by using different proportions of ingredients of concrete. The compressive strength of concrete is tested by testing standard cubes of size 150 mm × 150 mm × 150 mm. The crushing strength of this cube tested after curing it for 28 days is known as cube strength.

As per IS : 456 the concrete shall be in grades designated as per table 1-1. The *characteristic strength* of material is defined as the strength of the material below which not more than 5 per cent of the test results are expected to fall.

TABLE 1-1

GRADES OF CONCRETE

Grade designation	Specified characteristic strength of 28 days N/mm^2
M10	10
M15	15
M20	20
M25	25
M30	30
M35	35
M40	40

Note 1: In the designation of a concrete mix, letter M refers to the mix and number to the specified characteristic compressive strength of 15 cm cube at 28 days expressed in N/mm^2.

Note 2: M5 and M7·5 grades of concrete may be used for lean concrete bases and simple foundations for masonry walls. These mixes need not be designed.

Note 3: Grades of concrete lower than M15 shall not be used in reinforced concrete.

The proportion of ingredients of concrete to obtain a desired mix can be found out by laboratory methods of mix design. However, for normal uses as a guide line, the following proportions of ingredients may be used:

Mix	Proportion of ingredients cement : sand : kapchi
M5	1 : 5 : 10
M7·5	1 : 4 : 8
M10	1 : 3 : 6
M15	1 : 2 : 4
M20	1 : 1½ : 3
M25	1 : 1 : 2

M5, M7·5 and M10 mixes are used for lean concrete. M15 and M20 grade concretes are most popular concrete for structural work. Higher grades of concrete may be used in heavy design e.g. lower columns in multistorey building. For prestressed concrete structures much higher grades of concrete are used.

1-3. Properties of concrete: Some important properties of concrete are discussed below:

(*1*) *Unit weight:* According to IS : 456, "Unless more accurate calculations are warranted, the unit weights of plain concrete and reinforced concrete with sand and gravel or crushed natural stone aggregate may be taken as 24000 N/m^3 and 25000 N/m^3 respectively".

(*2*) *Increase in strength with age:* In design, the concrete strength at 28 days is considered. The chemical action of cement with water is faster initially and in seven days concrete attains approximately two-third strength of that of 28 days strength. This is true when ordinary portland cement is used. When portland pozzolana cement is used, the chemical action is slow and the rate of gaining strength upto 14 days is slower. However, if properly cured, at 28 days both the types of cement give required strength. Even after 28 days, the hydration is continued and concrete gains the strength with age at a much slower rate. This property of concrete may be used in design.

According to IS : 456, "Where it can be shown that a member will not receive its full design load/stress within a period of 28 days after the casting of the member

(for example, in foundations and lower columns in multistorey buildings), the characteristic compressive strength given in table 1-1 may be increased by multiplying by the factors given below:

Minimum age of member when full design load/stress is expected (months)	Age factor
1	1·00
3	1·10
6	1·15
12	1·20

Note 1: No increase in respect of age at loading should be allowed where high alumina cement concrete is used.

Note 2: Where members are subjected to lower direct load during construction, they should be checked for stresses resulting from combination of direct load and bending during construction.

Note 3: The permissible stresses or design strength shall be based on the increased value of compressive strength".

(3) *Tensile strength of concrete:* In reinforced concrete construction it is usually considered that concrete resists compression and steel reinforcement resists tension. However, concrete has to resist tension due to many reasons such as temperature changes, shrinkage of concrete, etc.

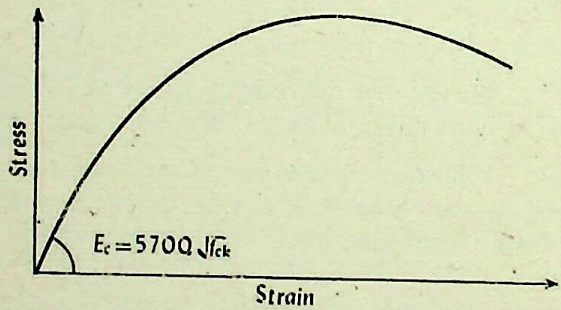
When the designer wishes to use an estimate of tensile strength from the compressive strength, according to IS : 456, the following formula may be used:

flexural strength $f_{cr} = 0{\cdot}7\sqrt{f_{ck}}$ N/mm²

where f_{ck} is the characteristic compressive strength of concrete.

(4) *Elastic deformation:* The modulus of elasticity of concrete is primarily influenced by the elastic properties of the aggregate and to a lesser extent by the conditions of curing and age of the concrete, the mix proportions and type of cement. The modulus of elasticity is normally related to the compressive strength of concrete.

A concrete cube is subjected to uniaxial compression and the stresses and corresponding strains (using dial gauges or electrical strain gauges) are found out for uniformly increasing loads. A graph of stress v/s strain is drawn, which is a parabolic curve. The value of modulus of elasticity found from initial tangent at the origin is known as the short term static modulus of elasticity. Modulus of elasticity may be measured in tension also . Usually modulus of elasticity in compression and tension are equal. A typical stress-strain curve for concrete is shown in fig. 1-1.



Typical stress-strain curve for concrete

FIG. 1-1

In the absence of test data, the modulus of elasticity for structural concrete may be assumed as follows:

$$E_c = 5700\sqrt{f_{ck}}$$

where E_c is the *short term static modulus of elasticity* in N/mm^2 and f_{ck} is the characteristic cube strength of concrete in N/mm^2.

(5) *Shrinkage:* The shortening in length of a member or contraction of the concrete due to drying when concrete sets, is known as shrinkage. On drying the cement crystallizes and gives out the free water and shrinks. This is irreversible process.

The total shrinkage of concrete depends upon the constituents of concrete, size of the member and environmental conditions. For a given environment, the total shrinkage of concrete is most influenced by the total amount of water present in the concrete at the time of mixing and to a lesser extent, by the cement content.

In the absence of test data, the approximate value of the total shrinkage strain for design may be taken as 0·0003.

(6) *Creep:* Under the sustained stress, the plastic flow occurs in concrete. This strain is known as creep. Creep takes place only under stress.

Creep of concrete depends on constituents of concrete, size of member, environmental conditions, the stress in concrete, the age at loading and the duration of loading. As long as the stress in concrete does not exceed one-third of its characteristic compressive strength, creep may be assumed to be proportional to the stress.

In the absence of experimental data and detailed information on the effect of the variables, the ultimate creep strain may be estimated from the following values of creep coefficient where

$$\text{creep coefficient} = \frac{\text{ultimate creep strain}}{\text{elastic strain at the age of loading}}$$

Age at loading	Creep coefficient
7 days	2·2
28 days	1·6
1 year	1·1

Note: The ultimate creep strain, estimated as described above does not include the elastic strain.

The *long term modulus of elasticity* for concrete including creep value is given by:

$$E_{ce} = \frac{E_c}{1 + \theta} \text{ where}$$

E_{ce} = long term modulus of elasticity of concrete

E_c = short term static modulus of elasticity of concrete

θ = creep coefficient.

1-4. Strength test of concrete: To assure for the quality of concrete, it has to be tested in laboratory. Samples from fresh concrete shall be taken as per IS : 1199-1959 (Methods of sampling and analysis of concrete), and cubes shall be made, cured and tested at 28 days in accordance

with IS: 516-1959 (Methods of tests for strength of concrete). For relatively small and unimportant buildings and works in which quantity of concrete is less than 15 m^3, the strength tests may be waived by the engineer-in-charge at his discretion.

In order to get a relatively quicker idea of the quality of concrete, optional tests on beam for modulus of rupture at 72 ± 2 hours or at 7 days, or compressive strength tests at 7 days may be carried out in addition to 28 days compressive strength tests. For this purpose, the values given in table 1-2 may be taken for general guidance in the case of concrete made with ordinary portland cement. In all cases, the 28 days compressive strength shall alone be the criterion for acceptance or rejection of the concrete.

TABLE 1-2

OPTIONAL TEST REQUIREMENTS OF CONCRETE

Grade of concrete	Compressive strength of 15 cm cubes, min, at 7 days N/mm^2	Modulus of rupture by beam test, min at 72 ± 2h N/mm^2	Modulus of rupture by beam test, min at 7 days N/mm^2
M10	7·0	1·2	1·7
M15	10·0	1·5	2·1
M20	13·5	1·7	2·4
M25	17·0	1·9	2·7
M30	20·0	2·1	3·0
M35	23·5	2·3	3·2
M40	27·0	2·5	3·4

1-5. Reinforcement: In reinforced concrete, concrete being weak in tension, steel bars are used to carry the tension. However the steel bars are used to carry compression also in beams and columns. The steel bars reinforce the concrete and is known as *reinforcement*. Different types of reinforcement used are: mild steel, medium tensile steel, hot rolled deformed bars, cold-twisted high yield steel and hard drawn steel wire fabric. The bars may be plain or twisted. In all cases the modulus of elasticity of steel shall be taken as 200 kN/mm^2.

The combination of concrete and steel is ideal because when concrete sets, it contracts and thus, it grips the reinfor-

cement. Because of this adhesion, steel and concrete can work together as a single material.

In practice, three types of steel reinforcement are normally used. They are mild steel reinforcement of grade Fe 250, tor steel reinforcement of grade Fe 415 and tor steel reinforcement of grade Fe 500, where Fe denotes ferrous materials and number denotes the yield strength of steel in N/mm^2.

1-6. Structural elements: Primary elements of a concrete structure are slabs, beams, columns and foundations. Slabs and beams carry the loads from floor and are primarily flexural members. The column carries an axial load and is primarily a compression member. The loads from floors are carried by slabs and beams and transmitted to columns. The column load is transmitted to ground through foundations. Fig. 1-2 illustrates the structural elements of a concrete structure.

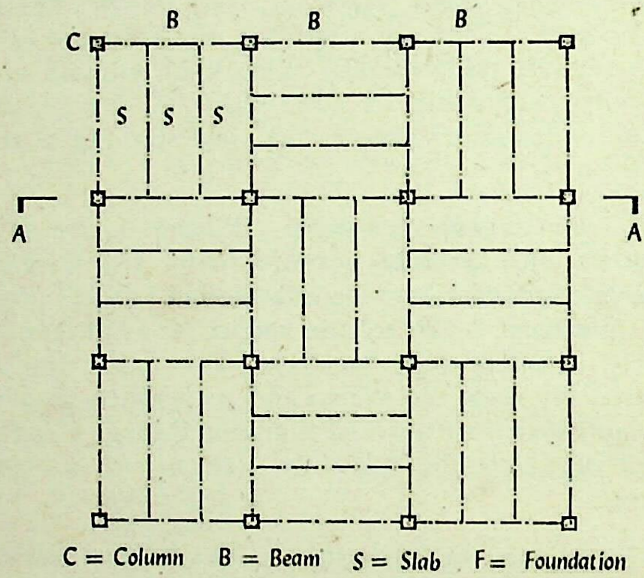
1-7. Loads on structure: The correct estimation of loads on a structure or a part of the structure leads to an economical and safe design. It is very important that no load which is to be borne by the structure is overlooked. The procedure of correct estimation of loads consists of:

(1) Estimation of different types of loads expected to be borne by the structure throughout its life. Different kinds of loads may be estimated using respective Indian Standard codes of practice.

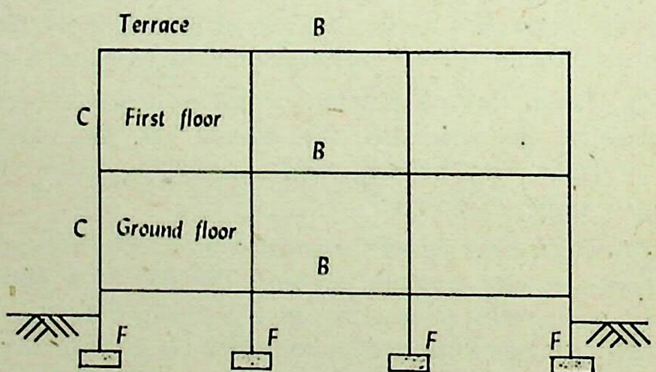
(2) Determination of the worst combination of loads that may occur at one time throughout the life of a structure. The standard codes of practice give guide lines for this. All the loads are not expected at the same time. For example, earthquake loads and wind loads are never expected at the same time.

In general the loads on structure are classified as vertical or gravity loads, horizontal loads and longitudinal loads. The vertical loads are further classified as dead loads, live loads and impact loads. The horizontal loads are classified as wind loads and earthquake loads. The

longitudinal loads are considered in some special cases. The following is a brief discussion on various types of loads:



(a) Plan



(b) Section AA

Structural plan and section of a building showing elements of a concrete structure

FIG. 1-2

Dead loads are loads due to the self weight of the structure or structural members. The dead loads are static loads and

remain reasonably constant throughout the life of the structure. These are also due to partition walls, flooring, roofs, false ceiling, fixtures, etc. The magnitude of dead loads can be calculated if the unit weights of different materials are known.

At the beginning of the design, the sizes of members are not known, therefore an estimation of size has to be made and dead weights are calculated. After solving some problems, a designer will be able to estimate the correct size of the member. If necessary, after completing the trial, a design shall be repeated.

Other dead loads like partition walls, flooring etc. can be correctly estimated as the size and unit weights are known. The unit weights of different materials may be taken from IS: 1911-1967 (Schedule of unit weights of building materials).

Live loads are the loads which are not steady. Unlike the dead loads, they change their magnitudes. This includes moving loads like persons, car etc. and also movable loads like furniture. Usually the live loads are assumed to be the uniformly distributed loads specified by the standard codes of practice.

Impact loads are the loads caused by the vibration of live loads. There is a difference between a person simply walking and a soldier marching. The person produces a live load while the soldier produces impact loads. When live loads cause impact, it is usual to increase the live load by some percentage depending on the type of the impact.

For further information of live loads and impact loads, the reference may be made to IS : 875-1964 (Code of practice for structural safety of buildings: loading standards). The live loads on floors and roofs are given in table I and table II of IS : 875 and are given in Appendix B.

Wind loads are the lateral loads and depend on the velocity of the wind. In different parts of our country, the velocity of wind that can be estimated is different at different places. For one particular place also, the wind velocity is different at different heights from the ground. Considering all these possibilities, the country is divided in some zones. The wind pressure is converted into equivalent horizontal uniformly

distributed loads. For further information a reference shall be made to IS : 875-1964.

Earthquake loads are also the horizontal loads caused by earthquake. The country is divided into some zones according to the intensity of the earthquake. The earthquake forces on the structure shall be calculated in accordance with IS : 1893-1975 (Criteria for earthquake resistant design of structures).

Longitudinal loads are caused by sudden stopping of moving loads. A moving crane, a moving truck etc. when stopped causes longitudinal loads. For further details, reference shall be made to IS : 875-1964.

The combination of loads shall be as given in IS : 875-1964.

1-8. Methods of design: IS : 456 permits three methods of design. They are limit state method, working stress method and methods based on experimental investigations.

When a design is made by the methods based on experimental investigations on models or full size structure or element, load tests shall be carried out as per clause 18 of IS : 456. For further details the code shall be consulted.

In the following paragraphs, working stress method (or elastic theory) and limit state method are briefly discussed. Working stress method being the basic method, is discussed first.

(*a*) *Working stress method:* It is assumed in this method that concrete and steel are elastic. At the worst combination of working loads, the stresses in materials are not exceeded than permissible values. The permissible stresses are found out by using a suitable factor of safety to the material strength e.g. for concrete in compression due to bending, a factor of safety equal to 3 is considered on 28 days cube strength and a factor of safety equal to 1·8 is considered on the yield strength for mild steel reinforcement in tension due to bending. The permissible stresses for different grades of concrete and steel are given in tables 15 and 16 of IS : 456 and are reproduced in tables 1-3 and 1-4. The modular ratio of steel and concrete is defined as follows:

$$m = \frac{\text{modulus of elasticity of steel}}{\text{modulus of elasticity of concrete}}$$ and this can be obtained by the formula

$m = \dfrac{280}{3\sigma_{cbc}}$ where σ_{cbc} is permissible compressive stress due to bending in concrete in N/mm² as specified in table 1-3.

The expression given for m partially takes into account the long term effects such as creep. Therefore this m is not the same as the modular ratio derived based on the value of $E_c = 5700\ \sqrt{f_{ck}}$.

The working stress method is treated in chapters 2 to 8.

TABLE 1-3

PERMISSIBLE STRESSES IN CONCRETE

Grade of concrete (1)	All values in N/mm² Permissible stress in compression: Bending (2) σ_{cbc}	Direct (3) σ_{cc}	Permissible stress in bond (average) for plain bars in tension (4) τ_{bd}
M10	3·0	2·5	—
M15	5·0	4·0	0·6
M20	7·0	5·0	0·8
M25	8·5	6·0	0·9
M30	10·0	8·0	1·0
M35	11·5	9·0	1·1
M40	13·0	10·0	1·2

Note 1: The values of permissible shear stresses in concrete are given in table 3-1.

Note 2: The bond stress given in column 4 shall be increased by 25 per cent for bars in compression.

(*b*) *Limit state method:* "The acceptable limit for the safety and serviceability requirements before failure occurs is known as limit state". In this method of design, the structure is designed to withstand safely all loads liable to act on it throughout its life. The structure also has to be checked for the serviceability requirements such as limitations on deflection and cracking.

TABLE 1-4

PERMISSIBLE STRESSES IN STEEL REINFORCEMENT

Sr. No.	Type of stress in steel reinforcement	Permissible stresses in N/mm^2		
		Mild steel bars confirming to grade I of IS : 432 (part I) - 1966 or Deformed Mild steel bars conforming to IS : 1139 - 1966	Medium tensile steel confirming to IS : 432 (part I) - 1966 or Deformed Medium tensile steel bars conforming to IS : 1139 - 1966	High yield strength Deformed bars confirming to IS : 1139 - 1966 or IS : 1786 - 1979 (Grade Fe 415).
(1)	(2)	(3)	(4)	(5)
(i)	Tension (σ_{st} or σ_{sv}):			
	(a) Upto and including 20 mm	140	Half the guaranteed yield stress subject to a maximum of 190	230
	(b) Over 20 mm	130		230
(ii)	Compression in column bars (σ_{sc})	130	130	190
(iii)	Compression in bars in a beam or slab when the compressive resistance of the concrete is taken into account:	The calculated compressive stress in the surrounding concrete multiplied by 1·5 times the modular ratio or σ_{sc} whichever is lower		
(iv)	Compression in bars in a beam or slab where the compressive resistance of the concrete is not taken into account:			
	(a) Upto and including 20 mm	140	Half the guaranteed yield stress subject to a maximum of 190	190
	(b) Over 20 mm	130		190

IS : 456 states "The aim of design is to achieve acceptable probabilities that the structure will not become unfit for the use for which it is intended, that it will not reach a limit state".

The idea is to base the design on probability theory and to use the statistical methods in analysis and design. Thus, the design is based on characteristic values for material strengths and applied loads, which take into account the variations in the material strengths and in the loads to be supported.

The limit state method of design is treated in chapter 9.

1-9. Adaption of SI units: New editions of Indian Standards have adopted SI units. It is an abbreviation of the French 'Systéme International d'Unités'. IS : 456-1978 which is used throughout in this book has adopted SI units. *This book is written completely in SI units.* Students are expected to be conversant with these units.

1-10. Codes of practice: All designs in this book are in accordance with IS : 456-1978. Other codes required for the study of this book are IS : 1911-1967 (Schedule of unit weights of building materials), IS : 875-1964 (Code of practice for structural safety of buildings: Loading Standards) and SP-16 (Design aids to IS : 456-1978). While reading this book, it is extremely essential to obtain a copy of above IS publications and the book shall be read in conjunction with these publications.

2

Design for Flexure

2-1. Introductory: Before starting the design of an element for flexure, a study of practical requirements is necessary. Important practical aspects are: Size of the beam, cover to the reinforcement and spacing of bars. These are discussed below.

2-2. Size of the beam: In most cases the size of a beam depends on architectural requirements e.g. width of the beam may be governed by the thickness of a wall running parallel to the beam so that a beam can flush with the wall. Depth of the beam may be governed by the clear height required under the beam.

Where the depth is decided by design loads and moments, it is necessary to see that it is not varied from beam to beam e.g. in one big hall containing ten beams, if ten different sizes of beam are chosen, the formwork will be very costly and will not look aesthetic. A designer having sufficient knowledge and practice will be able to select width and depth of beam fulfilling all architectural and structural requirements.

2-3. Cover to the reinforcement: A concrete cover shall have to be provided to the reinforcement for the following reasons:

(1) to protect the reinforcement from weather and fire, and

(2) to ensure the grip of concrete over reinforcement so that they act as one and resist the loads.

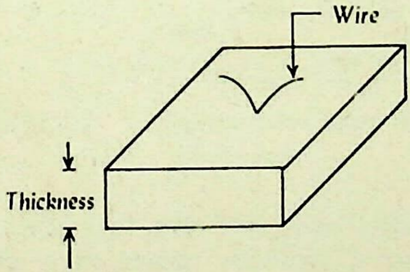
The thickness of cover shall be different for different elements. According to IS : 456 clause 25·4, the reinforcement shall have concrete cover and the thickness of such cover (exclusive of plaster or other decorative finish) shall be as follows:

(1) At each end of reinforcing bar not less than 25 mm, nor less than twice the diameter of such bar.

(2) For longitudinal reinforcing bar in a beam not less than 25 mm, nor less than the diameter of such bar (fig. 2-3).

The concrete cover to the reinforcement can be provided using *supports/spacers* at the bottom and sides of the reinforcement throughout. This will keep a space equal to the thickness of spacer between the concrete and formwork. When concrete is placed, the reinforcement will get the concrete cover.

In cheaper constructions, the pieces of tiles are used as spacers in beams and slabs. In slabs, sometimes coarse aggregates (kapchi) are also used as spacers. This is not the correct way of providing adequate cover throughout.

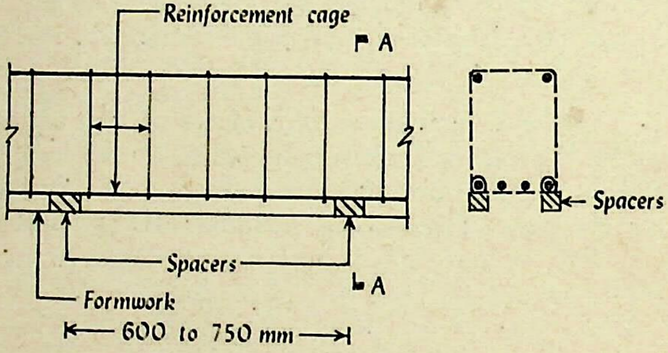


Reinforcement support/spacer
FIG. 2-1

In good constructions, the spacer is prepared from cement - sand mortar. A typical spacer is shown in fig. 2-1.

A cement - sand mortar of the proportion 1 : 2 is prepared using sufficient quantity of water for mixing. It is then laid on a steel plate which is previously oiled. Thickness of this layer is as per requirement e.g. if it is used for slab, 15 mm thick layer is used or if it is used for beam, 25 mm thick layer is prepared. With a sharp knife, then, the mortar is cut in squares of approximate size 50 mm × 50 mm. At the centre of the square, a twisted binding wire is inserted. This is allowed to set. Next day all the pieces are taken and immersed in water. After a curing of 28 days, it gets sufficient strength and can be used as a spacer.

Where the use of spacer is required, the wire of it is tied with reinforcement of beam, column or slab as the case may be, at a spacing of 600 to 750 mm c/c. This will ensure the required space between formwork and reinforcement and will give adequate concrete cover when concreting is done. A typical arrangement of spacers for the beam is shown in fig. 2-2. Now-a-days ready-made spacers manufactured from plastic are available in the market.



(a) Reinforcement cage with spacers (b) Section A-A′

Fig. 2-2

In the interest of standard work, it is advisable to adopt the above mentioned methods rather than using tiles or kapchi as spacer.

2-4. Spacing of bars: The bars shall be placed in such a way that it allows the concrete to enter when poured or a vibrator can be immersed. The minimum spacing of bars is described in clause 25.3.1 of IS : 456. The requirements are:

(1) The horizontal distance between two parallel main reinforcing bars shall usually be not less than the greatest of the following:

— the diameter of the bar if the diameters are equal,

— the diameter of the larger bar if the diameters are unequal, and

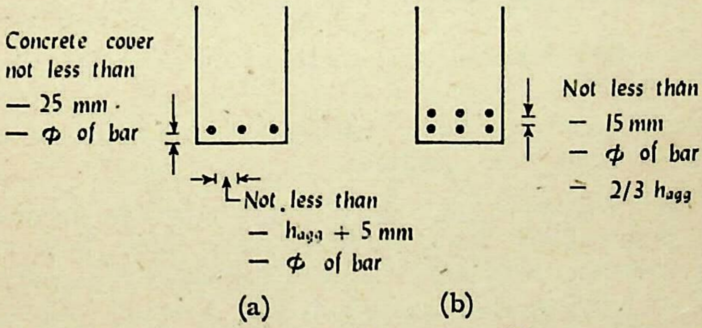
— 5 mm more than the nominal maximum size of coarse aggregate.

Note: This does not preclude the use of larger size of aggregates beyond the congested reinforcement in the same member; the size of aggregates may be reduced around congested reinforcement to comply with this provision.

(2) Greater horizontal distance than the minimum specified in (1) above should be provided wherever possible. However when needle vibrators are used the horizontal distance between bars of a group may be reduced to two-thirds the nominal maximum size of the coarse aggregate, provided that sufficient space is left between groups of bars to enable the vibrator to be immersed.

(3) Where there are two or more rows of bars, the bars shall be vertically in line and minimum vertical distance between the bars shall be 15 mm, two-thirds the nominal maximum size of aggregate (h_{agg}) or the maximum size of the bar, whichever is the greatest.

The above requirements are shown in fig. 2-3.



Concrete cover and spacing of bars

FIG. 2-3

2-5. Design of a beam: A beam is primarily a flexural member and therefore in most cases the design for flexure will govern the overall design of beam. A beam has to resist shear stresses and for the life time service, the beam has to be checked for deflection and cracking. These are all separate topics of design and considered separately in the chapters to follow. In this chapter the design of a beam for flexure is considered.

Three kinds of beams are considered. They are (i) singly reinforced beams, (ii) doubly reinforced beams and (iii) flanged beams.

SINGLY REINFORCED BEAMS

2-6. Assumptions: In singly reinforced beams, concrete resists compression and steel resists tension. The following assumptions are made for a section resisting moment in elastic theory.

(1) At any cross-section, plane sections before bending remain plane after bending.

(2) All tensile stresses are taken up by reinforcement and none by concrete, except as otherwise permitted.

(3) The stress-strain relationship of steel and concrete, under working loads, is a straight line.

(4) There exists a perfect bond between steel and concrete.

(5) The modular ratio m has the value $\dfrac{280}{3\sigma_{cbc}}$ where σ_{cbc} is permissible compressive stress due to bending in concrete in N/mm^2.

Considering the above assumptions, the following three types of design are possible:

(*1*) *Balanced design:* In this type of design the section is so proportioned that the steel and concrete both reach their maximum permissible value of stresses at the same time. Thus, at some value of loads, both the materials will fail at the same time.

(*2*) *Under-reinforced design:* In this type of design the steel provided is less than what it is required for the balanced design. Therefore at some loads, the steel reaches its maximum permissible value of stress and fails, while concrete stress is less than its permissible value.

(*3*) *Over-reinforced design:* In this type of design, the steel provided is more than what it is required for a balanced design. Therefore, at some loads, the concrete reaches its maximum permissible value of stress and fails, while stress in steel is less than its permissible value.

A beam if under-reinforced, gives notice before the failure as the steel yields and concrete in tension zone shows cracks. If it is over-reinforced, concrete fails first which does not yield. Thus, over-reinforced structure may collapse without giving a notice when over-loaded. Therefore, normally balanced or under-reinforced design is preferred.

In singly reinforced beams, concrete resists compression while steel resists tension. In doubly reinforced beams, steel bars are provided to give additional strength in compression. The theory will be derived for balanced section.

2-7. Derivation of formulae for balanced design: Consider a singly reinforced beam as shown in fig. 2-4(*a*). The strain diagram is shown in fig. 2-4(*b*) and the stress diagram is shown in fig. 2-4(*c*). Define:

σ_{cbc} = permissible stress in concrete in bending compression

σ_{st} = permissible stress in steel in tension

E_c = modulus of elasticity of concrete

E_s = modulus of elasticity of steel

ε_c = strain in concrete $= \dfrac{\sigma_{cbc}}{E_c}$

ε_{st} = strain in steel $= \dfrac{\sigma_{st}}{E_s} = \dfrac{\sigma_{st}}{mE_c}$

where m is the modular ratio

b = width of beam

d = *effective depth* which is defined as the distance from extreme compression fibre to the centre of tensile reinforcements

x = *depth of neutral axis* which is defined as the distance of neutral axis from extreme compression fibre

z = *lever arm* which is defined as the distance between centroid of compressive force to the centroid of tensile force.

*** *To find neutral axis***

From the strain diagram

$$\frac{x}{d-x}=\frac{\sigma_{cbc}/E_c}{\sigma_{st}/E_s}=\frac{m\sigma_{cbc}}{\sigma_{st}}$$

solving for x gives

$$x=\left[\frac{m\sigma_{cbc}}{m\sigma_{cbc}+\sigma_{st}}\right]d$$

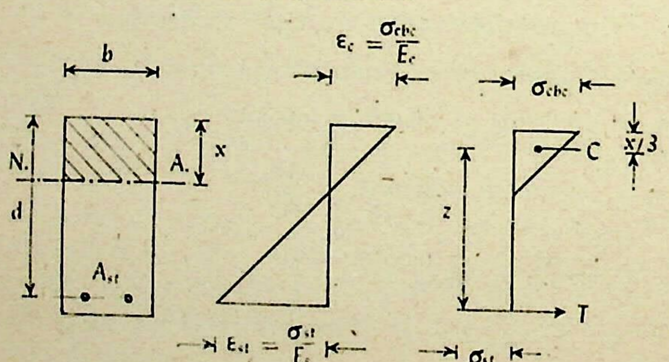
$$=\left[\frac{1}{1+\frac{\sigma_{st}}{m\sigma_{cbc}}}\right]d \quad \text{.......................(2-1}a\text{)}$$

$$=kd$$

where the constant

$$k=\frac{1}{1+\frac{\sigma_{st}}{m\sigma_{cbc}}} \quad \text{.............................(2-1}b\text{)}$$

and is known as *neutral axis constant.*



(a) Section (b) Strain diagram (c) Stress diagram

Singly reinforced balanced section

FIG. 2-4

*** *To find lever arm***

From the stress diagram

$$z=d-\frac{x}{3}=d-\frac{kd}{3}$$

$$=\left(1-\frac{k}{3}\right)d$$

$$=jd \quad \text{.......................................(2-2}a\text{)}$$

where the constant

$$j = (1 - \frac{k}{3}) \quad \ldots\ldots (2\text{-}2b)$$

and is known as *lever arm constant.*

***** *To find total forces*

Define C = total compression and

T = total tension

then $C = \frac{1}{2} \times \sigma_{cbc} \times b \times x = \frac{bx\ \sigma_{cbc}}{2}$ ......(2-3a)

and $T = \sigma_{st}\ A_{st}$ ......(2-3b)

***** *To find moment of resistance of section*

Capacity of a section to resist the moment is known as its moment of resistance. This is equal to

total compressive force × lever arm

OR

total tensile force × lever arm, whichever is smaller.

For a balanced section both will have the same value. Considering the compressive forces,

$$\text{M.R.} = \text{total compression} \times \text{lever arm}$$

$$= (\frac{1}{2} \cdot \sigma_{cbc} \cdot b.x) \times (jd)$$

$$= \frac{1}{2}\,\sigma_{cbc}.b.\ kd.\ jd$$

$$= (\frac{1}{2}\,\sigma_{cbc}\,kj)\ bd^2$$

$$\therefore \quad M = Q\ bd^2 \quad \ldots\ldots (2\text{-}4a)$$

where the constant

$$Q = \frac{1}{2}\,\sigma_{cbc}\ kj \quad \ldots\ldots (2\text{-}5)$$

and is known as *moment of resistance factor for balanced rectangular section.*

Considering the tensile forces

$$\text{M.R.} = \text{total tension} \times \text{lever arm}$$

$$M = (A_{st} \cdot \sigma_{st}) \times (jd) \quad \ldots\ldots (2\text{-}4b)$$

**To find steel area*

For a balanced section

$$\text{M.R.} = \sigma_{st}\, A_{st}\, jd$$

$$\therefore \quad A_{st} = \frac{\text{M.R.}}{\sigma_{st} \times jd}$$

define $p_t = \dfrac{100\, A_{st}}{bd}$

where p_t is percentage steel.

$$\therefore \quad p_t = 100 \times \frac{\text{M.R.}}{\sigma_{st}\, jd} \times \frac{1}{bd}.$$

For a balanced section

$$p_{t,bal} = \frac{100 \times \frac{1}{2}\, \sigma_{cbc} \times k \times j \times bd^2}{\sigma_{st} \times jd \times bd}$$

$$= \frac{50\, \sigma_{cbc} \times k}{\sigma_{st}}$$

$$\therefore \quad p_{t,bal} = \frac{50\, k\sigma_{cbc}}{\sigma_{st}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(2-6)}$$

**To design balanced section*

For a given design moment, if width b of the beam is assumed

$$d = \sqrt{\frac{M}{Qb}}$$

and steel area

$$A_{st} = \frac{M}{\sigma_{st} \cdot j \cdot d}$$

Example 2-1.

Calculate the design constants for following materials considering the balanced design for singly reinforced section. The materials are grade M15 concrete and mild steel reinforcements.

Solution:

For $M15$ mix $\sigma_{cbc} = 5\ \text{N/mm}^2$

for mild steel $\sigma_{st} = 140\ \text{N/mm}^2$

$$m = \frac{280}{3\sigma_{cbc}} = \frac{280}{3 \times 5} = 18{\cdot}66.$$

Then, neutral axis constant $k = \dfrac{1}{1 + \dfrac{\sigma_{st}}{m\sigma_{cbc}}} = \dfrac{1}{1 + \dfrac{140}{18{\cdot}66 \times 5}}$

$= 0{\cdot}4$

lever arm constant $j = 1 - \dfrac{k}{3} = 1 - \dfrac{0{\cdot}4}{3} = 0{\cdot}866$ say $0{\cdot}87$.

M.R. constant $Q = \frac{1}{2} \times \sigma_{cbc} \times k \times j$

$= \frac{1}{2} \times 5 \times 0{\cdot}4 \times 0{\cdot}866 = 0{\cdot}866$ say $0{\cdot}87$

$$p_{t,bal} = \frac{50\ \sigma_{cbc}k}{\sigma_{st}} = \frac{50 \times 5 \times 0{\cdot}4}{140} = 0{\cdot}71.$$

Design constants for some other materials are given in table 2-1.

TABLE 2-1

DESIGN CONSTANTS FOR BALANCED SECTION

Concrete grade	Steel grade	σ_{cbc}	σ_{st}	k	j	Q	$p_{t,bal}$
M15	Fe250	5	140	0·4	0·87	0·87	0·71
	Fe415	5	230	0·29	0·90	0·65	0·31
M20	Fe250	7	140	0·4	0·87	1·21	1·00
	Fe415	7	230	0·29	0·90	0·91	0·44

Note: These are the most common materials used in practice.

Example 2-2.

A simply supported rectangular beam of 4 m span carries a uniformly distributed load of 20 kN/m. The width of the beam is 230 mm. Find the depth and steel area for balanced design. Use M15 grade concrete and mild steel reinforcements.

Solution:

$$M = 20 \times \frac{4^2}{8} = 40 \text{ kNm}.$$

For balanced section $Q = 0{\cdot}87$

effective depth required $d = \sqrt{\frac{M}{Qb}}$

$$= \sqrt{\frac{40 \times 10^6}{0{\cdot}87 \times 230}} = 447 \text{ mm}$$

and steel area $A_{st} = \frac{M}{\sigma_{st}\, jd}$

$$= \frac{40 \times 10^6}{140 \times 0{\cdot}87 \times 447} = 735 \text{ mm}^2.$$

Provide 4 no. 16 mm diameter bars giving area of 804 mm².

Overall depth of beam = 447 + 8 + 25 (cover)
= 480 mm.

The designed section is shown in fig. 2-5. If overall depth is not a whole number, round it upon higher side.

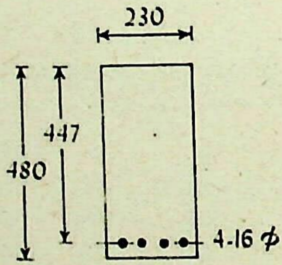


Fig. 2-5

For convenience in design, areas of group of bars are given in table 2-2.

TABLE 2-2
AREA OF GROUP OF BARS (mm²)

Diameter (mm)	Number of bars in group							
	1	2	3	4	5	6	7	8
6	28	56	84	113	141	169	197	226
8	50	100	150	201	251	301	351	402
10	78	157	235	314	392	471	549	628
12	113	226	339	452	565	678	791	904
16	201	402	603	804	1005	1206	1407	1608
20	314	628	942	1256	1570	1884	2198	2512
25	491	981	1472	1963	2453	2944	3435	3926

2-8. Transformed area method: A beam section as shown in fig. 2-6(a) is subjected to a moment M. Find out the maximum stresses in concrete and steel. Fig. 2-6(b), (c) and (d) represents the strain diagram, the stress diagram and the transformed section. "A *transformed section* is a section in which the steel area is replaced by the equivalent concrete area".

At centroid of steel reinforcement, the surrounding concrete being elastic and having perfect bond with steel,

strain in steel = strain in concrete.

Let f_{st} and f_{cb} be the stresses in steel and concrete respectively.

$$\text{Force in steel} = A_{st}\, f_{st}$$
$$= A_{st}.m.f_{cb}.$$

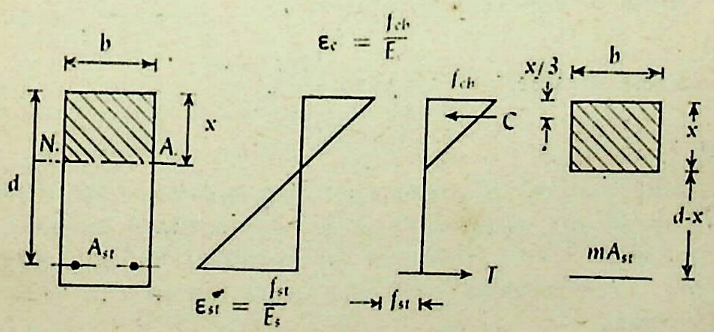
If this steel is to be replaced by an equivalent concrete area (transformed area), the equivalent concrete will carry the same force.

Thus,

force in equivalent concrete

$$= \text{transformed area} \times f_{cb}$$
$$= A_{st}.m.f_{cb}$$

$\therefore$ Transformed area $= m.A_{st}$.



(a) Section (b) Strain diagram (c) Stress diagram (d) Transformed section

Transformed area method — Singly reinforced beam

FIG. 2-6

* *To find neutral axis*

Taking moments of transformed areas about N.A.

$$b \cdot x \cdot \frac{x}{2} = mA_{st} \ (d - x)$$

solution of this equation gives value of x.

* *Method 1:*

$$\text{lever arm} = d - \frac{x}{3}$$

$$\text{stress in steel} = \frac{M}{A_{st} \ (d - \frac{x}{3})} = f_{st}.$$

Stress in concrete:

from strain diagram $\dfrac{f_{cb}/E_c}{f_{st}/E_s} = \dfrac{x}{d - x}$

$$\therefore \qquad {}_cf_b = \frac{f_{st}}{E_s/E_c} \cdot \frac{x}{d - x} = \frac{f_{st}}{m} \times \frac{x}{d - x}$$

* *Method 2* (classic flexure formula):

Find out moment of inertia of beam.

$$I_{xx} = \frac{bx^3}{3} + mA_{st} \ (d - x)^2$$

The stresses in the concrete and steel are given by,

Stress in concrete $f_{cb} = \dfrac{M.x}{I_{xx}}$

Stress in steel $\qquad f_{st} = m. \dfrac{M(d - x)}{I_{xx}}$

Example 2-3.

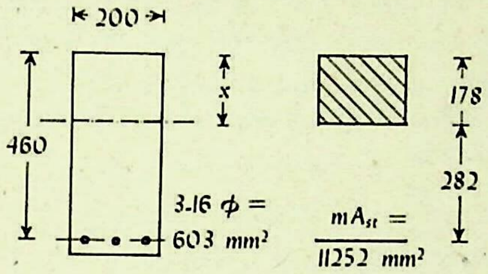
The dimensions of a rectangular beam section and the reinforcing steel provided are shown in fig. 2-8. The section is subjected to a moment of 30 kNm. Determine the maximum stresses in steel and concrete. The materials are M15 grade concrete and mild steel reinforcement.

Solution:

For M15 Mix and Mild steel

$$m = 18{\cdot}66$$

transformed area of steel $= 18 \cdot 66 \times 603$
$= 11252 \text{ mm}^2$.



(a) Section　　(b) Transformed section

FIG. 2-7

To find neutral axis, taking moments about N.A.

$$200\, x \cdot \frac{x}{2} = 11252\ (460 - x)$$

$$100\, x^2 + 11252\, x - 5175920 = 0$$

$$x^2 + 112 \cdot 52x - 51759 = 0$$

which gives $x = 178$ mm.

Method 1:

$$\text{Lever arm} = 460 - \frac{178}{3} = 400 \cdot 66 \text{ mm}$$

$$\text{Steel stress} = \frac{30 \times 10^6}{603 \times 400 \cdot 66} = 124 \cdot 17 \text{ N/mm}^2$$

$$\text{Concrete stress} = \frac{f_{st}}{m} \times \frac{x}{d - x}$$

$$= \frac{124 \cdot 17}{18 \cdot 66} \times \frac{178}{282}$$

$$= 4 \cdot 2 \text{ N/mm}^2.$$

Method 2:

$$I_{xx} = \frac{1}{3} \times 200 \times 178^3 + 11252 \times 282^2 = 1 \cdot 27 \times 10^9 \text{ mm}^4.$$

$$\text{Concrete stress } f_{cb} = \frac{30 \times 10^6 \times 178}{1 \cdot 27 \times 10^9} = 4 \cdot 2 \text{ N/mm}^2$$

$$\text{Steel stress } f_{st} = \frac{30 \times 10^6 \times 282}{1 \cdot 27 \times 10^9} \times 18 \cdot 66$$

$$= 124 \cdot 3 \text{ N/mm}^2.$$

2-9. Types of problems: In singly reinforced beams the following types of problems occur:

Type 1: To find out the depth of neutral axis for a given section and specifying the type of beam.

(a) If the section and actual stresses in the materials are given, find out depth of neutral axis using equation (2-1) for actual stresses.

$$x = kd$$

where $$k = \frac{1}{1 + \dfrac{f_{st}}{mf_{cb}}}$$

(b) If the section and steel area provided are given, find out neutral axis by taking moment of transformed area about neutral axis.

$$b \,.\, x \,.\, \frac{x}{2} = m \,.\, A_{st} \;(d - x)$$

(c) Find out the depth of neutral axis for balanced section, also known as *depth of critical neutral axis* using equation (2-1) for permissible stresses of steel and concrete.

$$x = kd$$

where $$k = \frac{1}{1 + \dfrac{\sigma_{st}}{m\sigma_{cbc}}}$$

(d) If $x_{actual} < x_{critical}$, the concrete is not fully stressed and the beam is under-reinforced.

(e) If $x_{actual} > x_{critical}$, the steel is not fully stressed and the beam is over-reinforced.

Type 2: To find the moment of resistance for a given section.

(a) Find the position of actual neutral axis and critical neutral axis as explained in type 1.

(b) If $x_{actual} < x_{critical}$, the section is under-reinforced and moment of resistance is given by,

$$\text{M.R.} = A_{st} \,.\, \sigma_{st} \,.\, \left(d - \frac{x}{3}\right).$$

(c) If $x_{actual} > x_{critical}$, the section is over-reinforced and moment of resistance is given by,

$$M.R. = b \cdot x \cdot \frac{\sigma_{cbc}}{2} \cdot \left(d - \frac{x}{3}\right).$$

Type 3: For the given moment and section of beam, to check the stresses.

This is explained in art 2-8.

Type 4: To design the section for a given moment.

(a) If the sectional dimensions are not given, fix the width and find depth required from the equation:

$$d = \sqrt{\frac{M}{Q\,b}}$$

and steel area is given by

$$A_{st} = \frac{M}{\sigma_{st}\, j d}.$$

(b) If the size of beam is given, find out the moment of resistance of critical section for the beam by the equation,

$$M.R. = Q\, bd^2.$$

Here two cases are possible.

(1) $M <$ M.R. In this case the section is designed as under-reinforced.

$$M.R. = \sigma_{st} A_{st} \left(d - \frac{x}{3}\right) \quad \ldots\ldots (1)$$

Taking moments about neutral axis,

$$b \cdot x \cdot \frac{x}{2} = mA_{st}\,(d - x)$$

$$\therefore \quad A_{st} = \frac{bx^2}{2m\,(d-x)} \quad \ldots\ldots (2)$$

Substituting in (1)

$$M.R. = \frac{\sigma_{st} . bx^2 \left(d - \frac{x}{3}\right)}{2m\,(d-x)} \quad \ldots\ldots (3)$$

The solution of equation (3) gives value of x. Substituting this value in equation (2), A_{st} can be found out.

The method explained here, gives exact amount of steel. This also can be found out using available design aids as explained in art. 2-10.

However for speedy calculations, when $M <$ M.R. i.e. when under-reinforced section is to be designed, steel area may be found from the equation,

$$A_{st} = \frac{M}{\sigma_{st}\, jd}$$

where j of balanced section may be considered.

(2) $M >$ M.R. In this case the section can be designed as over-reinforced section.

Find depth of neutral axis using the equation

$$M = bx \cdot \frac{\sigma_{cbc}}{2} \left(d - \frac{x}{3}\right).$$

Area of steel can be found out by taking moments about neutral axis,

$$b \cdot x \cdot \frac{x}{2} = m\ A_{st}\ (d - x).$$

As discussed earlier, the over-reinforced design is not followed in practice. Therefore in such cases, the section is designed as doubly reinforced beam where the reinforcements are provided in compression to give additional strength to the concrete.

Example 2-4.

Determine the position of neutral axis of a reinforced concrete beam 230 mm wide and 460 mm effective depth, if the stresses developed in concrete and steel are 4·2 N/mm² and 98 N/mm² respectively. The materials are M15 grade concrete and mild steel reinforcement. Also state the type of the beam.

Solution:

For M15 mix, $m = 18{\cdot}66$.

Using equation (2-1) for actual stresses,

$$k = \frac{1}{1 + \dfrac{f_{st}}{m\ f_{cb}}}$$

$$= \frac{1}{1 + \dfrac{98}{18{\cdot}66 \times 4{\cdot}2}}$$

$$= 0{\cdot}44$$

and $$x = kd = 0{\cdot}44 \times 460 = 202{\cdot}4 \text{ mm}$$

depth of critical N.A. $= 0{\cdot}4 \times 460 = 184$ mm

$x_{actual} > _{critical}$

$\therefore$ The beam is over-reinforced.

Example 2-5.

An R.C.C. beam, 350 mm wide and 460 mm effective depth is reinforced with 4 nos. 12 mm dia. bars in tension. Find out the depth of neutral axis and state the type of the beam. The materials are M15 grade concrete and tor steel reinforcement of grade Fe415.

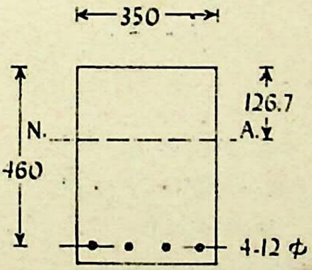


Fig. 2-8

Solution:

For M15 mix, $m = 18{\cdot}66$

$A_{st} = 4 \times 113 = 452 \text{ mm}^2$.

Let x be the depth of neutral axis.

Taking moments about neutral axis,

$$b \cdot x \cdot \frac{x}{2} = m A_{st} (d - x)$$

$$\frac{350}{2} x^2 = 18{\cdot}66 \times 452 \, (460 - x)$$

which gives $x = 126{\cdot}7$ mm

depth of critical N.A. $= 0{\cdot}29 \times 460 = 133{\cdot}4$ mm

$x_{actual} < x_{critical}$

$\therefore$ The beam is under-reinforced.

Example 2-6.

Find the moment of resistance of the beam as shown in fig. 2-9. Also state whether the beam is under-reinforced or over-reinforced. The materials used are grade M15 concrete and Fe415 grade tor steel reinforcement.

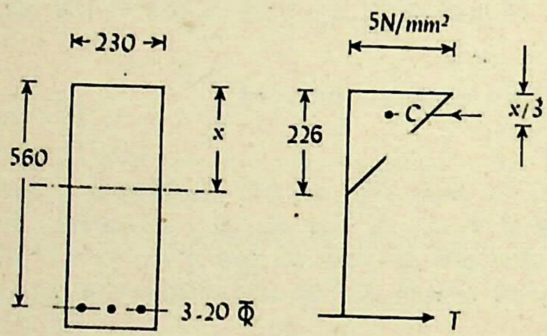


FIG. 2-9

Solution:

For M15 mix $\sigma_{cbc} = 5\ \text{N/mm}^2$

Fe 415 steel $\sigma_{st} = 230\ \text{N/mm}^2$

$m = 18{\cdot}66$

$A_{st} = 3 \times 314 = 942\ \text{mm}^2$.

Let x be the depth of neutral axis.

Taking moments about N.A.,

$$bx \cdot \frac{x}{2} = m\ A_{st}\ (d - x)$$

$$230 \cdot x \cdot \frac{x}{2} = 18{\cdot}66 \times 942\ (560 - x)$$

$$115x^2 = 9843523 - 17578\ x$$

$\therefore\ x^2 + 152{\cdot}85\ x - 85596 = 0$

which gives $x = 226$ mm

depth of critical neutral axis

$kd = 0{\cdot}29 \times 560 = 162{\cdot}4 < 226$ mm.

$\therefore$ Section is over-reinforced and concrete will fail first.

$$\therefore \quad \text{M.R.} = \frac{1}{2}\,\sigma_{cbc}\,.\,b\,.\,x\,\left(d - \frac{x}{3}\right)$$

$$= \frac{1}{2} \times 5 \times 230 \times 226\,\left(560 - \frac{226}{3}\right) \times 10^{-6}$$

$$= 62{\cdot}98 \text{ kNm.}$$

$\therefore$ M.R. of the section is 62·98 kNm.

Example 2-7.

Find the moment of resistance of beam section as shown in fig. 2-10. Also state whether the beam is under-reinforced or over-reinforced. The materials used are grade 15 concrete and mild steel reinforcement.

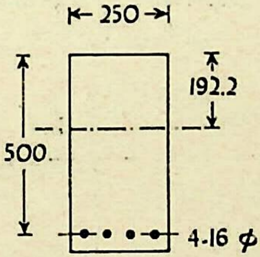


Fig. 2-10

Solution:

For M15 mix $m = 18{\cdot}66$

$A_{st} = 4 \times 201 = 804$ mm².

Let x be the depth of neutral axis.

Taking moments about neutral axis,

$$b\,.\,x\,.\,\frac{x}{2} = m\,A_{st}\,(d - x)$$

$$\frac{250}{2}\,x^2 = 18{\cdot}66 \times 804\,(500 - x)$$

$$x^2 = 60010 - 120{\cdot}02\,x$$

which gives $x = 192{\cdot}2$ mm

depth of critical neutral axis

$$= 0{\cdot}4 \times 500 = 200 \text{ mm}$$

$x_{actual} < x_{critical}$

$\therefore$ Under-reinforced section.

$$\text{M.R.} = A_{st} \cdot \sigma_{st} \cdot (d - \frac{x}{3})$$

$$= 804 \times 140 \ (500 - \frac{192{\cdot}2}{3}) \times 10^{-6}$$

$$= 49{\cdot}07 \text{ kNm.}$$

$\therefore$ M.R. $= 49{\cdot}07$ kNm.

Example 2-8.

Find the moment of resistance of the beam section as shown in fig. 2-11. The permissible stress in concrete in bending compression and steel in tension are respectively 5·6 N/mm² and 210 N/mm².

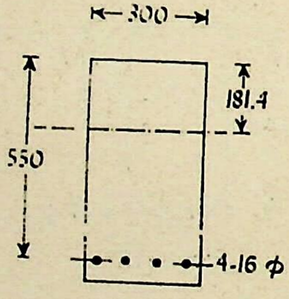


FIG. 2-11

Solution:

For a given concrete mix,

$$m = \frac{280}{3 \times 5{\cdot}6} = 16{\cdot}66.$$

$A_{st} = 4 \times 201 = 804 \text{ mm}^2.$

Let x be the depth of neutral axis.

Taking moments about neutral axis,

$$300 \ x \cdot \frac{x}{2} = 16{\cdot}66 \times 804 \ (550 - x)$$

$$x^2 = 49115 - 89{\cdot}3 \ x$$

which gives $x = 181{\cdot}4$ mm.

Now the depth of critical neutral axis

$$= kd$$

$$= \frac{1}{1 + \frac{\sigma_{st}}{m\sigma_{cbc}}} \cdot d$$

$$= \frac{1}{1 + \frac{210}{16{\cdot}66 \times 5{\cdot}6}} \times 550$$

$$= 169{\cdot}2 \text{ mm}$$

$$x_{actual} > x_{critical}$$

The beam is over-reinforced and concrete will fail first.

$$\text{M.R.} = b \cdot x \cdot \frac{\sigma_{cbc}}{2} \left(d - \frac{x}{3}\right)$$

$$= 300 \times 181{\cdot}4 \times \frac{5{\cdot}6}{2} \left(550 - \frac{181{\cdot}4}{3}\right) \times 10^{-6}$$

$$= 74{\cdot}6 \text{ kNm.}$$

Example 2-9.

A beam of size 230 mm × 600 mm overall depth is reinforced with 4 nos. 12 mm dia. bars. Find the safe uniformly distributed load on the beam in addition to its self weight on a span of 4 m. The materials are M20 grade concrete and Fe415 grade steel reinforcement.

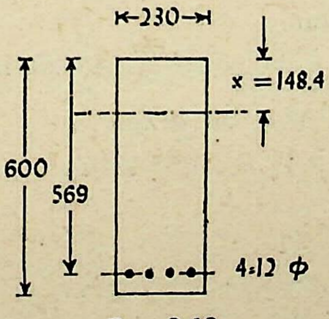


Fig. 2-12

Solution:

For M20 grade concrete,

$$m = \frac{280}{3 \times 7} = 13{\cdot}33$$

$A_{st} = 4 \times 113 = 452 \text{ mm}^2$

$d = 600 - 25 - 6 = 569.$

Let x be the depth of neutral axis.

Taking moments about N.A.,

$$230\, x \cdot \frac{x}{2} = 13 \cdot 33 \times 452\ (569 - x)$$

$$x^2 = 29811 - 52 \cdot 4\, x$$

which gives $x = 148 \cdot 4$ mm.

Depth of critical N.A. $= 0 \cdot 29 \times 569 = 165 \cdot 01$ mm.

$x_{actual} < x_{critical}$

$\therefore$ Beam is under-reinforced and steel will fail first.

$$\text{M.R.} = 452 \times 230\ (569 - \frac{148 \cdot 4}{3}) \times 10^{-6}$$

$$= 54 \cdot 01 \text{ kNm.}$$

If the load on beam is w kN/m,

$$M = \frac{w \times 4^2}{8} = 54 \cdot 01$$

$$w = 27 \text{ kN/m.}$$

Self weight of beam

$$= 0 \cdot 23 \times 0 \cdot 6 \times 25 = 3 \cdot 45 \text{ kN/m.}$$

Additional safe U.D.L. on beam

$$= 27 - 3 \cdot 45 = 23 \cdot 55 \text{ kN/m.}$$

Example 2-10.

A simply supported beam over a span of 4·5 m is reinforced with tension reinforcement only. The beam is 250 mm wide and has an effective depth of 610 mm. It is reinforced with 4 nos. 20 mm dia. bars. Calculate the stresses in both the materials at the centre of span when a beam carries a uniformly distributed load of 22 kN/m inclusive of self weight. The materials are M15 grade concrete and mild steel reinforcement.

Solution :

For M15 mix, $m = 18 \cdot 66$

$A_{st} = 4 \times 314 = 1256 \text{ mm}^2$.

Transformed area of steel $= 18 \cdot 66 \times 1256$
$= 23436 \text{ mm}^2$.

To find neutral axis, taking moments of transformed area about neutral axis,

$$250 \; x \cdot \frac{x}{2} = 23436 \; (610 - x)$$

$$x^2 = 114375 - 187 \cdot 5 \, x$$

which gives $x = 257 \cdot 2$ mm.

For the beam, maximum moment at centre

$$= 22 \times \frac{4 \cdot 5^2}{8} = 55 \cdot 69 \text{ kNm.}$$

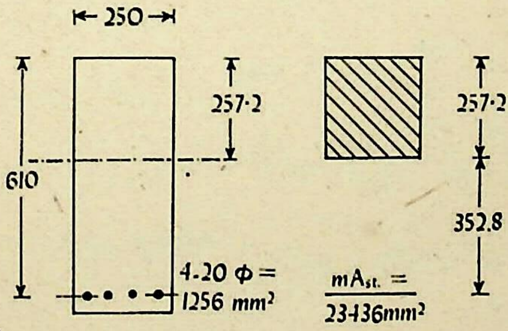


FIG. 2-13

Method 1 :

$$\text{Lever arm} = 610 - \frac{257 \cdot 2}{3} = 524 \cdot 2 \text{ mm.}$$

$$\text{Steel stress} = \frac{55 \cdot 69 \times 10^6}{1256 \times 524 \cdot 2} = 84 \cdot 58 \text{ N/mm}^2.$$

$$\text{Concrete stress} = \frac{f_{st}}{m} \times \frac{x}{d - x} = \frac{84 \cdot 58}{18 \cdot 66} \times \frac{257 \cdot 2}{352 \cdot 8}$$

$$= 3 \cdot 3 \text{ N/mm}^2.$$

Method 2:

$$I_{xx} = \tfrac{1}{3} \times 250 \times 257{\cdot}2^3 + 23436 \times 352{\cdot}8^2$$
$$= 4{\cdot}33 \times 10^9 \text{ mm}^4.$$

$$\text{Concrete stress} = \frac{55{\cdot}69 \times 10^6 \times 257{\cdot}2}{4{\cdot}33 \times 10^9}$$
$$= 3{\cdot}3 \text{ N/mm}^2.$$

$$\text{Steel stress} = \frac{55{\cdot}69 \times 10^6 \times 352{\cdot}8}{4{\cdot}33 \times 10^9} \times 18{\cdot}66$$
$$= 84{\cdot}67 \text{ N/mm}^2.$$

Example 2-11.

A simply supported beam of 5 m span carries a U.D.L. of 26 kN/m inclusive of its self weight. Find out the steel area for balanced section, if it is reinforced in tension only. The width of beam is 230 mm. The materials are M15 grade concrete and mild steel reinforcement.

Solution:

$$M = 26 \times \frac{5^2}{8} = 81{\cdot}25 \text{ kNm}$$

$$d = \sqrt{\frac{81{\cdot}25 \times 10^6}{0{\cdot}87 \times 230}} = 637{\cdot}2 \text{ mm}$$

$$A_{st} = \frac{81{\cdot}25 \times 10^6}{140 \times 0{\cdot}87 \times 637{\cdot}2} = 1046{\cdot}8 \text{ mm}^2.$$

Example 2-12.

A simply supported beam of 6 m span carries a U.D.L. of 10 kN/m inclusive of self weight. The beam is 230 mm wide and effective depth of 580 mm. Find the steel area. The materials are M15 grade concrete and tor steel reinforcement of grade Fe415.

Solution:

$$M = \frac{10 \times 6^2}{8} = 45 \text{ kNm}$$

M.R. of balanced section

$$= 0{\cdot}65 \times 230 \times 580^2 \times 10^{-6}$$
$$= 50{\cdot}29 \text{ kNm} > 45 \text{ kNm}.$$

$\therefore$ Design as under-reinforced section.

Alternatively, depth required $= \sqrt{\dfrac{45 \times 10^6}{0{\cdot}65 \times 230}}$

$= 548{\cdot}6 \text{ mm} < 580 \text{ mm provided.}$

$\therefore$ Design as under-reinforced section..

Exact method:

$$\text{M.R.} = \sigma_{st} \,.\, A_{st} \left(d - \frac{x}{3}\right)$$

$$45 \times 10^6 = 230\, A_{st} \left(d - \frac{x}{3}\right) \text{ which gives,}$$

$$586956 = A_{st}\,(3d - x) \quad \ldots\ldots(1)$$

Taking moments about neutral axis,

$$230 \times x \times \frac{x}{2} = 18{\cdot}66\, A_{st}\,(d - x)$$

$$\therefore \quad A_{st} = \frac{6{\cdot}16\, x^2}{d - x} \quad \ldots\ldots(2)$$

Substituting in equation (1),

$$586956 = \frac{6{\cdot}16\, x^2}{(580 - x)} \times (3 \times 580 - x)$$

which on simplifying yields,

$$x^3 - 1740\, x^2 - 95285x + 55265338 = 0.$$

The solution of this equation gives,

$$x = 159{\cdot}2 \text{ mm.}$$

Substituting in (2),

$$A_{st} = \frac{6{\cdot}16 \times 159{\cdot}2^2}{(580 - 159{\cdot}2)} = 371 \text{ mm}^2.$$

Approximate method:

Provided depth > required depth..............(O.K.)

$$A_{st} = \frac{45 \times 10^6}{230 \times 0{\cdot}9 \times 580}$$

$$= 374{\cdot}8 \text{ mm}^2.$$

From above two calculations, it can be observed that for practical purpose, approximate method may be followed and is conservative for under-reinforced section. However, if the exact method is to be followed, one may use the available design tables. This is explained in art. 2-10.

2-10. Use of design aids: Referring fig. 2-4, to find neutral axis, take moments about N.A.

$$b.\ kd.\frac{kd}{2} = m\ A_{st}\ (d - kd)$$

putting $A_{st} = \dfrac{p_t\ bd}{100}$

$$bd^2 \cdot \frac{k^2}{2} = bd^2\ \frac{p_t m}{100}\ (1-k)$$

$$\therefore \qquad k^2 = \frac{p_t\ m}{50}\ (1-k)$$

$$\therefore \qquad k^2 + \frac{p_t\ mk}{50} - \frac{p_t\ m}{50} = 0.$$

The positive root of this equation gives

$$k = \frac{-p_t\, m}{100} + \sqrt{\frac{p_t^2 m^2}{(100)^2} + \frac{p_t m}{50}} \qquad \ldots\ldots\ldots\ldots(2\text{-}7)$$

Now the moment of resistance of the under-reinforced section is given by,

$$\text{M.R.} = A_{st}.\ \sigma_{st} \times (d - \frac{kd}{3})$$

$$= \frac{p_t \cdot bd}{100} \times \sigma_{st}\ d\,(1 - \frac{k}{3})$$

$$\therefore \qquad \frac{M}{bd^2} = \frac{p_t\ \sigma_{st}}{100}\ (1 - \frac{k}{3}) \qquad \ldots\ldots\ldots\ldots(2\text{-}8)$$

Values of $\dfrac{M}{bd^2}$ have been tabulated against p_t in tables 68 to 71 of SP: 16.

Example 2-13.

A simply supported rectangular beam of 4 m span carries a uniformly distributed load including self weight of 20 kN/m. If the

beam is 250 mm wide × 450 mm effective depth, find the steel area required at mid-span. The concrete is M15 grade and mild steel reinforcements are used.

Solution:

$$\text{Design moment } M = \frac{20 \times 4^2}{8} = 40 \text{ kNm}$$

$$\frac{M}{bd^2} = \frac{40 \times 10^6}{250 \times 450^2} = 0{\cdot}79.$$

From table 68, SP : 16

$$\frac{100\ A_{st}}{bd} = 0{\cdot}65$$

and
$$A_{st} = \frac{0{\cdot}65 \times 250 \times 450}{100} = 731 \text{ mm}^2.$$

Provide 4 nos. 16 mm ϕ = 804 mm^2.

Example 2-14.

Check the section of example 2-3 using design tables.

Solution:

The section can be checked by two measures.

(a) Maximum stress in steel and concrete shall be less than permissible value. This is done in example 2-3.

(b) The moment of resistance of the beam shall be greater than applied moment.

$$p_t = \frac{100\ A_{st}}{bd} = \frac{100 \times 603}{200 \times 460} = 0{\cdot}655.$$

From table 68, SP : 16

$$\frac{M}{bd^2} = 0{\cdot}7985$$

$$\therefore \quad \text{M.R.} = 0{\cdot}7985 \times 200 \times 460^2 \times 10^{-6} = 33{\cdot}8 \text{ kNm}.$$

The moment of resistance of the given section is greater than applied moment. Thus the section is safe.

DOUBLY REINFORCED BEAMS

2-11. Introductory: For a design moment M, if the size of the rectangular section is fixed and moment of resistance of a singly reinforced section is less than M, there are *two* methods to design such beams:

(1) Increase the concrete mix to increase the capacity of the section.

(2) Reinforcements are provided in compression zone to give additional strength to the concrete in compression. Such beams are called *doubly reinforced beams*.

A concrete structure when loaded, undergoes elastic as well as plastic deformation. The plastic deformation is termed as *creep*. Elastic deformation is an instantaneous process, while creep is a long process. Thus, total strain increases with time. Therefore the value of modulus of elasticity of concrete will decrease. This means that the modular ratio which is defined as ratio of modulus of elasticity of steel to the modulus of elasticity of concrete will increase with time. IS : 456 in table no. 16 (table no. 1-4 of this book) states that the permissible stress for compression in bars in a beam or slab when the compressive resistance of the concrete is taken into account, shall be taken as the calculated compressive stress in the surrounding concrete multiplied by 1·5 times the modular ratio or σ_{sc} whichever is lower where σ_{sc} is the permissible stress in compression in column bars.

2-12. Derivation of formulae for balanced design: A doubly reinforced beam subjected to a moment M can be expressed as a rectangular section with tension reinforcement A_{st_1} reinforced for balanced condition giving moment of resistance M_1 + an auxiliary section reinforced with compression reinforcement A_{sc} and tensile reinforcement A_{st_2} giving moment of resistance M_2.

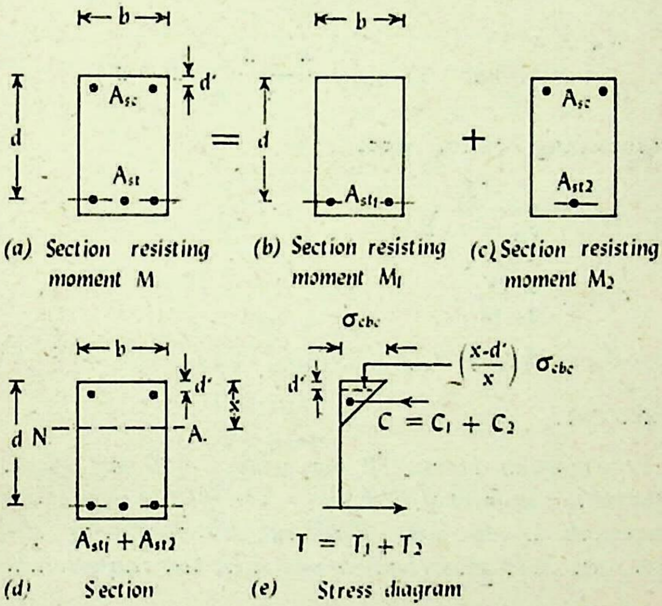
Consider a rectangular doubly reinforced beam as shown in fig. 2-14(a). This is equivalent to the section resisting M_1 shown in (b) + section resisting M_2 shown in (c) where $M_1 + M_2 = M$. The stress diagram is shown in (e).

Transformed area of compression zone in terms of concrete area

$$= 1{\cdot}5 \text{ m } A_{sc} + bx - A_{sc}$$

$$= (1{\cdot}5 \text{ m} - 1)\, A_{sc} + bx \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2\text{-}9)$$

The first term represents equivalent concrete area of compressive steel and second term represents the effective concrete area in compression.



Doubly reinforced section
FIG. 2-14

Stress in concrete at the level of steel in compression zone

$$= \frac{x - d'}{x} \sigma_{cbc}.$$

Taking moments of compressive forces about centroid of tensile forces to get M.R. of a beam.

$$\text{M.R.} = (1{\cdot}5m - 1)\, A_{sc} \,.\, \frac{x - d'}{x}\, \sigma_{cbc} \,.\, (d - d')$$

$$+ b \,.\, x \,.\, \frac{\sigma_{cbc}}{2}\left(d - \frac{x}{3}\right)$$

$$= (1{\cdot}5m - 1) A_{sc} \,.\, \frac{x - d'}{x} \,.\, \sigma_{cbc} \,.\, (d - d') + Qbd^2 \text{ where}$$

$Qbd^2 = M_1$ as defined.

Now $M = M_1 + M_2$

$$\therefore \quad M_2 = (1{\cdot}5m - 1)\ A_{sc}\ \sigma_{cbc}\ \left(\frac{x - d'}{x}\right)\ (d - d')$$

$$\therefore \quad A_{sc} = \frac{M_2}{(1{\cdot}5m - 1)\ \sigma_{cbc} \cdot \left(\frac{x - d'}{x}\right)\ (d - d')} \quad \ldots\ldots (2\text{-}10a)$$

Corresponding tension steel

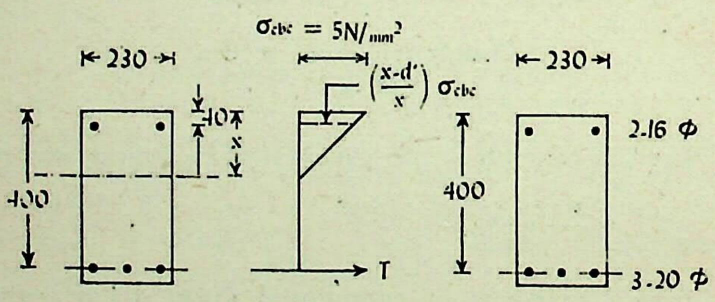
$$A_{st2} = \frac{M_2}{\sigma_{st}\ (d - d')} \quad \ldots\ldots (2\text{-}10b)$$

$$A_{st1} = \frac{M_1}{\sigma_{st}\ jd} \quad \ldots\ldots (2\text{-}10c)$$

$$A_{st} = A_{st1} + A_{st2} \quad \ldots\ldots (2\text{-}10d)$$

Example 2-15.

A rectangular beam 230 mm wide × 400 mm effective depth is subjected to a moment of 42 kNm. The effective cover of compressive reinforcements is 40 mm. Find out the reinforcing steel. The materials are M15 grade concrete and mild steel reinforcements.



(a) Section (b) Stress diagram (c) Reinforcement

FIG. 2-15

Solution:

For a balanced section,

$$M_1 = Qbd^2 = 0{\cdot}87 \times 230 \times 400^2 \times 10^{-6} = 32 \text{ kNm}$$

$$A_{st1} = \frac{32 \times 10^6}{140 \times 0{\cdot}87 \times 400} = 657 \text{ mm}^2.$$

Depth of N.A. $x = kd = 0{\cdot}4 \times 400 = 160$ mm

$$M_2 = 42 - 32 = 10 \text{ kNm}$$

$$m = \frac{280}{3 \times 5} = 18{\cdot}66$$

$$A_{sc} = \frac{M_2}{(1{\cdot}5m - 1)\ \sigma_{cbc} \times \dfrac{(x - d')}{x} \times (d - d')}$$

$$= \frac{10 \times 10^6}{(1{\cdot}5 \times 18{\cdot}66 - 1) \times 5 \times \left(\dfrac{160 - 40}{160}\right)(400 - 40)}$$

$$= 274 \text{ mm}^2.$$

Corresponding $A_{st2} = \dfrac{10 \times 10^6}{140\ (400 - 40)}$

$$= 198 \text{ mm}^2$$

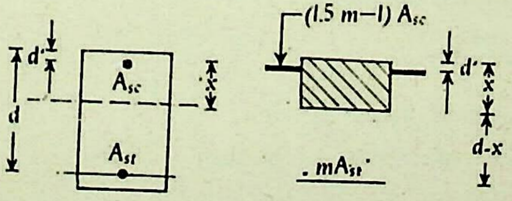
$$A_{sc} = 274 \text{ mm}^2$$

$$A_{st} = 657 + 198 = 855 \text{ mm}^2.$$

Provide $2 - 16\ \phi$ top bars $= 402$ mm²

$3 - 20\ \phi$ bottom bars $= 942$ mm².

2-13. Transformed area method: A doubly reinforced beam and its transformed section is shown in fig. 2-16.



(a) Section (b) Transformed section

Doubly reinforced beam

FIG. 2-16

Transformed area of compression zone

$$= (1{\cdot}5m - 1)\ A_{sc} + bx.$$

Transformed area of tension zone

$$= m\ A_{st}.$$

Position of neutral axis can be found out by taking moments of areas about N.A.

$$bx \cdot \frac{x}{2} + (1{\cdot}5m - 1)A_{sc}(x - d') = mA_{st}\ (d - x) \ldots (2\text{-}11a)$$

Solution of this equation gives depth of neutral axis x.

Method 1:

Taking moments of compressive forces about tensile steel and equating to external B.M.

$$M = (1{\cdot}5m - 1)A_{sc} \times \frac{x - d'}{x} f_{cb} \times (d - d')$$

(transformed area) (Stress in concrete at level of compressive steel) (distance from tensile steel)

$$+ bx \cdot \frac{f_{cb}}{2}\ (d - \frac{x}{3}) \ldots\ldots\ldots\ldots (2\text{-}11b)$$

where f_{cb} is the stress in top fibres of concrete.

Stress in compression steel

$$= (1{\cdot}5m - 1)\ (\frac{x - d'}{x})\ f_{cb} \ldots\ldots\ldots\ldots (2\text{-}11c)$$

Stress in tensile steel

$$= mf_{cb}\ (\frac{d - x}{x}) \ldots\ldots\ldots\ldots (2\text{-}11d)$$

Method 2: Classic flexure formula:

Find moment of inertia of section

$$I_{xx} = \tfrac{1}{3}\ bx^3 + (1{\cdot}5m - 1)A_{sc}\,(x - d')^2 + mA_{st}\,(d - x)^2 \ldots (2\text{-}12a)$$

Then stresses in concrete and steel can be found out by,

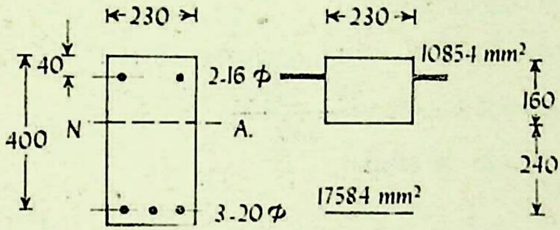
$$\text{Stress in concrete} = \frac{M.x}{I_{xx}} \ldots\ldots\ldots\ldots (2\text{-}12b)$$

$$\text{Stress in compression steel} = (1{\cdot}5m - 1) \times \frac{M(x - d')}{I_{xx}} \ldots\ldots (2\text{-}12c)$$

$$\text{Stress in tension steel} = m \cdot \frac{M(d - x)}{I_{xx}} \ldots\ldots (2\text{-}12d)$$

Example 2-16.

A rectangular beam is reinforced as shown in fig. 2-17(a). Find out the maximum stress in concrete and steel if it is subjected to a moment of 42 kNm. The materials are M15 grade concrete and mild steel reinforcements.



(a) Section		(b) Transformed area

FIG. 2-17

Solution:

For M15 grade concrete $m = \dfrac{280}{3 \times 5} = 18{\cdot}66$

$$\text{compression steel } A_{sc} = 2 \times 201 = 402 \text{ mm}^2$$
$$\text{transformed area} = (1{\cdot}5 \times 18{\cdot}66 - 1) \times 402$$
$$= 10854 \text{ mm}^2$$
$$\text{tension steel } A_{st} = 3 \times 314 = 942 \text{ mm}^2$$
$$\text{transformed area} = 942 \times 18{\cdot}66 = 17584 \text{ mm}^2.$$

To find the position of neutral axis, taking moments of transformed area about N.A.

$$\frac{1}{2} \times 230 \times x^2 + 10854\ (x - 40) = 17584\ (400 - x)$$
$$\therefore\ 115x^2 + 10854x - 434160 - 7033600 + 17584x = 0$$
$$x^2 + 247{\cdot}28x - 64937 = 0 \text{ which gives}$$
$$x = 160 \text{ mm.}$$

Method 1:

Taking moments of compressive forces about tensile steel and equating to external B.M. using equation (2-11*b*)

$$42 \times 10^6 = (1{\cdot}5 \times 18{\cdot}66 - 1) \times 402 \times \frac{160 - 40}{160} f_{cb}$$
$$\times (400 - 40) + 230 \times 160 \left(400 - \frac{160}{3}\right) . \frac{f_{cb}}{2}$$

$$= 2930580\, f_{cb} + 6378666\, f_{cb} \text{ which gives}$$
$$f_{cb} = 4{\cdot}51 \text{ N/mm}^2.$$

Stress in compression steel using equation (2-11*c*)

$$f_{sc} = (1{\cdot}5 \times 18{\cdot}66 - 1)\left(\frac{160 - 40}{160}\right) \times 4{\cdot}51$$
$$= 91{\cdot}33 \text{ N/mm}^2.$$

Stress in tension steel using equation (2-11*d*)

$$f_{st} = 18{\cdot}66 \times 4{\cdot}51\left(\frac{400 - 160}{160}\right)$$
$$= 126{\cdot}23 \text{ N/mm}^2.$$

Method 2:

$$I_{xx} = \tfrac{1}{3} \times 230 \times 160^3 + 10854 \times 120^2 + 17584 \times 240^2$$
$$= 1{\cdot}48 \times 10^9 \text{ mm}^4.$$

$$\text{Concrete stress} = \frac{42 \times 10^6 \times 160}{1{\cdot}48 \times 10^9} = 4{\cdot}54 \text{ N/mm}^2.$$

Stress in compression steel

$$= \frac{(1{\cdot}5 \times 18{\cdot}66 - 1) \times 42 \times 10^6 \times 120}{1{\cdot}48 \times 10^9}$$
$$= 91{\cdot}95 \text{ N/mm}^2.$$

Stress in tension steel

$$= \frac{18{\cdot}66 \times 42 \times 10^6 \times 240}{1{\cdot}48 \times 10^9}$$
$$= 127 \text{ N/mm}^2.$$

Example 2-17.

A rectangular beam is reinforced as shown in fig. 2-18(a). Find out the moment of resistance of the section. The materials are M20 grade concrete and tor steel reinforcement of grade Fe 415.

Solution:

For M20 grade concrete, $\sigma_{cbc} = 7$ N/mm²

$$m = \frac{280}{3 \times 7} = 13{\cdot}33.$$

Transformed area of compression steel

$$= (1{\cdot}5m - 1)A_{sc}$$
$$= (1{\cdot}5 \times 13{\cdot}33 - 1) \times 2 \times 314 = 11932 \text{ mm}^2.$$

Transformed area of tension steel

$$= 3 \times 491 \times 13{\cdot}33 = 19640 \text{ mm}^2.$$

To find neutral axis, taking moments about N.A.

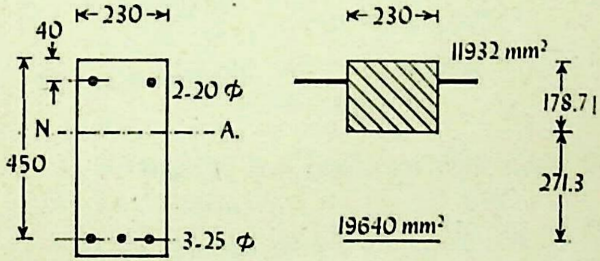
$$230 \frac{x^2}{2} + 11932 (x - 40) = 19640 (450 - x)$$

$$115x^2 + 11932x - 477280 = 8838000 - 19640x$$

$$115x^2 + 31572x - 9315280 = 0$$

$$x^2 + 274{\cdot}54x - 81002 = 0$$

which gives $x = 178{\cdot}7$ mm.



(a) Section (b) Transformed area

FIG. 2-18

Depth of balanced neutral axis referring table 2-1

$$= 0{\cdot}29 \times 450 = 130{\cdot}5 < 178{\cdot}7.$$

The section is over-reinforced and concrete stress reaches the maximum value first, then

$$\text{M.R.} = M_1 + M_2 \text{ where}$$

$$M_1 = Qbd^2$$

$$= 0{\cdot}91 \times 230 \times 450^2 \times 10^{-6} = 42{\cdot}38 \text{ kNm}$$

$$M_2 = (1{\cdot}5m - 1)A_{sc} \cdot \sigma_{cbc} \cdot \left(\frac{x - d'}{x}\right) (d - d')$$

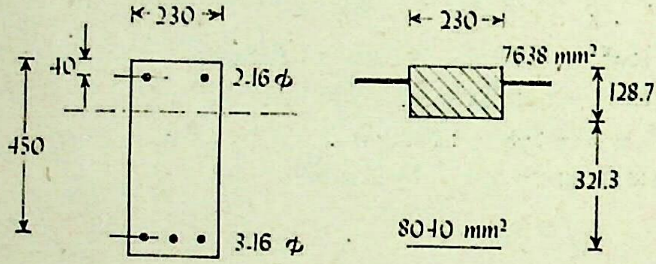
$$= (1{\cdot}5 \times 13{\cdot}33 - 1) \times 628 \times 7 \times \left(\frac{178{\cdot}7 - 40}{178{\cdot}7}\right) \times (450 - 40) \times 10^{-6}$$

$$= 26{\cdot}58 \text{ kNm}$$

$$M = M_1 + M_2 = 42{\cdot}38 + 26{\cdot}58 = 68{\cdot}96 \text{ kNm}.$$

Example 2-18.

A rectangular beam is reinforced as shown in fig. 2-19(a). Find out the moment of resistance of the section. The materials are M20 grade concrete and tor steel reinforcement of grade Fe 415.



(a) Section (b) Transformed area

FIG. 2-19

For M20 grade concrete, $\sigma_{cbc} = 7$ N/mm²

$$m = \frac{280}{3 \times 7} = 13{\cdot}33$$

$$A_{sc} = 2 \times 201 = 402 \text{ mm}^2$$

$$A_{st} = 3 \times 201 = 603 \text{ mm}^2.$$

Transformed area of compression steel

$$= (1{\cdot}5m - 1)\ A_{sc}$$

$$= (1{\cdot}5 \times 13{\cdot}33 - 1) \times 402 = 7638 \text{ mm}^2.$$

Transformed area of tension steel

$$= mA_{st} = 13{\cdot}33 \times 603 = 8040 \text{ mm}^2.$$

To find neutral axis, taking moments about N.A.

$$230\,\frac{x^2}{2} + 7638\ (x - 40) = 8040\ (450 - x)$$

On simplification, this yields

$$x^2 + 136{\cdot}3x - 34118 = 0.$$

Solving for x,

$$x = 128{\cdot}7 \text{ mm}.$$

Depth of balanced neutral axis

$$= 0{\cdot}29 \times 450 = 130{\cdot}5 > 128{\cdot}7.$$

∴ Beam is under-reinforced. Therefore the steel will fail first.

To find lever arm, c.g. of the compressive forces is to be found out.

Taking moment of compressive forces about the top fibre,

$$\bar{y} = \frac{7638 \times 40 + 230 \times 128{\cdot}7 \times 64{\cdot}35}{7638 + 230 \times 128{\cdot}7}$$

$$= 59{\cdot}35 \text{ mm.}$$

$$\text{Lever arm} = d - \bar{y} = 450 - 59{\cdot}35$$
$$= 390{\cdot}65 \text{ mm.}$$

Moment of resistance

$$= A_{st} \cdot \sigma_{st} \cdot jd$$
$$= 603 \times 230 \times 390{\cdot}65 \times 10^{-6}$$
$$= 54{\cdot}18 \text{ kNm.}$$

2-14. Use of design aids: Referring fig. 2-14,

$$M = M_1 + M_2$$
$$= M_{bal} + A_{st2} \times \sigma_{st}(d - d')$$

and $$A_{st} = A_{st1} + A_{st2}$$

where $$A_{st1} = p_{t,bal} \times \frac{bd}{100}$$

$$A_{st2} = \frac{M_2}{\sigma_{st}\,(d - d')}$$

and $$A_{sc} = \frac{M_2}{(1{\cdot}5m - 1)\,\sigma_{cbc} \cdot \dfrac{x - d'}{x} \cdot (d - d')}$$

The compression reinforcements can be expressed as a ratio of additional tensile reinforcement area A_{st2}.

$$\frac{A_{sc}}{A_{st2}} = \frac{\sigma_{st}\,(d - d')}{(1{\cdot}5m - 1)\,\sigma_{cbc}\left(\dfrac{x - d'}{x}\right)\,(d - d')}$$

$$= \frac{\sigma_{st}}{\sigma_{cbc}} \cdot \frac{1}{(1{\cdot}5m - 1)\,\left(1 - \dfrac{d'}{kd}\right)} \quad \text{where } x = kd.$$

Values of this ratio have been tabulated for different values of $\frac{d'}{d}$ and σ_{cbc} in table M of SP : 16. Values of p_t and p_c for different four values of $\frac{d'}{d}$ have been tabulated against $\frac{M}{bd^2}$ in tables 72 to 79 of SP : 16.

Example 2-19.

A rectangular beam of size 230 mm × 560 mm effective depth has to resist a moment of 92 kNm. Find out the reinforcements. The materials are M15 grade concrete and mild steel reinforcements. Use design tables.

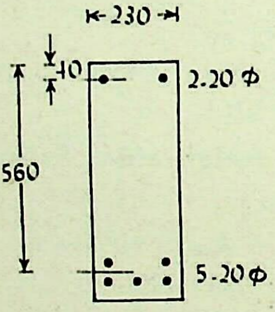


FIG. 2-20

Solution:

$$\frac{M}{bd^2} = \frac{92 \times 10^6}{230 \times 560^2} = 1{\cdot}28 > 0{\cdot}87$$

design as a doubly reinforced beam.

$$\frac{d'}{d} = \frac{40}{560} = 0{\cdot}09 \quad \text{use } \frac{d'}{d} = 0{\cdot}1.$$

From table 72 of SP : 16

$$p_t = 1{\cdot}042 \quad \therefore \quad A_{st} = \frac{1{\cdot}042 \times 230 \times 560}{100} = 1342 \text{ mm}^2.$$

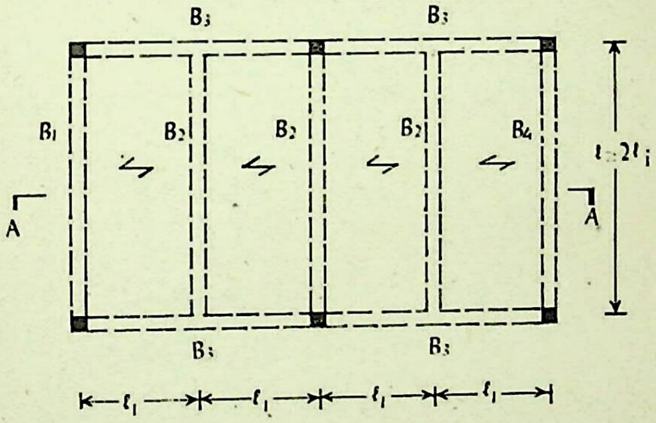
$$p_c = 0{\cdot}454 \quad \therefore \quad A_{sc} = \frac{0{\cdot}454 \times 230 \times 560}{100} = 585 \text{ mm}^2.$$

Provide 2-20 ϕ compressive steel = 628 mm²

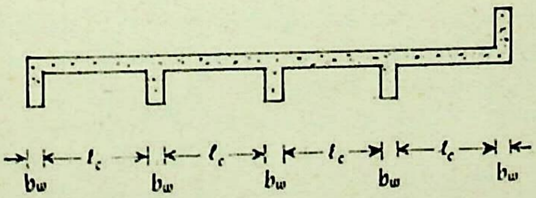
and 5-20 ϕ tensile steel = 1570 mm².

FLANGED BEAMS

2-15. Introductory: In monolithic construction, the slabs and beams are casted together. In such cases for design of beams, the compressive resistance provided by concrete of slab is also considered. These beams are known as flanged beams. Flanged beams may be "*Tee*" beams or "*Ell*" beams. The T beams and L beams are illustrated in fig. 2-21.



(a) Plan



(b) Section

T and L beams

Fig. 2-21

Note that beams B_1 and B_3 are L beams while beams B_2 are T beams. (For further discussion refer art. 5-17.) For some architectural reasons, beam B_4 has been inverted. The beam B_4 shall be designed as rectangular beam. The reason is that the slab is lying in a tension zone and slab concrete is not useful in resisting compression.

When the spacing of beams is increased (i.e. l_1 is increased), the concrete of slabs, far from the centre line of the beam will become uneffective in resisting the compression. Similarly if the spacing is decreased, all the concrete of slab may be effective in resisting compression. Thus, always all the concrete of flange cannot be considered resisting compression. IS : 456 gives following formulae for finding out the effective flange width of *T* or *L* beams.

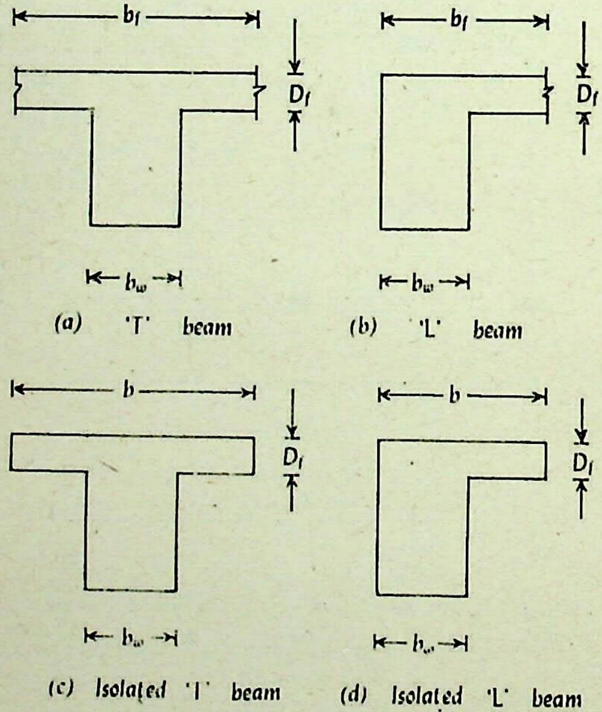


FIG. 2-22

In the absence of more accurate determination the effective width of flange may be taken as following but in no case greater than the breadth of web plus half the sum of the clear distances to the adjacent beams on either side.

Referring fig. 2-22(a) and (b),

For T beams,

(a) $$b_f = \frac{l_o}{6} + b_w + 6D_f.$$

For L beams,

(b) $$b_f = \frac{l_o}{12} + b_w + 3D_f.$$

(c) For isolated beams, the effective flange width shall be obtained as below but in no case greater than the actual width.

Referring fig. 2-22(c) and (d),

For T beam,

$$b_f = \frac{l_o}{\left(\frac{l_o}{b}\right) + 4} + b_w$$

For L beam,

$$b_f = \frac{0 \cdot 5\ l_o}{\left(\frac{l_o}{b}\right) + 4} + b_w$$

where

b_f = effective flange width,

l_o = distance between points of zero moments in the beam. This is equal to the effective span of the beam for simply supported beams and may be assumed as 0·7 times the effective span of the beam for continuous beams,

b_w = width of web,

D_f = thickness of flange, and

b = actual width of flange.

However in any case the value of b_f shall not exceed the actual width of flange.

2-16. Moment of resistance of a flanged beam: The flanged beam may be a singly reinforced or doubly reinforced beam. Doubly reinforced flanged beams are rare in practice and will not be considered here. However, this can be designed in a way similar to the doubly reinforced rectangular beam.

In flanged beams, two possibilities can be there:

(a) the N.A. lies in the flange, and

(b) the N.A. lies in the web.

These are discussed as below:

(*a*) *Neutral axis lies in flange:*

In this case the beam acts as a rectangular beam of width b_f and moment of resistance of the section is given by,

$$\text{M.R.} = Q\, b_f d^2 \quad \ldots\ldots \quad (2\text{-}13a)$$

Value of Q in this case will be higher than the case of rectangular beam as large compression area is concentrated here.

Alternatively, if the area of steel reinforcement is known, the depth of neutral axis can be found out by taking moment about N.A.

$$\frac{b_f \cdot x^2}{2} = mA_{st}\,(d - x) \quad \ldots\ldots \quad (2\text{-}13b)$$

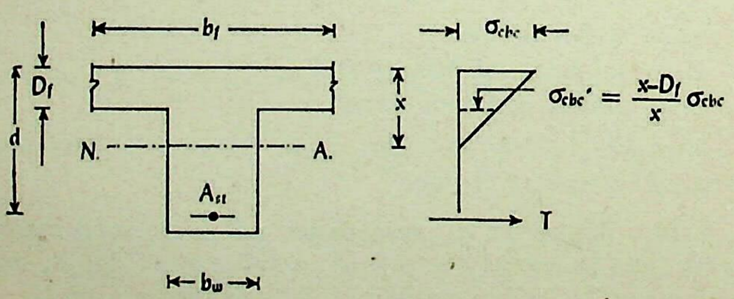
x can be found out from this equation then,

$$\text{M.R.} = b_f . x . \frac{\sigma_{cbc}}{2} . \left(d - \frac{x}{3}\right) \quad \ldots\ldots \quad (2\text{-}13c)$$

M.R. with respect to tensile force

$$= A_{st} \cdot \sigma_{st} \cdot (d - x/3) \quad \ldots\ldots \quad (2\text{-}13d)$$

(*b*) *Neutral axis lies in web:*



(a) Section (b) Stress diagram

Singly reinforced T beam

Fig. 2-23

When the N.A. lies in the web, the compression taken by web is very small as compared to the compression taken by flange and is usually neglected.

Fig. 2-23 shows a singly reinforced T beam where N.A. lies in the web.

Depth of N.A. can be found by taking moments about N.A.

$$b_f D_f \left(x - \frac{D_f}{2}\right) = m\, A_{st}\, (d - x)$$

The solution of this equation gives depth of neutral axis x.

Let $\bar{y}$ be the distance of c.g. of compressive forces from top of the flange. Taking moments about top fibres,

$$\bar{y} = \frac{\sigma_{cbc} \cdot \frac{D_f}{2} \cdot \frac{D_f}{3} + \sigma_{cbc}' \cdot \frac{D_f}{2} \cdot \frac{2D_f}{3}}{\sigma_{cbc} \cdot \frac{D_f}{2} + \sigma_{cbc}' \cdot \frac{D_f}{2}}$$

$$= \frac{D_f}{3} \times \frac{\sigma_{cbc} + 2\,\sigma_{cbc}'}{\sigma_{cbc} + \sigma_{cbc}'}$$

Now $\sigma_{cbc}' = \dfrac{x - D_f}{x}\, \sigma_{cbc}$

$$\therefore \quad \bar{y} = \frac{\sigma_{cbc} + 2\,\sigma_{cbc}\left(\frac{x - D_f}{x}\right)}{\sigma_{cbc} + \sigma_{cbc}\left(\frac{x - D_f}{x}\right)} \times \frac{D_f}{3}$$

$$= \frac{D_f}{3} \times \frac{1 + 2\left(\frac{x - D_f}{x}\right)}{1 + \left(\frac{x - D_f}{x}\right)} \quad \ldots\ldots\ldots\ldots\ldots\ldots (2\text{-}14)$$

lever arm $= d - \bar{y}$

M.R. with respect to compression

$$= b_f \cdot D_f \left(\frac{\sigma_{cbc} + \sigma_{cbc}'}{2}\right) (d - \bar{y}) \quad \ldots\ldots\ldots\ldots (2\text{-}15a)$$

and M.R. with respect to tension

$$= A_{st}\, \sigma_{st}\, (d - \bar{y}) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2\text{-}15b)$$

2-17. Types of problems:

Type 1: To find the neutral axis.

(a) If the section and actual stresses in materials are given, find out depth of neutral axis using equation (2-1) as for singly reinforced rectangular beam.

$$x = kd$$

where $$k = \frac{1}{1 + \dfrac{f_{st}}{m\, f_{cb}}}$$

If depth of critical neutral axis is to be found out replace f_{st} by σ_{st} and f_{cb} by σ_{cbc}.

(b) If the section and steel area provided are given, first decide whether the N.A. lies in flange or in web as follows:

Take moments of transformed area of concrete flange (M_{tc}) and transformed area of steel (M_{ts}) about the bottom of flange. Then,

(1) If $M_{tc} > M_{ts}$ N.A. lies in flange
(2) If $M_{tc} = M_{ts}$ N.A. lies at bottom of flange, and
(3) If $M_{tc} < M_{ts}$ N.A. lies in web.

When neutral axis lies in flange, the depth of N.A. can be found out by taking moments about N.A.

$$b_f \,.\, x \,.\, \frac{x}{2} = m\, A_{st}\ (d - x).$$

When neutral axis lies in web, the depth of N.A. can be found out by taking moments about N.A. neglecting small concrete area in web portion,

$$b_f \,.\, D_f \left(x - \frac{D_f}{2}\right) = m\, A_{st}\ (d - x)$$

Type 2: To find out the moment of resistance of given section.

(1) Find out the depth of actual neutral axis and critical neutral axis as explained in type 1.

(2) Decide the type of failure i.e. *compression failure* (over-reinforced) or *tension failure* (under-reinforced). If $x_{actual} < x_{critical}$, the beam is under-reinforced and if $x_{actual} > x_{critical}$, the beam is over-reinforced.

(3) If N.A. lies in flange, M.R. is given by,

M.R. $= A_{st} \cdot \sigma_{st} \left(d - \frac{x}{3}\right)$ using equation (2-13*d*), when under-reinforced, and

M.R. $= Q\, b_f\, d^2$ using equation (2-13*a*), when over-reinforced.

(4) If N.A. lies in web, M.R. is given by,

M.R. $= A_{st} \cdot \sigma_{st} (d - \bar{y})$ using equation (2-15*b*) when under-reinforced, and

M.R. $= b_f\, D_f \left(\frac{\sigma_{cbc} + \sigma_{cbc}'}{2}\right) (d - \bar{y})$ using equation (2-15*a*) when over-reinforced.

Type 3: For the given moment and section of beam, to check the stresses.

Method 1:

(1) Find out the neutral axis of beam as explained in type 1.

(2) Find out the lever arm. If N.A. lies in flange, lever arm $= d - \frac{x}{3}$ and if N.A. lies in web, lever arm $= d - \bar{y}$ where $\bar{y}$ is a distance of c.g. of compressive forces from extreme compression fibre.

(3) The stresses are found out as follows:

$$\text{Stress in steel} = f_{st} = \frac{M}{A_{st} \times \text{lever arm}}$$

$$\text{Stress in concrete} = f_{cb} = \frac{f_{st}}{m} \cdot \frac{x}{d - x}$$

Method 2:

(1) Find out N.A. of beam as explained in type 1.

(2) Find out moment of inertia of beam neglecting concrete in web portion.

$I_{xx} = \frac{bx^3}{3} + m\, A_{st}\, (d - x)^2$ when N.A. lies in flange,

and, $I_{xx} = \frac{1}{12} b_f D_f^3 + b_f D_f \left(x - \frac{D_f}{2}\right)^2 + m A_{st} (d-x)^2$ when N.A. lies in web.

The stresses in concrete and steel are given by,

Stress in concrete

$$f_{cb} = \frac{Mx}{I_{xx}} \text{ and,}$$

Stress in steel

$$f_{st} = m \cdot \frac{M (d-x)}{I_{xx}}.$$

Type 4: To design singly reinforced T beam for a given moment.

The resistance to compression of concrete slab in a flanged beam is very high as compared to a rectangular beam. This gives much less depth required to resist a given bending moment. The depth of T beam or L beam in practice depends on many factors such as the cost of concrete, steel and formwork. The architectural requirements also may decide the depth. Providing depth only using moment as a criteria will increase the steel requirement. The amount of shear reinforcement also increases as discussed in chapter 3. This may prove uneconomical.

To design a flanged beam, the following pattern may be followed:

(1) Fix the width of the beam using architectural considerations. Also the width shall be sufficient to accomodate the reinforcement fulfilling the practical requirements of spacing of bars as explained in art. 2-4.

(2) Assume overall depth of beam $D = \frac{1}{12}$ to $\frac{1}{10}$ of the span and substracting effective concrete cover from over all depth, find out effective depth d.

(3) Assume as a first trial, approximate lever arm

$$= d - \frac{D_f}{2}.$$

(4) Reinforcement area shall be found out from the equation,

$$A_{st} = \frac{M}{\sigma_{st}\left(\dfrac{d - D_f}{2}\right)}.$$

(5) Check the section either for moment of resistance or actual stresses as explained in type 2 and type 3.

Example 2-20.

A tee beam with rib width of 230 mm flange width of 1600 mm, thickness of flange 100 mm and effective depth of 500 mm is reinforced with 4 no. 20 mm diameter bars as tension reinforcement. Find out the depth of neutral axis. The materials are M15 grade concrete and mild steel reinforcement.

Solution:

For M15 mix $m = 18{\cdot}66$. To find the position of neutral axis, compare the moment of transformed areas of flange and reinforcement about bottom of flange.

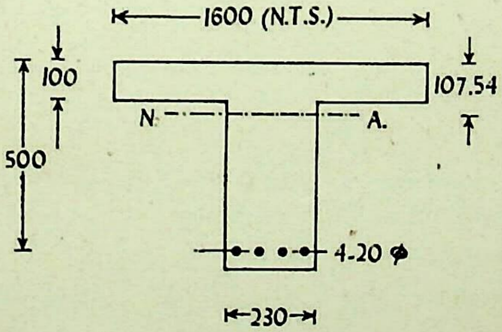


FIG. 2-24

$A_{st} = 4 \times 314 = 1256 \text{ mm}^2$

$M_{tc} = 1600 \times 100 \times 50 = 8 \times 10^6$

$M_{ts} = 18{\cdot}66 \times 1256 \times 400 = 9{\cdot}37 \times 10^6$

$M_{ts} > M_{tc}$

$\therefore$ N.A. lies in web.

Taking moments about neutral axis,

$$\therefore \quad b\ D_f \left(x - \frac{D_f}{2}\right) = m\ A_{st}\ (d - x)$$

$$1600 \times 100\ (x - 50) = 18{\cdot}66 \times 1256\ (500 - x)$$

which on solving gives,

$$x = 107{\cdot}54 \text{ mm.}$$

Example 2-21.

Find the moment of resistance of a tee beam as shown in fig. 2-25. The materials are M15 grade concrete and tor steel reinforcement of grade Fe 415.

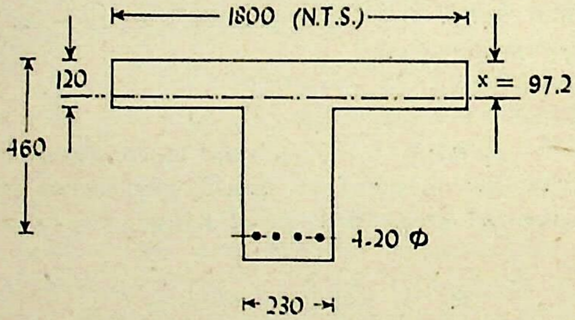


Fig. 2-25

Solution:

Taking moments of transformed areas of flange and reinforcement about bottom of flange,

$$M_{tc} = 1800 \times 120 \times 60 = 1{\cdot}296 \times 10^7$$

$$M_{ts} = 18{\cdot}66 \times 1256 \times (460 - 120) = 0{\cdot}797 \times 10^7$$

$$M_{tc} > M_{ts}$$

$\therefore$ N.A. lies in flange.

Taking moments about N.A.

$$1800\ x \times \frac{x}{2} = 18{\cdot}66 \times 1256\ (460 - x)$$

$$\therefore \quad x^2 = 11979 - 26{\cdot}04\ x$$

which gives $x = 97{\cdot}2$ mm.

Depth of critical N.A. $= 0{\cdot}29 \times 460 = 133{\cdot}4$

$$\therefore\ x_{actual} < x_{critical}$$

$\therefore$ Under-reinforced beam.

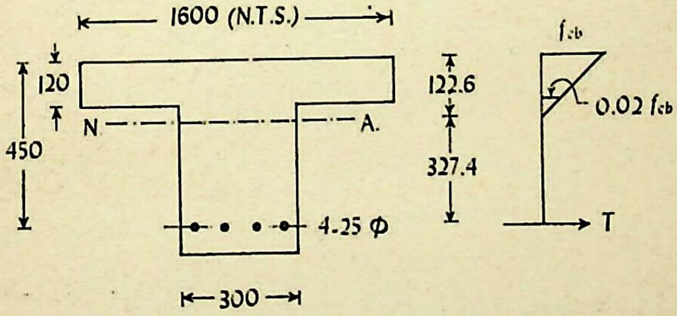
$$\text{M.R.} = A_{st} \cdot \sigma_{st} \cdot \left(d - \frac{x}{3}\right)$$

$$= 1256 \times 230 \left(460 - \frac{97{\cdot}2}{3}\right) \times 10^{-6}$$

$$= 123{\cdot}52 \text{ kNm.}$$

Example 2-22.

A tee beam as shown in fig. 2-26 is subjected to a moment of 120 kNm. Find out the maximum stresses in the materials. The materials are M15 grade concrete and mild steel reinforcement.



(a) Section (b) Stress diagram

Fig. 2-26

Solution:

$$A_{st} = 4 \times \frac{\pi}{4} \times 25^2 = 1963 \text{ mm}^2.$$

Taking moment of transformed area of flange and reinforcement about bottom of flange,

$$M_{tc} = 1600 \times 120 \times 60 = 1{\cdot}152 \times 10^7$$

$$M_{ts} = 18{\cdot}66 \times 1963 \times 330 = 1{\cdot}209 \times 10^7$$

$$M_{ts} > M_{tc}$$

$\therefore$ N.A. lies in web.

Taking moments about N.A. neglecting small concrete in web,

$$1600 \times 120 \times (x - 60) = 18{\cdot}66 \times 1963 \; (450 - x)$$

which on solving gives $x = 122{\cdot}6$ mm.

Method 1:

The c.g. of compressive force from extreme compression fibre,

$$\bar{y} = \frac{f_{cb} \cdot \dfrac{120}{2} \times \dfrac{120}{3} + 0{\cdot}02\, f_{cb} \times \dfrac{120}{2} \times \dfrac{2 \times 120}{3}}{f_{cb} \times \dfrac{120}{2} + 0{\cdot}02\, f_{cb} \times \dfrac{120}{2}}$$

$$= 40{\cdot}78 \text{ mm}$$

lever arm $= 450 - 40{\cdot}78 = 409{\cdot}22$ mm.

$$\text{Stress in steel} = \frac{120 \times 10^6}{1963 \times 409{\cdot}22} = 149{\cdot}38 \text{ N/mm}^2$$

$$\text{Stress in concrete} = \frac{149{\cdot}38}{18{\cdot}66} \times \frac{122{\cdot}6}{327{\cdot}4}$$

$$= 3{\cdot}0 \text{ N/mm}^2.$$

Method 2:

$$I_{xx} = \frac{1}{12} \times 1600 \times 120^3 + 1600 \times 120 \; (122{\cdot}6 - 60)^2 + 18{\cdot}66 \times 1963 \; (327{\cdot}4)^2$$

$$= 4{\cdot}91 \times 10^9 \text{ mm}^4.$$

Stress in concrete

$$f_{cb} = \frac{120 \times 10^6 \times 122{\cdot}6}{4{\cdot}9 \times 10^9} = 3 \text{ N/mm}^2.$$

Stress in steel

$$f_{st} = 18{\cdot}66 \times \frac{120 \times 10^6 \times 327{\cdot}4}{4{\cdot}91 \times 10^9} = 149{\cdot}3 \text{ N/mm}^2.$$

Example 2-23.

A tee beam (beam B_2 of fig. 2-21) of a concrete floor is loaded with 25 kN/m load inclusive of self weight. The tee beams are spaced at 3 m centres. The span of beam is 6 m and simply supported. Thickness of slab is 120 mm. Design the beam for flexure. The materials are M15 grade concrete and mild steel reinforcement.

Solution:

$$M = \frac{25 \times 6^2}{8} = 112{\cdot}5 \text{ kNm.}$$

Use $b_w = 230$ mm and $D = \frac{1}{10}$ of span $= 600$ mm.

Using two layers of 20 mm diameter bars,

$$d = 600 - 25 - 20 - 10 = 545.$$

Assume lever arm $= d - \frac{D_f}{2} = 545 - 60 = 485$ mm

$$A_{st} = \frac{112{\cdot}5 \times 10^6}{140 \times 485} = 1657 \text{ mm}^2.$$

Use 4 no. 20 mm diameter bars plus 2 no. 16 mm diameter bars.

$$A_{st} = 4 \times 314 + 2 \times 201 = 1658 \text{ mm}^2.$$

The designed section shall now be checked for moment of resistance or for actual stresses.

Actual width of flange $= 3000$ mm,

and $b_f = \frac{l_o}{6} + b_w + 6D_f$

$$= \frac{6000}{6} + 230 + 6 \times 120 = 1950 \text{ mm.}$$

Use $b_f = 1950$ mm.

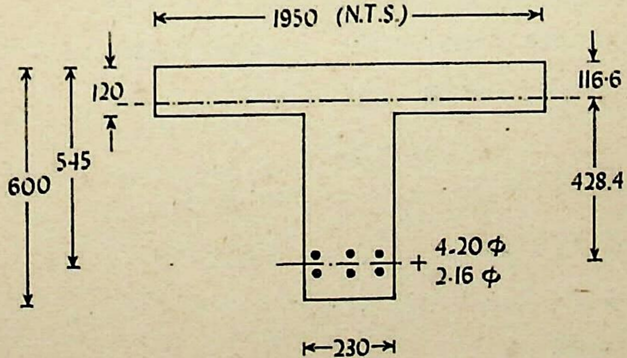


FIG. 2-27

$$M_{tc} = 1950 \times 120 \times 60 = 1{\cdot}404 \times 10^7$$

$$M_{ts} = 18{\cdot}66 \times 1658\ (545 - 120) = 1{\cdot}315 \times 10^7$$

$$M_{tc} > M_{ts}$$

$\therefore$ N.A. lies in flange.

Taking moments about neutral axis to find its position,

$$1950\ x \times \frac{x}{2} = 18{\cdot}66 \times 1658\ \ (545 - x).$$

On simplification this gives,

$$x^2 + 31{\cdot}73\ x - 17294 = 0$$

which on solving gives,

$$x = 116{\cdot}6 \text{ mm.}$$

Method 1:

Depth of critical neutral axis

$$= 0{\cdot}4 \times 545 = 218 \text{ mm}$$

$$x_{actual} < x_{critical}$$

$\therefore$ Under-reinforced beam.

$$\text{M.R.} = A_{st} \,.\, \sigma_{st}\,(d - \frac{x}{3})$$

$$= 1658 \times 140 \times (545 - \frac{116{\cdot}6}{3}) \times 10^{-6}$$

$$= 117{\cdot}48 \text{ kNm} > 112{\cdot}5 \text{ kNm}............ \text{(O.K.)}$$

Method 2:

If f_{cb} is the stress at the extreme compression fibre,

compressive force $= 1950 \times 116{\cdot}6 \times \frac{f_{cb}}{2}$

$$= 113685\ f_{cb}.$$

Equating M.R. of section to external B.M.

$$113685\ f_{cb}\ \ (545 - \frac{116{\cdot}6}{3}) = 112{\cdot}5 \times 10^6$$

$\therefore\ f_{cb} = 1{\cdot}96 \text{ N/mm}^2 < 5 \text{ N/mm}^2$................(O.K.)

Stress in tensile steel

$$f_{st} = m\, f_{cb} \,.\, \frac{d - x}{x}$$

$$= 18{\cdot}66 \times 1{\cdot}96 \times \frac{545 - 116{\cdot}6}{116{\cdot}6}$$

$$= 134{\cdot}38 \text{ N/mm}^2 < 140 \text{ N/mm}^2. \qquad \text{(O.K.)}$$

Method 3:

$$I_{xx} = \tfrac{1}{3} \times 1950 \times 116{\cdot}6^3 + 18{\cdot}66 \times 1658 \times 428{\cdot}4^2$$
$$= 6{\cdot}7 \times 10^9 \text{ mm}^4.$$

$$f_{cb} = \frac{112{\cdot}5 \times 10^6 \times 116{\cdot}6}{6{\cdot}7 \times 10^9}$$
$$= 1{\cdot}96 \text{ N/mm}^2 < 5 \text{ N/mm}^2 \ldots\ldots\ldots\ldots\ldots \text{(O.K.)}$$

$$f_{st} = \frac{112{\cdot}5 \times 10^6 \times 428{\cdot}4}{6{\cdot}7 \times 10^9} \times 18{\cdot}66$$
$$= 134{\cdot}22 \text{ N/mm}^2 < 140 \text{ N/mm}^2 \ldots\ldots\ldots\ldots \text{(O.K.)}$$

2-18. Slabs: Slabs are plate elements having the depth *D* much smaller than its span and width. They are usually carrying uniformly distributed loads from floors and supported on walls or beams. Slabs are primarily flexural members and designed in the same way as the beams. They are treated separately in chapter 6.

EXAMPLES II

[*Note:* Assume suitable data wherever necessary.]

(1) Answer the following:

(a) How many 20 mm dia. tension bars can be adjusted in a beam of 200 mm width in one layer?

(b) Why is the over-reinforced design not advisable?

(c) What is the transformed area?

(d) Explain how modulur ratio changes with time.

(e) Explain why the actual width of tee beam is not always effective in carrying compression.

(f) In some type of concrete, permissible stress in compression in bending is 6·2 N/mm². Find out value of *m*.

(2) Determine the position of neutral axis of a reinforced concrete beam 250 mm wide × 360 mm effective depth, if the stresses developed in concrete and steel are 4·6 N/mm^2 and 106 N/mm^2 respectively. The materials are M15 grade concrete and mild steel reinforcement. Also determine the type of the beam.

(3) An R.C.C. beam 200 mm wide × 460 mm effective depth is reinforced with 3 no. 16 mm dia. bars. Find out the position of neutral axis and state the type of the beam. The materials are M15 grade concrete and tor steel reinforcement of grade Fe 415.

(4) A concrete beam 300 mm wide × 420 mm effective depth is reinforced with a steel reinforcement having a safe tensile stress of 115 N/mm^2. If the concrete grade M20 is used, find the depth of critical neutral axis.

(5) An R.C.C. beam of size 230 mm wide × 660 mm effective depth is reinforced with 4 no. 20 mm dia. bars. Find out the moment of resistance of beam. Also state whether the beam is under-reinforced or over-reinforced. The materials are M20 grade concrete and mild steel reinforcement.

(6) Solve the above Example (5) if the concrete mix of M15 grade is used.

(7) An R.C.C. beam of size 300 mm wide × 620 mm effective depth is reinforced with 5 no. 12 mm dia. bars. Find the moment of resistance of the beam. The materials are M15 grade concrete and tor steel reinforcement of grade Fe 415.

(8) An R.C.C. beam of size 200 mm wide × 560 mm effective depth is reinforced with 4 no. 16 mm dia. bars. Find out the moment of resistance of the beam if the permissible stresses in concrete in bending compression and steel in tension are respectively 5·2 N/mm^2 and 215 N/mm^2.

(9) A rectangular R.C.C. beam of size 350 mm wide × 450 mm effective depth is reinforced with 4 no. 12 mm dia. bars. Find the safe concentrated central point load on a span of 3·6 m, which the beam can resist in addition to its self weight. The materials are M15 grade concrete and tor steel reinforcement of grade Fe 415.

(10) A simply supported rectangular beam over a span of 3·6 m is reinforced in tension only. The beam is 230 mm wide and has an effective depth of 510 mm. It is reinforced with 4 no. 16 mm dia. bars. Calculate the stresses in both materials at the centre of span when a beam carries a uniformly distributed load of 26 kN/m inclusive of self weight. The materials are M15 grade concrete and mild steel reinforcement.

(11) Solve the above Example (10), if M20 grade concrete and tor steel reinforcement of grade Fe 415 are used.

(12) A simply supported beam of 4·8 m span is carrying a uniformly distributed load of 28 kN/m inclusive of its self weight. Find out steel area for balanced section, if it is reinforced in tension only. The width of beam is 230 mm and the materials are M15 grade concrete and mild steel reinforcement.

(13) A simply supported rectangular beam of 5 m span carries a uniformly distributed load of 6 kN/m inclusive of its self weight. It also carries a central point load of 16 kN. Find the depth and steel area for balanced design. The materials are M15 grade concrete and tor steel reinforcement of grade Fe 415.

(14) A simply supported rectangular beam of 6·2 m span carries a uniformly distributed load including self weight of 7·5 kN/m. If the beam is 230 mm wide × 460 mm effective depth, find the steel required at mid span. Use design tables. The materials are M15 grade concrete and mild steel reinforcement.

(15) A beam of size 250 mm × 600 mm effective depth is reinforced with 4 no. 12 mm dia. bars. It is subjected to a bending moment of 60 kNm. Check the section using design tables. The materials are M15 grade concrete and tor steel reinforcement of grade Fe 415.

(16) A rectangular beam 200 mm wide × 460 mm effective depth is subjected to a moment of 45 kNm. The effective cover of compressive reinforcement is 40 mm. Find out the reinforcing steel. The materials are M15 grade concrete and tor steel reinforcement of grade Fe 415.

(17) A doubly reinforced rectangular beam of over all size 230 mm width × 550 mm depth is reinforced with 2 no. 20 mm dia. bars at top and 4 no. 20 mm dia. bars at bottom. Find out the moment of resistance of the beam. If this beam is subjected to a moment of 72 kNm, find the stresses in concrete and steel. The materials are M20 grade concrete and tor steel reinforcement of grade Fe 415.

(18) A rectangular beam of size 230 mm wide × 610 mm effective depth is reinforced with 2 no. 20 mm dia. bars at top and 5 no. 20 mm dia. bars at bottom. Find out maximum stresses developed in concrete and steel if it is subjected to a moment of 120 kNm. The materials are M15 grade concrete and mild steel reinforcement.

(19) A rectangular beam of size 350 mm × 500 mm effective depth is reinforced with 3 no. 20 mm dia. bars at top and 5 no. 20 mm dia. bars at bottom. Find out the moment of resistance of the section. The materials are M15 grade concrete and tor steel reinforcement of grade Fe 415.

(20) A rectangular beam of size 230 mm width × 565 mm effective depth is subjected to a negative bending moment (tension at top) of 95 kNm. Find the reinforcements. The materials are M15 grade concrete and mild steel reinforcement. Use design aids.

(21) A tee beam with rib width 230 mm, flange width of 1200 mm, thickness of flange 75 mm and effective depth of 460 mm is reinforced with 4 no. 16 mm dia. bars in tension. Find out the depth of neutral axis. The materials are M15 grade concrete and mild steel reinforcement. Also determine whether the beam is under-reinforced or over-reinforced.

(22) A tee beam with rib width of 300 mm, flange width of 1650 mm, thickness of flange 100 mm and effective depth of 365 mm is reinforced with 4 no. 20 mm dia. bars in tension. Find out the position of neutral axis and state the type of beam. The materials are M15 grade concrete and tor steel reinforcement of grade Fe 415.

(23) Solve the above Example (22), if now the beam is reinforced with 4 no. 25 mm dia. bars.

(24) A tee beam with rib width 350 mm, flange width of 1700 mm, thickness of flange 100 mm and effective depth of 410 mm is reinforced with 4 no. 20 mm dia. bars in tension. Find out the moment of resistance of the section. Also, find out the M.R. if it is reinforced with 5 no. 25 mm dia. bars. The materials are M15 grade concrete and tor steel reinforcement of grade Fe 415.

(25) A tee beam with rib width 300 mm, flange width 1650 mm, thickness of flange 100 mm and effective depth of 460 mm is reinforced with 4 no. 25 mm dia. bars in tension. Find out the maximum stresses in materials if it is subjected to a bending moment of 180 kNm. The materials are M15 grade concrete and tor steel reinforcement of grade Fe 415.

(26) An isolated *T* beam has an actual width of 1500 mm and web width of 350 mm. Thickness of flange is 120 mm and overall depth is 600 mm. The span of beam is 5 m and carries a uniformly distributed load inclusive of self weight of 20 kN/m. Find the steel area required. The materials are M15 grade concrete and mild steel reinforcement.

(27) A *L* beam of a concrete floor is loaded with 22 kN/m load inclusive of self weight. The beams are spaced at 3·2 m centres. The span of beam is 6 m and simply supported. Thickness of flange is 115 mm. Design the beam for flexure. The materials are M15 grade concrete and mild steel reinforcement.

(28) An isolated *T* beam of a combined footing has an actual width of flange 1800 mm and web width of 500 mm. Thickness of flange is 300 mm and the span of beam is 4·2 m. The slab is at bottom of the beam and the loadings are upwards. The net soil pressure on beam is 160 kN/m. Design the beam for flexure. The materials are M15 grade concrete and tor steel reinforcement of grade Fe 415.

(29) An isolated *L* beam of a foot-bridge has actual flange width of 900 mm, rib width of 250 mm, depth of slab 120 mm and overall depth of 500 mm is subjected to a load of 8 kN/m inclusive of self weight. The span of the beam is 6 m and simply supported. Desig? the reinforcement for flexure.

3

Shear and Development Length

SHEAR

3-1. Shear in homogeneous beam: Shear in a beam is induced due to the change of bending moment along the span. For homogeneous beams, the shear stress distribution is given by the well-known formula,

$\tau = \dfrac{V.A.y}{b.I_{xx}}$ where referring fig. 3-1, it is required to find out shear stress at level xx,

$V =$ Shear force at the section

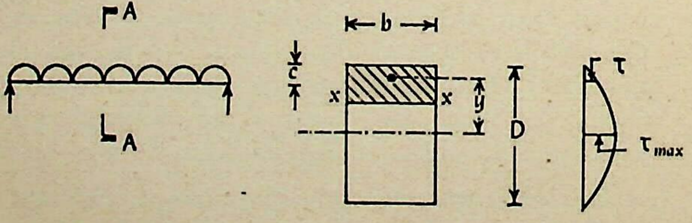
$A =$ Area outside the section where the shear stress is required $= bc$

$y =$ Distance from the centroid of A to the centre of the section $= \dfrac{1}{2}(D - c)$

$I_{xx} =$ Moment of inertia of beam $= \dfrac{1}{12} bD^3$

$b =$ breadth of the section

$D =$ depth of the section



(a) Beam (b) Section A-A (c) Shear stress distribution

Shear in homogeneous beam

FIG. 3-1

For a rectangular homogeneous beam, the shear stress distribution is parabolic. The maximum shear stress is at centre and is given by,

$$\tau_{max} = \frac{3V}{2\,bd}.$$

3-2. Shear in reinforced concrete beams — Elastic theory: Consider a short length δx of an R.C.C. beam as shown in fig. 3-2. Let M and V be the moment and shear at section 1-1 and $M + \delta M$ and $V + \delta V$ be the moment and shear at section 2-2.

Taking the moment of forces about side 2-2,

$M + V.\delta x - \dfrac{w\delta x^2}{2} - M - \delta M = 0$ as section is in equilibrium.

$\dfrac{w\delta x^2}{2}$ is a very small quantity and can be neglected.

$\therefore \quad V\delta x - \delta M = 0 \qquad$ or $\qquad V\delta x = \delta M.$

Now moment of resistance with respect to compression

$= \delta C \,.\, z$ where z is lever arm;

and moment of resistance with respect to tension

$= \delta T \,.\, z$

for the balanced section $\delta C \,.\, z = \delta T \,.\, z.$

Now external moment = internal moment of resistance

$\therefore \quad \delta M = \delta T \,.\, z = \delta C \,.\, z = V \,.\, \delta x$

$$\delta C = \frac{V\delta x}{z} \text{ and } \delta T = \frac{V\delta x}{z} \qquad \text{(3-1)}$$

These longitudinal forces induce horizontal shear stresses which can be given as follows:

At neutral axis,

$$\text{shear stress} = \frac{\delta C}{b.\delta x} = \frac{V.\delta x}{z.b\delta x} = \frac{V}{bz}$$

Now $z = jd$ = lever arm

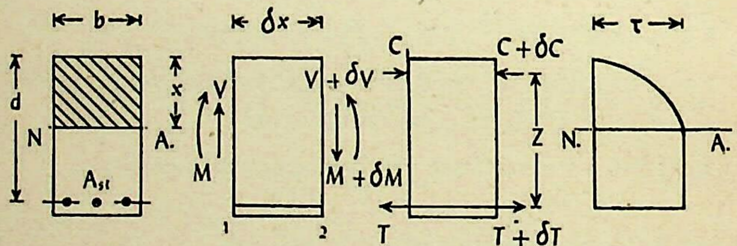
$$\therefore \quad \tau = \frac{V}{bjd} \qquad \text{.........} \quad (3\text{-}2a)$$

The shear stress distribution in R.C.C. beam is zero at top and parabolic upto neutral axis. At neutral axis the value is maximum and is equal to V/bjd.

Below the neutral axis as concrete is considered ineffective in tension, the change in the longitudinal forces remains constant and is equal to δC or δT.

$$\text{Shear stress} = \frac{\delta T}{b\delta x} = \frac{V\delta x}{bz\delta x} = \frac{V}{bjd} \qquad \text{.........} \quad (3\text{-}2b)$$

At c.g. of bars the compressive forces causing shear (δC) are neutralised by equal and opposite force δT and hence, shear stress drops to zero. The shear distribution is shown in fig. 3-2(d).



(a) Section (b) Short length δ_x (c) Forces on δ_x (d) Shear stress distribution

Shear in R.C.C. beam

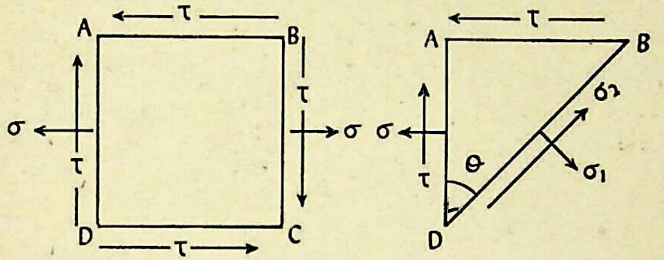
FIG. 3-2

3-3. Diagonal tension and diagonal compression: Consider a small element along the length of the beam. This is subjected to the shear stress (τ) parallel to four sides and the tensile stress (σ) along the length of the beam as shown in fig. 3-3.

The principal stresses on this element are given by,

σ_1 or $\sigma_2 = \frac{\sigma}{2} \pm \sqrt{\left(\frac{\sigma}{2}\right)^2 + (\tau)^2}$ and the inclination of principal planes is given by,

$$\tan 2\theta = \frac{2\tau}{\sigma}$$



(a) Small element along the length of beam (b) Free body diagram

Forces on small element along the length of beam

FIG. 3-3

The major principal stress is tensile and is equal to $\sigma_1 = \frac{\sigma}{2} + \sqrt{\left(\frac{\sigma}{2}\right)^2 + (\tau)^2}$ and is known as *diagonal tension.* The minor principal stress is compressive and is equal to $\sigma_2 = \frac{\sigma}{2} - \sqrt{\left(\frac{\sigma}{2}\right)^2 + (\tau)^2}$ and is known as *diagonal compression.*

Two important cases are discussed below:

(1) If B.M. $= 0$ i.e. $\sigma = 0$ then,

$$\sigma_1 = \tau \text{ and } \sigma_2 = -\tau$$

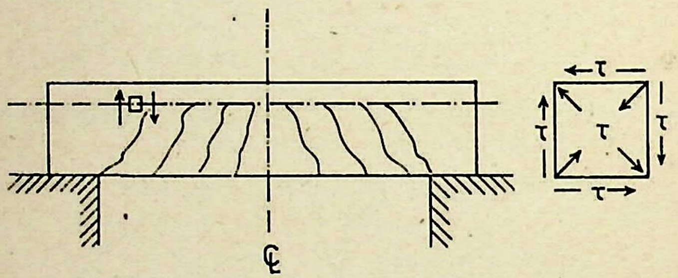
$$\tan 2\theta = \infty \text{ and } \theta = 45° \text{ or } 135°.$$

This means that near the support for a simply supported beam, where B.M. is zero, the principal tension is equal to shear stress and is inclined at 45°. This is known as diagonal tension. As the concrete is weak is tension, the concrete near the support cracks at 45° with horizontal (i.e. perpendicular to diagonal tension) as shown in fig. 3-4(a). These are known as shear cracks. To avoid the shear cracks, the beam should be reinforced across the cracks (i.e. along the principal tension) and is explained in art. 3-6.

The other principal stress is inclined at 135° and is compressive. This is known as diagonal compression and is of the same value as the shear stress. The concrete is strong

in compression and for usual cases it is below the permissible value. However, if the shear stress is very high, precautions to avoid the diagonal compression failure also have to be taken. This is explained in art. 3-5.

The diagonal tension and compression near the support are shown in fig. 3-4(b). The shear cracks are shown in fig. 3-4(a).



(a) Shear cracks in R.C.C. beam (b) Small element near support

Fig. 3-4

(2) When B.M. is maximum at mid-span of a simply supported beam,

$$\tau = 0 \quad \text{and} \quad \theta = 0.$$

This means that principal tension acts in horizontal direction and shear cracks will be vertical as shown in fig. 3-4(a).

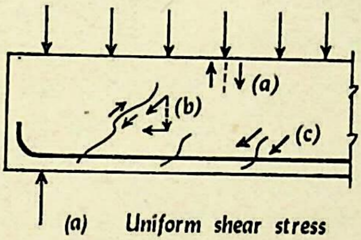
3-4. Limit state theory: Equation 3-2 for shear stress is derived using elastic theory. In this derivation the resistance to shear provided by reinforcement was ignored. In limit state theory this is considered. The mechanics of shear transfer when concrete cracks due to shear is illustrated in fig. 3-5. The shear is resisted by:

(1) Above neutral axis the shear resistance is provided by the uniform shear stress in uncracked concrete.

(2) Along the crack, the shear resistance is provided by the vertical component of force due to the interlocking of aggregates.

(3) At the tensile reinforcements, shear is resisted by the dowel action of the longitudinal bar.

The behaviour of concrete is thus complicated and the shear stress is found out in limit state theory by a simple formula similar to that of elastic theory where τ_v is defined as nominal shear stress.



Shear resistance of a cracked beam
Fig. 3-5

Then $\tau_v = \dfrac{V}{bd}$ .... (3-3)

where

τ_v = nominal shear stress

V = design shear

b = width of section. In the case of flanged beam, it is a width of rib

d = effective depth.

3-5. Permissible shear stresses: Even in elastic theory, according to IS : 456, the shear stress is found out by eq. (3-3) i.e. limit state behaviours are adopted. The permissible shear stresses are given in table 17 of IS : 456 and are reproduced in table 3-1.

The close observation of table 3-1 shows that the permissible shear stress τ_c in concrete depends on the percentage of tension steel. This is because:

(1) When amount of tension steel increases, the depth of neutral axis increases and thus, the depth of uncracked concrete increases. This increases the capacity of concrete in shear.

(2) When amount of tension steel increases, the cracks formed are smaller, which improves the aggregate interlock. Also because of larger steel area the dowel action is improved. This improves the capacity of section in shear.

TABLE 3-1

PERMISSIBLE SHEAR STRESS IN CONCRETE

$\frac{100A_s}{bd}$	Permissible shear stress in concrete, τ_c, N/mm² Grade of concrete					
	M15	M20	M25	M30	M35	M40
(1)	(2)	(3)	(4)	(5)	(6)	(7)
0·25	0·22	0·22	0·23	0·23	0·23	0·23
0·50	0·29	0·30	0·31	0·31	0·31	0·32
0·75	0·34	0·35	0·36	0·37	0·37	0·38
1·00	0·37	0·39	0·40	0·41	0·42	0·42
1·25	0·40	0·42	0·44	0·45	0·45	0·46
1·50	0·42	0·45	0·46	0·48	0·49	0·49
1·75	0·44	0·47	0·49	0·50	0·52	0·52
2·00	0·44	0·49	0·51	0·53	0·54	0·55
2·25	0·44	0·51	0·53	0·55	0·56	0·57
2·50	0·44	0·51	0·55	0·57	0·58	0·60
2·75	0·44	0·51	0·56	0·58	0·60	0·62
3·00 and above	0·44	0·51	0·57	0·60	0·62	0·63

Note: A_s is that area of longitudinal tension reinforcement which continues at least one effective depth beyond the section being considered except at supports where the full area of tension reinforcement may be used provided the detailing conforms code requirements.

When the longitudinal bars are not required to resist moment, they are usually curtailed or bent up. If the bars are curtailed, at the point of curtailment, this creates complicated stresses which reduce the shear capacity of the section. Therefore using table 3-1, any longitudinal bars which are terminated within a distance '*d*' of the section under consideration, shall not be considered for calculation of 100 A_s/bd. This is the reason of the foot-note of table 3-1.

If the nominal shear stress does not exceed the value of τ_c found from table 3-1, the section is safe for shear and shear reinforcements are not required theoretically. However, some minimum shear reinforcements have to be provided as discussed in art. 3-7. If the nominal shear stress exceeds the permissible shear stress τ_c, the section shall be suitably reinforced with shear reinforcements. This is explained in art. 3-6. Note that shear reinforcements are provided against diagonal tension. As the concrete is strong in compression, generally a beam is safe for diagonal compression. However, for any beam, suitably reinforced against diagonal tension, the nominal shear shall not exceed the values given in table 18 of IS: 456 and reproduced in table 3-2. By this provision the failure of the beam by diagonal compression is prevented. For any section if nominal shear stress exceeds the maximum permissible value, the section shall be redesigned either using higher concrete mix or providing more depth.

TABLE 3-2

MAXIMUM SHEAR STRESS, $\tau_{c\ max}$, N/mm²

Concrete grade	M15	M20	M25	M30	M35	M40
$\tau_{c\ max}$, N/mm²	1·6	1·8	1·9	2·2	2·3	2·5

3-6. Shear reinforcements in beams: As shown earlier, because of the shear stresses, there are cracks in concrete perpendicular to the diagonal tension. Therefore a beam shall be suitably reinforced along the diagonal tension i.e. across the crack. This can be achieved by using vertical stirrups or inclined bent bars crossing the cracks. According to IS: 456, the shear reinforcements shall be designed to carry the shear equal to

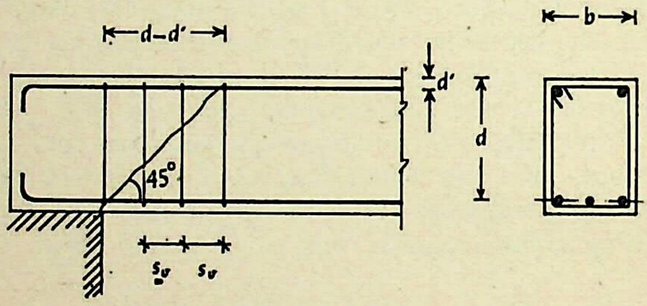
$$V - \tau_c\, bd,$$

where

V = shear force due to design loads

$\tau_c\, bd$ = shear resistance of concrete without shear reinforcement.

(*a*) *Vertical stirrups:* Stirrups are the commonly used shear reinforcements. Applied shear does not produce any stress in the shear reinforcement unless the concrete is cracked in diagonal tension. When the concrete starts cracking, any shear reinforcement crossing the crack is stressed. This means that any shear reinforcement not crossing the crack essentially remains unstressed. An R.C.C. beam reinforced in shear with vertical stirrups spaced at a distance s_v apart and crossing a 45° shear crack is shown in fig. 3-6.



(a) Elevation (b) Section

Vertical stirrup as shear reinforcement

FIG. 3-6

Let

A_{sv} = area of legs of stirrups

σ_{sv} = permissible tensile stress in shear reinforcement as given in table 16 of IS: 456 or table 1-4 of this book, and

V_s = the strength of shear reinforcement.

Now,

horizontal length of crack $= d - d' \simeq d$ (d' is neglected).

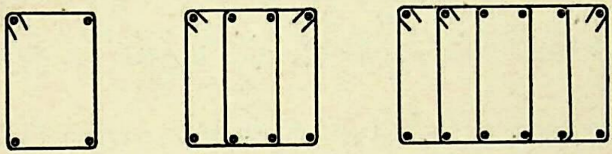
Then no. of stirrups crossing the crack $= \dfrac{d}{s_v}$ and shear taken by the stirrups $= \dfrac{d}{s_v} \times A_{sv} \times \sigma_{sv}$. This shall be equal to the required strength of shear reinforcement.

$$\therefore \qquad V_s = \frac{d}{s_v} \times A_{sv} \times \sigma_{sv}$$

$$\text{or} \qquad V_s = \frac{\sigma_{sv} \cdot A_{sv} \cdot d}{s_v} \qquad \text{(3-4)}$$

This formula is given in IS: 456.

The stirrups are usually two-legged. However four-legged, six-legged etc. stirrups also can be formed as shown in fig. 3-7. More than two-legged stirrups are used for heavy shear. Unless otherwise specified, stirrups are always two-legged.



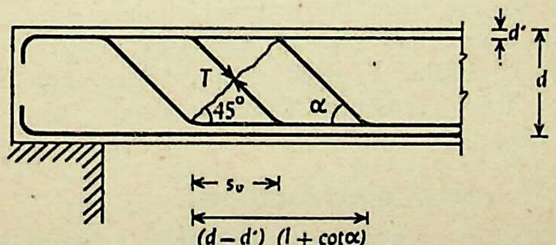
(a) Two-legged stirrup   (b) Four-legged stirrup   (c) Six-legged stirrup

Types of stirrups

FIG. 3-7

(*b*) *Inclined stirrups:* A series of inclined stirrups near the support crossing a 45° crack is shown in fig. 3-8. If A_{sv} is the area of inclined stirrup and σ_{sv} is the permissible tensile stress in the shear reinforcements, the tensile force in a bar or pair of bars

$$= \sigma_{sv} . A_{sv}.$$



Inclined stirrups as shear reinforcement

FIG. 3-8

No. of inclined stirrups crossing the crack

$$= \frac{(d-d')\ (1+\cot\alpha)}{s_v}$$

$$= \frac{(1+\cot\alpha)\ d}{s_v}$$ neglecting value of d.

Tension in bar

$$= \frac{(1 + \cot\alpha)\ d}{s_v} \times \sigma_{sv} \times A_{st}.$$

Component of this force effective in shear

$$= \frac{(1 + \cot\alpha)\ d}{s_v} \cdot \sigma_{sv} \cdot A_{sv} \cdot \sin\ \alpha.$$

This shall be equal to V_s, the strength of the stirrups required.

Thus for a series of inclined stirrups,

$$V_s = \frac{\sigma_{sv} \cdot A_{sv}\ \sin\alpha\ (1 + \cot\alpha)\ d}{s_v}$$

$$\therefore \quad V_s = \frac{\sigma_{sv}\ A_{sv} \cdot d}{s_v}\ (\sin\alpha + \cos\alpha) \qquad \text{(3-5)}$$

For a single bar or a group of bars, all bent up at the same section,

$$V_s = \sigma_{sv} \cdot A_{sv} \cdot \sin\alpha \qquad \text{(3-6)}$$

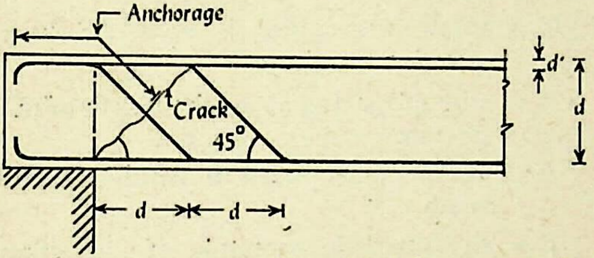
The formulae 3-5 and 3-6 are the same as given in IS : 456.

The bent bars alone are not satisfactory for the shear reinforcements. This is because the exact behaviour of bent bars in resisting shear is not clearly understood. Also the bent bars do not resist the reversal of shear force. IS : 456 states that when bent bars are provided, their contribution towards shear resistance shall not be more than half of the total shear reinforcement. The remaining shear shall be taken by stirrups.

3-7. Practical considerations:

(*a*) *Distance of first bent from support:* Usually the bars are bent at 45° or 60° to the longitudinal axis of the beam. In this book, unless specified, a bent up bar means that it is bent at 45°. The minimum and maximum limit of the horizontal distance of a bent bar is shown in fig. 3-9. Note that if a bar is bent at a distance from the support, less than d, the bent will enter the support and this is not practical. Therefore the first bent, if its contribution to shear at the support, is to be counted, should start at a distance from support between d and $2d$. Value of d' is neglected.

It is shown in art. 3-13 that the bent bar shall be anchored in compression zone for a length equal to development length of the bent bar. The anchorage is shown in fig. 3-9. Thus, a bar shall be bent such that it satisfies this criteria also.



Limits of distance of first bent from support

FIG. 3-9

Design engineer may choose this value satisfactory to him. One such method is to bent the first bar at a distance equal to 1·25 D from the face of the support where D is the overall depth of the beam. The use of overall depth is made to simplify the calculations and to have a uniformity for number of beams designed in one particular job.

Note that if a second bar or a group of bars is to be bent, this shall have to start at a distance d or less than d from the first bent bar as restricted by the code and is explained below. However, for the practical purposes distance d may be approximated to overall depth D.

(*b*) *Maximum spacing:* The shear reinforcements are provided to prevent the shear cracks in the beam. The horizontal distance between two successive cracks is approximately equal to the effective depth 'd'. A stirrup shall be provided such that it crosses the crack and also no crack shall remain unreinforced. To ensure this it is stated in clause no. 25.5.1.5 of IS: 456 that, "the maximum spacing of shear reinforcement measured along the axis of the member shall not exceed 0·75d for vertical stirrups and 'd' for inclined stirrups at 45° where d is the effective depth of the section under consideration. In no case shall the spacing exceed 450 mm".

(*c*) *Minimum shear reinforcement:* Minimum shear reinforcement in the form of vertical stirrups shall be provided such that

$$\frac{A_{sv}}{b_{sv}} \geqslant \frac{0{\cdot}4}{f_y}$$

where A_{sv} = total cross-sectional area of stirrup legs effective in shear

s_v = stirrup spacing along the length of the member in mm

b = width of the beam or width of rib of flanged beam

f_y = characteristic strength of the stirrup reinforcement in N/mm^2 which shall not be taken greater than 415 N/mm^2.

However, in the members of minor structural importance such as lintels or where the maximum shear stress is less than half the permissible value, this provision need not be complied with.

3-8. Use of design aids:

(*a*) *Vertical stirrups:* For vertical stirrups,

$$V_s = \frac{\sigma_{sv} \cdot A_{sv} \cdot d}{s_v}$$

$$\therefore \quad \frac{V_s}{d} = \frac{\sigma_{sv} \cdot A_{sv}}{s_v}$$

For a given bar type and diameter the value of $\sigma_{sv} \cdot A_{sv}$ is constant. Therefore a graph of $\frac{V_s}{d}$ against s_v can be drawn or for different values of $\frac{V_s}{d}$ the spacing s_v can be tabulated. This is done in table 3-3 for two-legged stirrups.

(*b*) *Inclined stirrups:* In practice a series of bent bars is rarely used. For a single bent bar,

$$V_s = \sigma_{sv} \cdot A_{sv} \cdot \sin\alpha.$$

For different bars and for $\alpha = 45°$ and $\alpha = 60°$, values of V_s are tabulated in table 82 of SP: 16 for ready reference.

TABLE 3-3

VALUES OF $\frac{V_s}{d}$ FOR TWO-LEGGLED STIRRUPS IN N/MM

Stirrup spacing s_v in mm	σ_{sv} = 140 N/mm² Diameter in mm				σ_{sv} = 230 N/mm² Diameter in mm			
	6	8	10	12	6	8	10	12
50	158·3	281·5	439·8	633·3	260·1	462·4	722·6	1040·5
60	131·4	234·6	366·5	527·8	216·8	385·4	602·1	867·1
70	113·1	201·1	314·2	452·4	185·8	330·3	516·1	743·2
80	99·0	175·9	274·9	395·8	162·6	289·0	451·6	650·3
90	88·0	156·4	244·3	351·9	144·5	256·9	401·4	578·1
100	79·2	140·7	219·9	316·7	130·1	231·2	361·3	520·2
110	72·0	127·9	199·9	287·9	118·2	210·2	328·4	473·0
120	66·0	117·3	183·3	263·9	108·4	192·7	301·2	433·5
130	60·9	108·3	169·2	243·6	100·0	177·9	277·9	400·2
140	56·5	100·5	157·1	226·2	92·0	165·2	258·0	371·6
150	52·8	93·8	146·6	211·1	86·7	154·1	240·9	346·8
160	49·5	88·0	137·4	197·9	81·3	144·5	225·8	325·2
170	46·6	82·8	129·4	186·3	76·5	136·0	212·5	306·0
180	44·0	78·2	122·2	175·9	72·3	128·5	200·7	289·0
190	41·7	74·1	115·7	166·7	68·5	121·7	190·1	273·8
200	39·6	70·4	110·0	158·3	65·0	115·6	180·6	260·1
250	31·7	56·3	88·0	126·7	52·0	92·5	144·5	208·1
300	26·4	46·9	73·3	105·6	43·2	77·1	120·4	173·4
350	22·6	40·2	62·8	90·5	37·2	66·1	103·2	148·6
400	19·8	35·2	55·0	79·2	32·5	57·8	90·3	130·1
450	17·6	31·3	48·9	70·4	28·9	51·4	80·3	115·6

(*c*) *Minimum shear reinforcements:* A close study of formula given by code shows that minimum shear reinforcements depend solely on the width of the beam. For different bars normally used as shear reinforcements and for different width of beams generally used in practice the maximum spacing of two-legged stirrups is tabulated in table 3-4.

TABLE 3-4
MINIMUM SHEAR REINFORCEMENTS (TWO LEGGED)

Bar dia ↓ | Beam width→	150	175	200	230	250	300	350	400	450
	Maximum spacing in mm for two legged stirrups								
Fe250									
6 ϕ	230	200	175	150	140	115	100	85	75
8 ϕ	415	350	300	270	250	200	175	150	135
10 ϕ	450	450	450	425	395	325	280	240	210
Fe415									
8 ϕ	450	450	450	450	415	345	295	250	230

Note: (a) Value of s_v in this table is rounded off to 5 mm.
(b) In any case $s_v \not> 0{\cdot}75\,d$ or $s_v \not> 450$ mm.

Example 3-1.

A simply supported tee beam 230 mm wide × 460 mm effective depth is reinforced with 5 no. 16 mm diameter bars as tension reinforcement. The beam is subjected to a shear of 35 kN at support. Check the shear stresses and design the shear reinforcement at support. The materials are M15 grade concrete and mild steel reinforcement.

Solution:

$$\text{At support } V = 35 \text{ kN}$$

$$A_{st} = 5 \times 201 = 1005 \text{ mm}^2, \; b = 230 \text{ mm}, \; d = 460 \text{ mm}.$$

$$\text{Shear stress } \tau_v = \frac{35 \times 10^3}{230 \times 460} = 0{\cdot}33 \text{ N/mm}^2.$$

$$\frac{100\,A_s}{bd} = \frac{100 \times 1005}{230 \times 460} = 0{\cdot}95.$$

Note that A_{st} denotes steel area at maximum bending moment while A_s denotes the area of steel which continues at least one effective depth beyond the section being considered for checking the shear.

For $\frac{100\ A_s}{bd} = 0{\cdot}95$ from table 5-1 τ_c shall be calculated by interpolation.

For $\frac{100\ A_s}{bd} = 0{\cdot}75$, $\tau_c = 0{\cdot}34$ N/mm²,

and $\frac{100\ A_s}{bd} = 1{\cdot}0$, $\tau_c = 0{\cdot}37$ N/mm².

$$\therefore \quad \text{For } \frac{100\ A_s}{bd} = 0{\cdot}95, \quad \tau_c = 0{\cdot}34 + \frac{0{\cdot}37 - 0{\cdot}34}{1 - 0{\cdot}75}(0{\cdot}95 - 0{\cdot}75)$$

$$= 0{\cdot}364 \text{ N/mm}^2$$

$$\therefore \quad \tau_c = 0{\cdot}364 \text{ N/mm}^2.$$

Now, $\tau_v < \tau_c$, therefore only nominal shear reinforcement is required.

Select 6 mm diameter M.S. bars for stirrups.

$A_{sv} = 2 \times 28 = 56$ mm² for two-legged stirrups.

For minimum shear reinforcement,

$$\frac{A_{sv}}{bs_v} \geqslant \frac{0{\cdot}4}{f_y}$$

$$\frac{56}{230 s_v} \geqslant \frac{0{\cdot}4}{250}$$

$$s_v \leqslant \frac{56 \times 250}{230 \times 0{\cdot}4} \quad \therefore \quad s_v \leqslant 152 \text{ mm.}$$

The spacing shall be lesser of

(a) $0{\cdot}75\ d = 0{\cdot}75 \times 460 = 345$ mm

(b) 450 mm

(c) 152 mm as calculated above.

Provide 6 mm dia. two-legged stirrups about 150 mm c/c.

Alternatively minimum shear reinforcement may be selected from table 3-4.

Example 3-2.

If the shear of above section is increased to 60 kN, check the shear stresses and find the spacing of 6 mm dia. stirrups at the support.

Solution:

$$V = 60 \text{ kN}, \qquad b = 230 \text{ mm}, \qquad d = 460 \text{ mm}.$$

$$\text{Shear stress } \tau_v = \frac{60 \times 10^3}{230 \times 460}$$

$$= 0{\cdot}567 \text{ N/mm}^2 < 1{\cdot}6 \text{ N/mm}^2$$

where 1·6 N/mm² is the maximum permissible shear stress for M15 mix.

$\tau_c = 0{\cdot}364$ N/mm² as calculated in Example 3-1.

Now, $\tau_v > \tau_c$, therefore shear reinforcement shall be designed.

Shear resistance of concrete

$$\tau_c\, bd = 0{\cdot}364 \times 230 \times 460 \times 10^{-3}$$

$$= 38{\cdot}5 \text{ kN}.$$

Shear to be resisted by stirrups

$$= 60 - 38{\cdot}5 = 21{\cdot}5 \text{ kN}.$$

Using 6 mm dia. two-legged stirrups

$$A_{sv} = 56 \text{ mm}^2$$

$$s_v = \frac{\sigma_{sv} \cdot A_{sv} \cdot d}{V_s} = \frac{140 \times 56 \times 460}{21{\cdot}5 \times 10^3}$$

$$= 167 \text{ mm}.$$

The spacing shall be lesser of

(a) $0{\cdot}75\ d = 0{\cdot}75 \times 460 = 345$ mm

(b) 450 mm

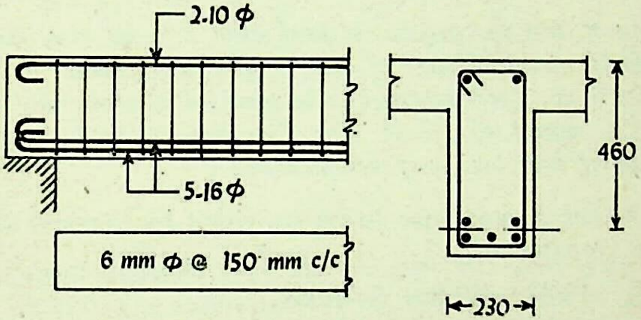
(c) minimum shear reinforcement required i.e. 152 mm as shown in Example 3-1.

(d) required spacing = 167 mm.

∴ Provide 6 mm ϕ two-legged stirrups about 150 mm c/c at support. This is shown in fig. 3-10. Use 2-10 ϕ as anchor bars.

Note: In Example 3-1, $\tau_v < \tau_c$ and minimum shear reinforcement was required. Therefore two-legged stirrups of 6 mm dia. about 150 mm c/c were provided. In Example 3-2, $\tau_v > \tau_c$ and still designed shear reinforcement are same as minimum required. This is because in the first case, the

contribution of stirrups was not considered while in second case it is considered. The difference between two cases is that in the first case $\tau_v < \tau_c$ and hence, *minimum shear reinforcement* was provided while in second case $\tau_v > \tau_c$ and *designed reinforcement* (*may be equal to but not less than minimum required*) is provided.



(a) Elevation (b) Section

FIG. 3-10

Example 3-3.

In Example 3-2, if 2 no. 16 mm dia. bars are bent up at 45° and shear is increased to 80 kN, find the spacing of 6 mm dia. stirrups at support.

Solution:

2 bars bent up $A_{sv} = 2 \times 201 = 402$ mm².

Shear resistance $= 402 \times 140 \sin 45 \times 10^{-3}$ kN
$= 39{\cdot}8$ kN.

For the remaining 3 bars,

$$\frac{100A_s}{bd} = \frac{100 \times 603}{230 \times 460} = 0{\cdot}57$$

$$\tau_c = 0{\cdot}304 \text{ N/mm}^2.$$

Shear resistance of concrete,

$$\tau_c\, bd = 0{\cdot}304 \times 230 \times 460 \times 10^{-3} = 32{\cdot}16 \text{ kN}.$$

Shear resistance to be provided by shear reinforcement
$= 80 - 32{\cdot}16 = 47{\cdot}84$ kN.

Now 2 bent bars can provide a shear of 39·8 kN, however according to IS: 456, "When bent up bars are provided, their contribution towards shear resistance shall not be more than half that of the total shear reinforcement". This means, shear resistance of bent up bars in this case shall not be considered more than $\frac{47 \cdot 84}{2}$ kN i.e. 23·92 kN. (*In some cases, full resistance of bent up bars can be used when it is not more than half the required shear resistance of total shear reinforcement. If suppose in this case the shear resistance to be provided by shear reinforcement is 85 kN instead of 47·84 kN, then full contribution of shear resistance of bent bars may be considered.*)

The shear resistance to be provided by vertical stirrups is then, greater of,

(a) $47 \cdot 84 - 39 \cdot 8 = 8 \cdot 04$ kN.

(b) $\frac{1}{2} \times 47 \cdot 84 = 23 \cdot 92$ kN.

Provide stirrups to resist a shear of 23·92 kN. Using 6 mm dia. two-legged stirrups,

$$A_{sv} = 56 \text{ mm}^2,$$

$$s_v = \frac{56 \times 140 \times 460}{23 \cdot 92 \times 10^3}$$

$$= 150 \cdot 8 \text{ mm.}$$

The spacing shall be lesser of

(a) $0 \cdot 75 \times 460 = 345$ mm

(b) 450 mm

(c) 152 mm (minimum shear reinforcement)

(d) 150·8 mm (required).

∴ Provide 6 mm dia. stirrups about 150 mm c/c.

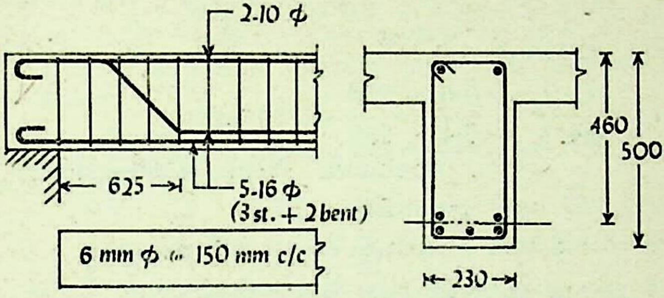
Note: It is not necessary to write "two-legged stirrups" always. Unless specified, the stirrups shall be considered as two-legged.

The bars are bent up at $1 \cdot 25D$, where

$D = 460 + 40$ (assumed cover) $= 500$ mm

i.e. $1 \cdot 25 \times 500 = 625$ mm.

The elevation and section of the beam is shown in fig. 3-11.



(a) Elevation (b) Section

FIG. 3-11

Example 3-4.

A simply supported tee beam of 8 m span is carrying a load of 25 kN/m. The section of beam is 230 mm wide × 500 mm effective depth. It is reinforced with 6 no. 20 mm dia. bars. The materials are M15 grade concrete and tor steel reinforcement of grade Fe 415. Design the shear reinforcement using vertical stirrups.

Solution:

$$\text{Max. S.F.} = 25 \times \frac{8}{2} = 100 \text{ kN.}$$

$$\text{Shear stress } \tau_v = \frac{100 \times 10^3}{230 \times 500} = 0{\cdot}87 \text{ N/mm}^2 < 1{\cdot}6 \text{ N/mm}^2.$$

$$\frac{100\ A_s}{bd} = \frac{100 \times 6 \times 314}{230 \times 500} = 1{\cdot}64$$

$$\tau_c = 0{\cdot}431 \text{ N/mm}^2.$$

Shear resistance of concrete

$$= 0{\cdot}431 \times 230 \times 500 \times 10^{-3} = 49{\cdot}5 \text{ kN.}$$

Shear resistance to be provided by stirrups

$$V_s = 100 - 49{\cdot}5 = 50{\cdot}5 \text{ kN.}$$

Using 8 mm φ tor steel stirrups $A_{sv} = 100$ mm².

$$s_v = \frac{\sigma_{sv} \cdot A_{sv} \cdot d}{V_s}$$

$$= \frac{230 \times 100 \times 500}{50{\cdot}5 \times 10^3} = 227 \text{ mm.}$$

The spacing shall be lesser of

(a) $0{\cdot}75\ d = 0{\cdot}75 \times 500 = 375$ mm

(b) 450 mm

(c) $\dfrac{A_{sv} \cdot f_y}{0{\cdot}4\ b} = \dfrac{100 \times 230}{0{\cdot}4 \times 230} = 250$ mm
(minimum shear reinforcement)

(d) 227 mm (required).

Provide 8 mm ϕ stirrups about 225 mm c/c.

To find zone of minimum shear reinforcement:

Shear resistance of concrete = 49·5 kN. Referring fig. 3-12(b), central portion *AB*, is having a shear force less than 49·5 kN and in this portion, theoretically no shear reinforcement is required. However minimum shear reinforcement as required by code shall be provided in portion *AB*. This is 8 mm ϕ about 250 c/c for this case. Zone *AB* may be called as zone of minimum shear reinforcement.

Just left to point *A* (or right to point *B*) towards the support, the shear stress exceeds τ_c and shear reinforcement shall be designed. Here now resistance of shear reinforcement is considered and upto certain distance 8 mm ϕ about 250 c/c stirrups are sufficient. Let us find out this distance.

Shear resistance provided by minimum shear reinforcement

$$V_s = \frac{\sigma_{sv} \cdot A_{sv} \cdot d}{s_v} = \frac{230 \times 100 \times 500}{250} \times 10^{-3}$$
$$= 46 \text{ kN.}$$

Shear resistance of concrete = 49·5 kN.

Total shear resistance of section with minimum shear reinforcement

$$= 46 + 49{\cdot}5 = 95{\cdot}5 \text{ kN.}$$

Distance from supports of a section with a shear of

$$95{\cdot}5 \text{ kN} = \frac{100 - 95{\cdot}5}{25} = 0{\cdot}18 \text{ m} = 180 \text{ mm.}$$

Thus in zone *PQ* designed stirrups are that minimum required. Therefore, stirrups designed at support shall be provided in zone *RP* or *SQ*.

Provide 8 mm ϕ stirrups about 225 c/c upto 450 mm. Then provide 8 mm ϕ stirrups about 250 c/c being minimum shear reinforcements. 450 mm distance is used to have at least 3 stirrups to create sufficient difference between two kinds of stirrups.

Beam loading, S.F. diagram, beam elevation and section are shown in fig. 3-12.

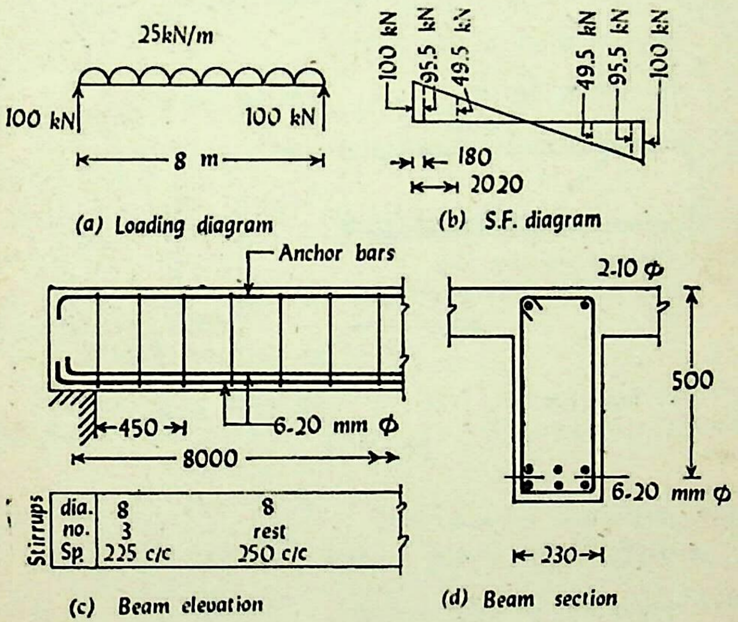


Fig. 3-12

Example 3-5.

The shear force diagram and section properties of a beam is shown in fig. 3-13. Design the shear reinforcement. The materials are M15 grade concrete and tor steel reinforcement of grade Fe 415.

At support, $V = 90$ kN.

$$\tau_v = \frac{90 \times 10^3}{200 \times 400} = 1{\cdot}125 \text{ N/mm}^2$$

$$< 1{\cdot}6 \text{ N/mm}^2 \ldots\ldots\ldots \text{(O.K.)}$$

At support $A_s = 4 \times 314 = 1256 \text{ mm}^2$

$$\frac{100\ A_s}{bd} = \frac{100 \times 1256}{200 \times 400} = 1{\cdot}57$$

$$\tau_c = 0{\cdot}425 \text{ N/mm}^2.$$

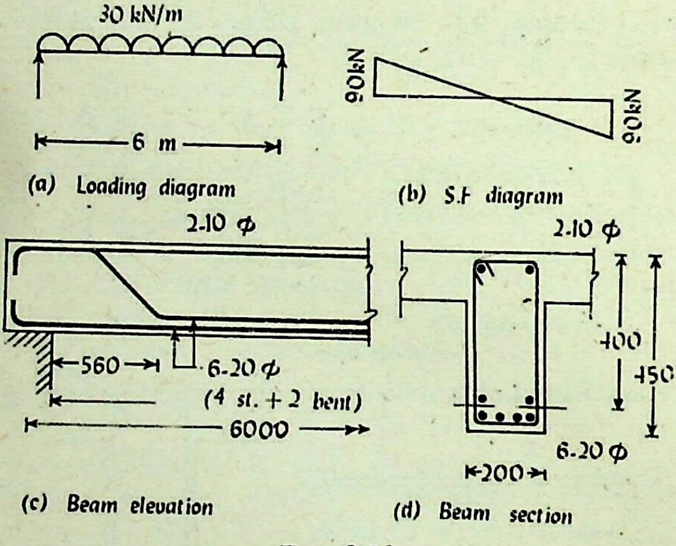


FIG. 3-13

Shear resistance of concrete

$$= 0{\cdot}425 \times 200 \times 400 \times 10^{-3} = 34 \text{ kN}.$$

Shear resistance of 2-20 ϕ bars

$$= 2 \times 314 \times 230 \times 0{\cdot}707 \times 10^{-3} = 102 \text{ kN}.$$

Shear resistance to be provided by shear reinforcement

$$= 90 - 34 = 56 \text{ kN}.$$

Bent bars provide 50% = 28 kN < 102 kN........(O.K.)

Stirrups provide 50% = 28 kN.

Using 8 mm Φ stirrups, $A_{sv} = 100 \text{ mm}^2$.

$$s_v = \frac{\sigma_{sv} \cdot A_{sv} \cdot d}{V_s} = \frac{230 \times 100 \times 400}{28 \times 10^3} = 328 \text{ mm}.$$

Note: Φ *is used to denote tor steel diameter. The notation consists the letters T, O and R.*

The spacing shall be lesser of

(a) $0{\cdot}75\ d = 0{\cdot}75 \times 400 = 300$ mm

(b) 450 mm

(c) required 328 mm

(d) $\dfrac{A_{sv} \cdot f_y}{0{\cdot}4\ b} = \dfrac{100 \times 230}{0{\cdot}4 \times 200} = 287{\cdot}5$ mm.

Provide 8 mm ϕ two-legged stirrups about 280 c/c. Thus minimum shear reinforcement is required. (While using SI units in drawing, it is customary to drop word mm. Thus 280 c/c means 280 mm c/c. In structural drawing a note for this is written as "All dimensions are in mm".)

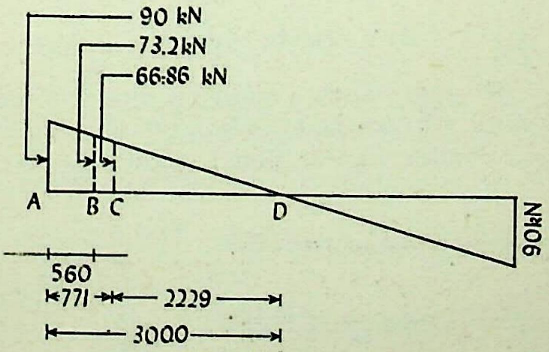
Now shear resistance provided by minimum shear reinforcement

$$= \frac{\sigma_{sv} \cdot A_{sv} \cdot d}{s_v} = \frac{230 \times 100 \times 400}{280} \times 10^{-3} = 32{\cdot}86 \text{ kN.}$$

Total resistance of concrete with minimum shear reinforcement

$$= 34 + 32{\cdot}86 = 66{\cdot}86 \text{ kN.}$$

Referring to S.F. diagram of fig. 3-14, upto 771 mm, the shear reinforcement more than minimum is required.



S.F. diagram

Fig. 3-14

Upto 560 mm from support, the contribution of bent bars is available and minimum shear reinforcements were proved sufficient. However at point B, there is a necessity of designing shear reinforcements.

At point B, S.F. $= 90 - 0{\cdot}56 \times 30 = 73{\cdot}2$ kN.

At this section there are total six bars, out of which two bars are bent at 45°. According to the foot-note of table 5-1 for calculation of $\frac{100\,A_s}{bd}$ at any section, A_s to be used is the area of continuing bars which continues at least one effective depth from the section under consideration. Therefore at point B, only 4 bars will be used for calculating $\frac{100\,A_s}{bd}$.

Shear resistance of concrete with 4 bars as calculated above $= 34$ kN, shear resistance to be provided by stirrups $= 73{\cdot}2 - 34 = 39{\cdot}2$ kN.

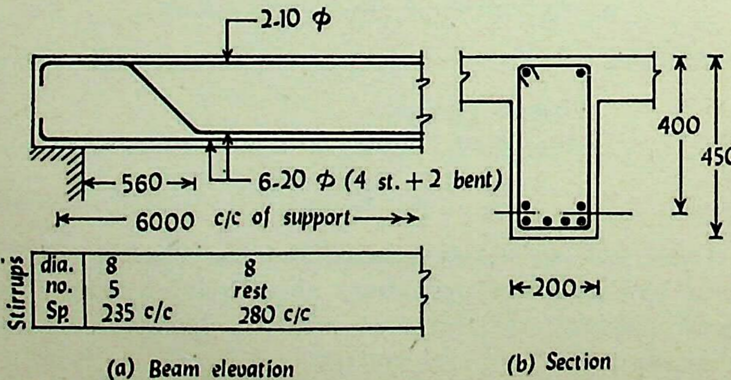
Using 8 ϕ stirrups,

$$s_v = \frac{230 \times 100 \times 400}{39{\cdot}2 \times 10^3} = 235 \text{ mm.}$$

Now this spacing is required only in the region BC i.e. $771 - 560 = 211$ mm.

Providing less shear reinforcement in portion AB than portion BC seems odd in practice. Therefore provide 8 mm ϕ two-legged stirrups about 235 mm c/c in portion AC. No. of stirrups required

$= \frac{771}{235} + 1 = 5$ (say) for a distance equal to $4 \times 235 = 940$ mm. Then provide 8 mm ϕ about 280 mm c/c two-legged stirrups being minimum shear reinforcement. The beam elevation and section is shown in fig. 3-15.



(a) Beam elevation (b) Section

Fig. 3-15

Example 3-6.

A rectangular beam is loaded with 40 kN/m inclusive of its own weight. Span of the beam is 6 m and size of the beam is 230 mm width × 450 mm effective depth. It is reinforced with 4 no. 25 mm dia. bars. Design shear reinforcement using stirrups. The materials are M15 grade concrete and tor steel reinforcements for main steel. Use design tables available.

Solution:

Shear force $V = 40 \times 3 = 120$ kN.

$$\text{Shear stress} = \frac{120 \times 10^3}{230 \times 450} = 1{\cdot}16 \text{ N/mm}^2 < 1{\cdot}6 \text{ N/mm}^2.$$

$$\frac{100\,A_s}{bd} = \frac{4 \times 491 \times 100}{230 \times 450} = 1{\cdot}9$$

$$\tau_c = 0{\cdot}44 \text{ N/mm}^2.$$

Shear resistance of concrete $= 0{\cdot}44 \times 230 \times 450 \times 10^{-3}$
$= 45{\cdot}54$ kN.

Shear resistance to be provided by stirrups

$$V_s = 120 - 45{\cdot}54 = 74{\cdot}46 \text{ kN}.$$

$$\frac{V_s}{d} = \frac{74{\cdot}46 \times 10^3}{450} = 165{\cdot}4 \text{ N/mm}.$$

From table 3-3 use 8 mm Φ two legged stirrups about 135 c/c where $V_s = 171{\cdot}6$ (value is interpolated).

From table 3-4 minimum shear reinforcement for 230 wide beam = 6 ϕ about 150 c/c, $\frac{V_s}{d} = 52{\cdot}78$ (ϕ = mild steel diameter).

$\therefore\quad V_s = 52{\cdot}78 \times 450 \times 10^{-3} = 23{\cdot}75$ kN.

Shear capacity of section with minimum shear reinforcements

$$= 45{\cdot}54 + 23{\cdot}75 = 69{\cdot}29 \text{ kN}$$

i.e. stirrups more than minimum are required from support for a distance equal to

$$\frac{120 - 69{\cdot}29}{40} = 1{\cdot}27 \text{ m}.$$

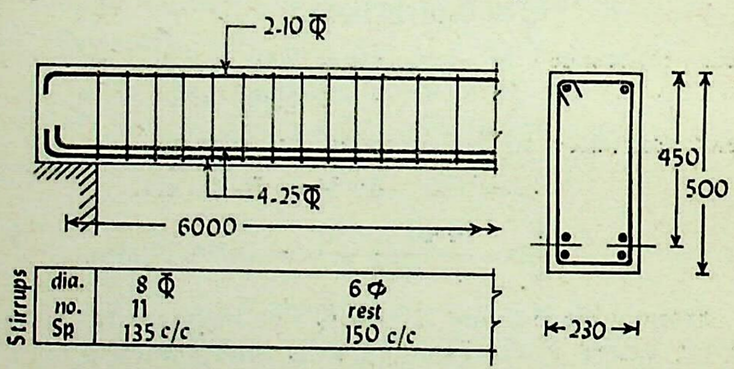
$$\text{No. of stirrups required} = \frac{1270}{135} + 1 = 11 \text{ (say)}.$$

Provide 8 mm Ⴔ two-legged stirrups about 135 c/c, 11 no. from support and then 6 mm ϕ (mild steel) about 150 c/c in remaining portion.

Note that for main steel, tor steel bars are used and for secondary steel (here stirrups), tor steel and mild steel are used. Different grade of steel for main and secondary reinforcement is permitted by clause 25·1 of IS : 456 where it is stated,

"Reinforcing steel of same type and grade shall be used as main reinforcement in a structural member. However, simultaneous use of two different types or grades of steel for main and secondary reinforcement respectively is permissible".

The reinforcements are shown in fig. 3-16.



(a) Elevation (b) Section

Fig. 3-16

Supplementary notes:

(1) If the beam is resting on masonary wall, the first stirrup may enter the support. However, when the beam is resting on column, because of the column bars and ties this is not possible. Therefore the first stirrup shall be placed at the face of support.

(2) Theoretically speaking, the spacing of stirrups from support to centre can be increased continuously. However, in practice the portion where more than minimum shear

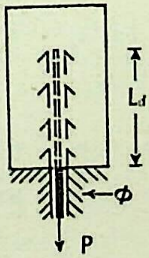
reinforcement required is divided in convenient parts and shear reinforcement is designed. This is done if this portion is sufficiently large and division leads to an economy.

(3) In zone of minimum shear reinforcement also, the spacing of stirrups can be increased where shear stress is less than $\frac{\tau_c}{2}$.

DEVELOPMENT LENGTH

3-9. Introductory: While analysing and designing the reinforced concrete structures, the basic assumption is that there is a perfect bond between concrete and steel i.e. there is absolutely no slippage between the concrete and steel. The grip of the reinforcement and concrete due to adhesion or bearing is termed as *bond*. A length of reinforcement embedded in concrete so that it can develop the stress by bond is termed as *development length.*

3-10. Development length: Pull out test: To find out the development length, consider a pull out test as indicated in fig. 3-17.



Pull out test

FIG. 3-17

A rod of diameter ϕ is embedded in concrete. An axial pull is applied and increased uniformly. Pull P is noted when the bar is pulled out. If diameter is known, the failure stress in the bar can be found out. Applying factors of safety at working load, the bond resistance of concrete and strength of the bar can be equated. The pull out test indicates that the maximum stress is induced in the materials where the bar enters the concrete and it diminishes to zero at the end of the bar. However, for all practical purposes average bond resistance is considered.

Let τ_{bd} = Average permissible bond stress in concrete

σ_s = working stress in bar

ϕ = diameter of bar.

Then equating bond resistance of concrete = strength of bar

$$\therefore \quad \tau_{bd} \times \pi\phi \times L_d = \sigma_s \cdot \frac{\pi\phi^2}{4}$$

$$\therefore \quad L_d = \frac{\phi\sigma_s}{4\tau_{bd}} \qquad \text{(3-7)}$$

where L_d is known as *development length.*

3-11. Code provisions: According to IS : 456, the bond stresses are assumed to be uniform over the effective surface area of the bar. Thus, according to clause 25.2 the calculated tension or compression in any bar at any section shall be developed on each side of the section by an appropriate development length or end anchorage or by a combination thereof, where the development length is defined as,

$$L_d = \frac{\phi\sigma_s}{4\tau_{bd}}$$

where

ϕ = nominal diameter of the bar

σ_s = stress in bar at the section considered at design load, and

τ_{bd} = permissible bond stress (average) given in table 15 of IS : 456 and reproduced in table 3-5.

TABLE 3-5

PERMISSIBLE STRESS IN BOND (AVERAGE) FOR PLAIN BARS IN TENSION

Grade of concrete	M10	M15	M20	M25	M30	M35	M40
τ_{bd} N/mm²	—	0·6	0·8	0·9	1·0	1·1	1·2

Note 1: τ_{bd} shall be increased by 25% for bars in compression.

Note 2: In case of deformed bars confirming to IS : 1139-1966 and IS: 1786-1979, the value of τ_{bd} shall be increased by 40%.

It is also stated in the same clause that development length includes the anchorage values of hook in tension reinforcements. For bars of section other than circular, the development length should be sufficient to develop the stress in the bar by bond.

In the above discussion, ϕ is defined as the nominal diameter of the bar. For the plain bars nominal diameter and actual diameter are same. However, for deformed bars the area is not same at each cross-section. Then average area is considered. Thus, for a particular diameter, say 10 mm ϕ bars, the weight of M.S. bar per metre and weight of deformed bar per metre would be the same. This average area of deformed bar is considered in design and corresponding diameter is known as nominal diameter.

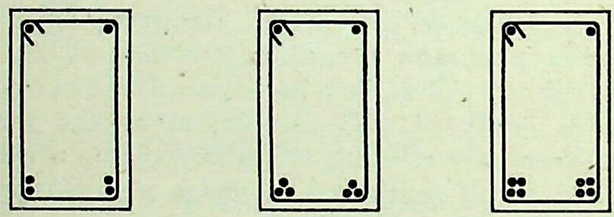
Note 1 of table 3-5 states that for bars in compression τ_{bd} shall be increased by 25%. This is because the end bearing of the bar helps in resisting compression and hence, the allowable bond stresses are increased.

Note 2 of table 3-5 states that for deformed bars τ_{bd} shall be increased by 40%. For deformed bars, the actual contact area of a bar with concrete is taken into account which is much more than contact area based on nominal diameter. This results in increase of permissible bond stress by 40%.

3-12. Use of bundled bars: The use of bundled bars is permitted by the standard. Using bundled bars, concrete can be compacted properly and also the large area of reinforcement can be made to concentrate at a point resulting in the increase of effective depth. According to clause 25.1.1 of IS : 456, "Bars may be arranged singly or in pairs in contact, or in groups of three or four bars bundled in contact. Bundles shall not be used in a member without stirrups. Bundled bars shall be tied together to ensure the bars remaining together. Bars larger than 36 mm diameter shall not be bundled except in columns". The bundled bars are illustrated in fig. 3-18.

The effective perimeter of a single bar in bond is its actual perimeter $\pi\phi$. When the bars are bundled, effective perimeter having a bond with concrete is decreased. This

is shown in fig. 3-19. Thus, total effective perimeter for a group of bars is less than the sum of perimeters of individual bars. Therefore it is stated in clause 25.2.1.2 of IS : 456 that, "The development length of each bar of bundled bars shall be that for the individual bar, increased by 10% for two bars in contact, 20% for three bars in contact and 33% for four bars in contact".



(a) Two-bars in contact (b) Three bars in contact (c) Four bars in contact

Bundled bars

Fig. 3-18

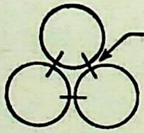


Fig. 3-19

To reduce the effective length of a bar to be anchored, hooks and bends are provided. The rules for anchoring the bars are given in clause 25.2.2 of IS : 456 and are explained below.

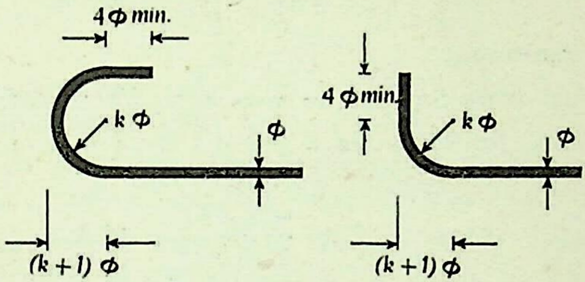
3-13. Anchoring reinforcements: The bars are anchored in tension, compression or shear as shown below:

(*a*) *Anchoring bars in tension:*

(1) Deformed bars may be used without end anchorages provided development length requirement is satisfied. Hooks should be normally provided for plain bars in tension.

(2) Bends: The anchorage value of bend shall be taken as 4 times the diameter of the bars for each 45° bend subject to a maximum of 16 times the diameter of the bar.

(3) Hooks: The anchorage value of a standard U type hook shall be equal to 16 times the diameter of the bar. Standard hook and bend are shown in fig. 3-20.



(a) Standard hook (b) Standard 90° bend

FIG. 3-20

The value of k is given as follows:

Type of steel	Minimum value of k
Mild steel	2
Cold worked steel	4

Note: These details are applicable to all grades of steel.

(*b*) *Anchoring bars in compression:* The anchorage length of straight bar in compression shall be equal to the development length of bars in compression. The projected length of hooks, bends and straight lengths beyond bends if provided for bars in compression, shall be considered for development length.

Example 3-7.

Calculate the anchorage length in tension and compression for a single mild steel bar of diameter ϕ in concrete of grade M15.

Solution:

(1) Tension:

Design stress for M.S. $\sigma_s = \sigma_{st} = 140$ N/mm^2

τ_{bd} for M15 mix $= 0{\cdot}6$ N/mm^2

anchorage length = development length

$$L_d = \frac{\phi\sigma_s}{4\tau_{bd}}$$

$$= \frac{\phi \times 140}{4 \times 0{\cdot}6} = 58{\cdot}3\ \phi.$$

(2) Compression:

Design stress for M.S. $= \sigma_s = \sigma_{sc} = 130$ N/mm².

τ_{bd} for M15 mix $= 0{\cdot}6 \times 1{\cdot}25$ (for compression) $= 0{\cdot}75$ N/mm²

$$L_d = \frac{\phi \times 130}{4 \times 0{\cdot}75} = 43{\cdot}3\ \phi.$$

This development length calculated for column bars in compression may be used for beam bars in compression as in doubly reinforced beam. In beam bars in compression the stress is limited to σ_{sc}. Therefore for all practical purposes, L_d calculated for column bars in compression may be used for beam bars in compression.

Example 3-8.

Calculate the anchorage length of a group of three bundled bars in contact and of equal diameter in tension. Concrete grade is M15 and mild steel reinforcement.

Solution:

$L_d = 58{\cdot}3\ \phi$ for single bar as shown in Example 3-7.

For three bars in bundle, the development length for each bar

$L_d = 58{\cdot}3\ \phi \times 1{\cdot}2 = 69{\cdot}96\ \phi = 70\ \phi$ (say).

Development length for different types of bars in different grades of concrete are given in table 3-6.

TABLE 3-6

(A) DEVELOPMENT LENGTH FOR SINGLE BAR IN TENSION

Tensile stress	Plain bars		Deformed bars	
in bar, N/mm²	M15	M20	M15	M20
130	54 ϕ	41 ϕ	39 ϕ	29 ϕ
140	58 ϕ	44 ϕ	42 ϕ	31 ϕ
190	—	—	57 ϕ	42 ϕ
230	—	—	69 ϕ	51 ϕ

(B) DEVELOPMENT LENGTH FOR SINGLE BAR IN COMPRESSION

Tensile stress in bar, N/mm^2	Plain bars M15	Plain bars M20	Deformed bars M15	Deformed bars M20
130	44 ϕ	33 ϕ	31 ϕ	24 ϕ
140	47 ϕ	35 ϕ	34 ϕ	25 ϕ
190	—	—	46 ϕ	34 ϕ

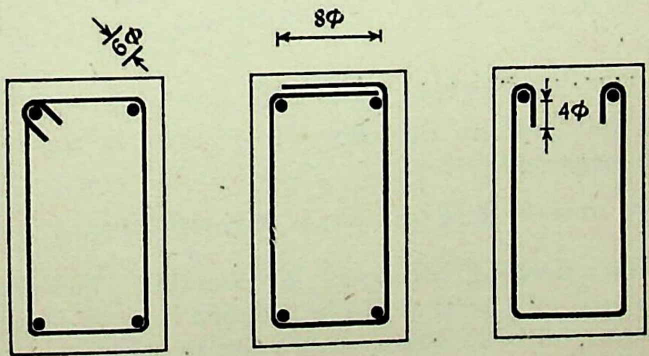
(c) *Anchoring bars in shear:*

(1) *Inclined bars:* The development length shall be as for bars in tension, this length shall be measured as under:

In the tension zone, from the end of the sloping or inclined portion of the bar and

In the compression zone, from the mid-depth of the beam.

(2) *Stirrups:* For the secondary reinforcements such as stirrups and transverse ties in column complete development lengths and anchorage shall be deemed to have been provided when the bar is bent through an angle of at least 90° round a bar of at least its own diameter and is continued beyond the end of the curve for a length of at least eight diameters or when the bar is bent through an angle of 135° and is continued beyond the end of the curve for a length of at least six diameters or when the bar is bent through an angle of 180° and is continued beyond the end of the curve for a length of at least four diameters. The above provisions are illustrated in fig. 3-21.



Anchoring vertical stirrups

FIG. 3-21

Example 3-9.

Check the anchorage of bent bar of Ex. 3-5. 2 bars are bent about 560 mm from support. In tension zone, sufficient length is available. In compression zone, the available anchorage is calculated in fig. 3-22.

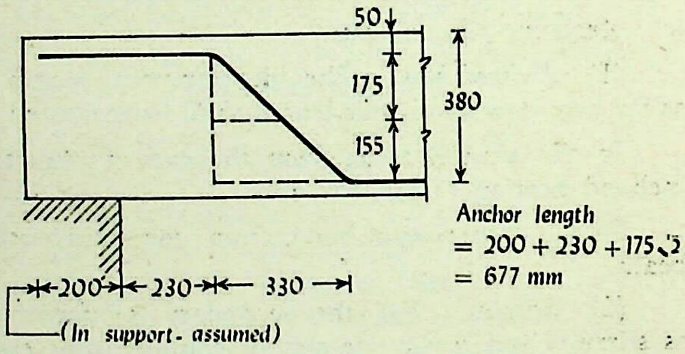


FIG. 3-22

Solution:

Shear taken by two bent bars = 28 kN.

$$\text{Stress in bar} = \frac{28 \times 10^3}{2 \times 314} = 44{\cdot}58 \text{ N/mm}^2.$$

$$L_d = \frac{\phi \times 44{\cdot}58}{4 \times 0{\cdot}6 \times 1{\cdot}4} = 13{\cdot}26\ \phi$$

$$= 13{\cdot}26 \times 20 = 265 \text{ mm.}$$

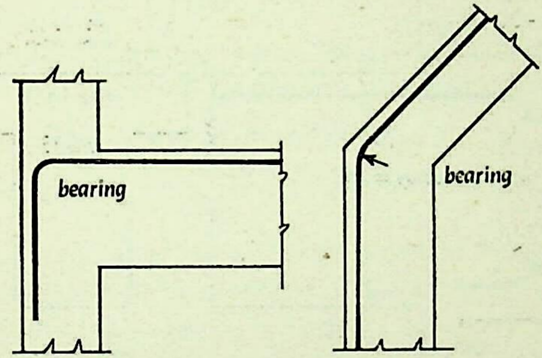
Anchorage provided = 677 mm .... (O.K.)

In this problem the effect of 45° bend in compression zone is neglected.

The anchorage of bent bar is thus satisfactory.

3-14. Bearing stresses at bends: The bearing stresses around a bend are to be checked in accordance with clause 25.2.2.5 of IS : 456, when a bar or a group of bars is bent in a stressed condition. This is illustrated in fig. 3-23.

According to code, "The bearing stress in concrete for bends and hooks described in IS : 2502-1963 (standard bends and hooks) need not be checked. The bearing stress inside a bend in any other bend shall be calculated as given below:



(a) Anchorage of beam bars in column (b) Stressed bar carried round a bend

FIG. 3-23

$$\text{Bearing stress} = \frac{F_{bt}}{r\,\phi} \text{ where}$$

F_{bt} = tensile force due to design loads in a bar or group of bars,

r = internal radius of the bend and

ϕ = size of the bar, or in bundle, the size of bar of equivalent area.

The bearing stress thus calculated shall not exceed

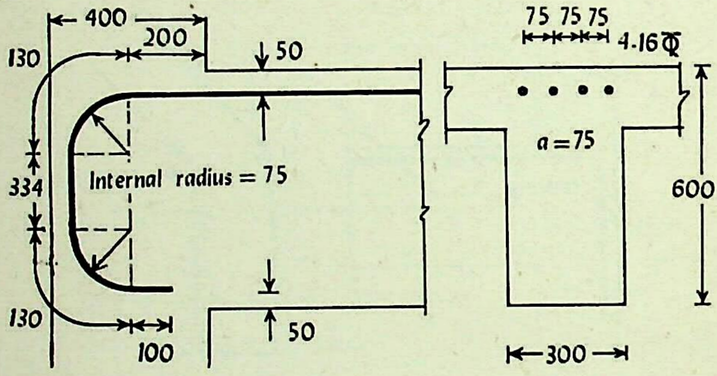
$$\frac{f_{ck}}{1 + 2\phi/a} \text{ where}$$

f_{ck} = characteristic strength of concrete

a = centre to centre distance between bars or group of bars perpendicular to the plane of the bend. For a bar or group of bars adjacent to the face of the member 'a' shall be taken as the cover plus size of bar (ϕ).

Example 3-10.

A beam fixed to the column requires 742 mm² tor steel reinforcements of grade Fe 415 for negative moment at face of column. The reinforcements are anchored in column as shown in fig. 3-24. Check the bearing stress at bend. The concrete is of M20 grade.



(a) Elevation (b) Section

Fig. 3-24

Solution:

A_{st} provided $= 4 \times 201 = 804$ mm²

$$\text{Anchorage required} = \frac{\phi \times 230}{4 \times 0{\cdot}8 \times 1{\cdot}4} \times \frac{742}{804}$$
$$= 47{\cdot}3\ \phi = 758 \text{ mm.}$$

$$\text{Anchorage provided} = 200 + 130 + 334 + 130 + 100$$
$$= 894 \text{ mm} \ldots\ldots\ldots\ldots\ldots \text{(O.K.)}$$

Anchorage provided is sufficient. However bearing stress at bend is to be checked.

From the figure $a = 75$ mm

$$\text{allowable bearing stress} = \frac{f_{ck}}{1 + 2\phi/a}$$
$$= \frac{20}{1 + \dfrac{2 \times 16}{75}} = 14{\cdot}02 \text{ N/mm}^2.$$

At the centre of the upper bend anchorage available

$$= 200 + \frac{130}{2} = 265 \text{ mm.}$$

Stress in bar at the face of column

$$= 230 \times \frac{742}{804} = 212{\cdot}26 \text{ N/mm}^2.$$

The stress in the bar varies linearly and diminishes to zero at 758 mm (required anchorage) from face of support.

Stress at the centre of upper bend

$$= 212{\cdot}26 \times \frac{758 - 265}{758} = 138{\cdot}05 \text{ N/mm}^2.$$

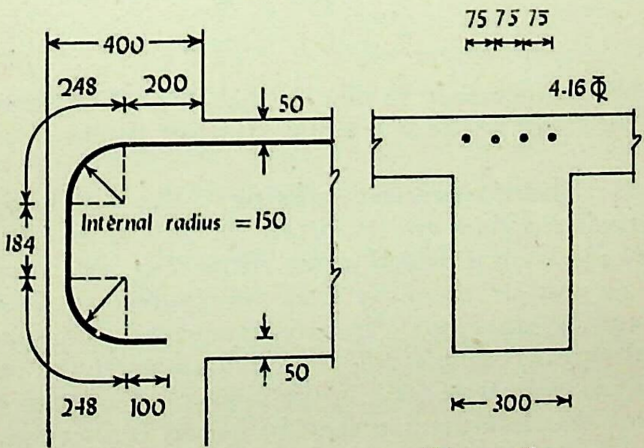
Tensile force in a bar at centre upper of bend

$F_{bt} = 201 \times 138{\cdot}05 \times 10^{-3} = 27{\cdot}75$ kN.

$r = 75 \qquad \phi = 16$

$$\text{Bearing stress} = \frac{F_{bt}}{r\phi}$$

$$= \frac{27{\cdot}75 \times 10^3}{75 \times 16} = 23{\cdot}12 \text{ N/mm}^2.$$



(a) Elevation (b) section

Fig. 3-25

The arrangement is thus not satisfactory; provide a new arrangement as shown in fig. 3-25.

Now anchorage required = 758 mm

anchorage provided $= 200 + 248 + 184 + 248 + 100$
$= 980$ mm

and allowable bearing stress $= 14{\cdot}02$ N/mm^2.

At centre of upper bend anchorage available

$$= 200 + \frac{248}{2} = 324 \text{ mm}$$

Stress in bar at face of column

$$= 230 \times \frac{742}{804} = 212{\cdot}26 \text{ N/mm}^2.$$

Stress in bar at centre of upper bend

$$= \frac{758 - 324}{758} \times 212{\cdot}26 = 121{\cdot}53 \text{ N/mm}^2.$$

Tensile force in a bar at centre of upper bend

$F_{bt} = 201 \times 121{\cdot}53 \times 10^{-3} = 24{\cdot}43$ kN

$r = 150$ $\phi = 16$

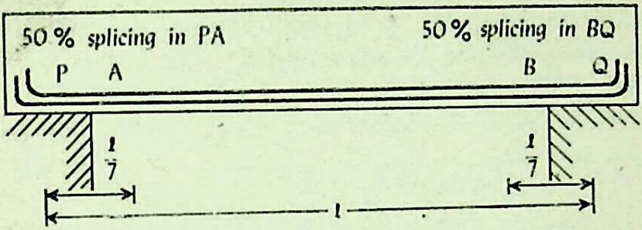
$$\text{Then bearing stress} = \frac{24{\cdot}43 \times 10^3}{150 \times 16}$$
$$= 10{\cdot}17 \text{ N/mm}^2 < 14{\cdot}02 \text{ N/mm}^2.$$

The arrangement is now satisfactory. The stresses at lower bend will be much less and need not be checked.

3-15. Reinforcement splicing: The requirements of reinforcement splices are set out in clause 25.2.5 of IS : 456. Where splices are provided in the reinforcing bars, they shall as far as possible be, away from the sections of maximum stress and be staggered. It is recommended that splices in flexural members should not be at sections where the bending moment is more than 50% of the moment of resistance, and not more than half the bars shall be spliced at a section.

Where more than one-half of the bars are spliced at a section or where splices are made at points of maximum stress, special precautions shall be taken, such as increasing the length of lap and/or using spirals or closely spaced stirrups around the length of the splice.

For simply supported beam loaded with uniformly distributed loads the above requirements are shown in fig. 3-26.



Splicing of reinforcement for simply supported beam

FIG. 3-26

The splices may be lap splice, welded splice or end bearing splice.

(*a*) *Lap splices:* This is covered in clause 25.2.5.1 of IS : 456 and is explained below:

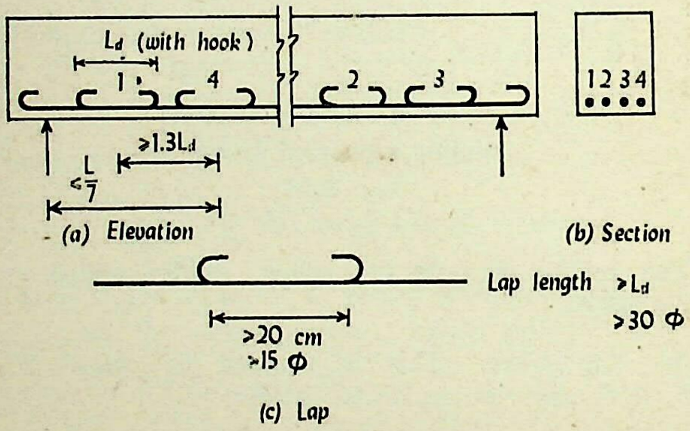
(1) Lap splices shall not be used for bars larger than 36 mm diameter; for larger diameters, bars may be welded such that the joint created is of the full strength of the bars connected. In cases where welding is not practicable, lapping of bars larger than 36 mm may be permitted, in which case additional spirals should be provided around the lapped bars. Splices in tension members shall be enclosed in spirals made of bars not less than 6 mm diameter with pitch not more than 100 mm. Hooks shall be provided at the ends of bars in tension members.

(2) Lap splices shall be considered staggered, if the centre to centre distance of the splices is not less than 1·3 times the lap length calculated as shown in (3).

(3) Lap length including anchorage value of hooks in flexural tension shall be L_d, the development length, or 30 ϕ whichever is greater and for direct tension $2L_d$ or 30 ϕ whichever is greater. The straight length of the lap shall not be less than 15 ϕ or 20 cm.

The above requirements are illustrated in fig. 3-27.

It should be noted and checked whether the above requirements can be satisfied for particular problem. If not, all four bars may not be spliced and only two shall be spliced. These are detailing rules and a site engineer must observe these rules seriously. Also at least a group leader of the bar benders must be sent with the knowledge of such rules.



Lap splices for tension bars

FIG. 3-27

(4) The lap length in compression shall be equal to the development length in compression but not less than 24 ϕ. The other requirements are as per tension bars.

(5) When bars of two different diameters are to be spliced, the lap length shall be calculated on the basis of diameter of the smaller bar.

(6) In case of bundled bars, lapped splices of bundled bars shall be made by splicing one bar at a time, such individual splices within a bundle shall be staggered.

(*b*) *Welded splices and mechanical connections:* The design strength of a welded splice or mechanical connection shall be taken as equal to 80 per cent of the design strength of the bar for tension splices and 100 per cent of the design strength for compression splices. However, 100 per cent of the design strength may be assumed in tension when the spliced area forms not more than 20 per cent of the total area of steel at

the section and the splices are staggered at least 60 cm from each other.

(*c*) *End bearing splices:* End bearing splices shall be used only for bars in compression. The ends of the bars shall be square cut and concentric bearing is ensured by suitable devices.

EXAMPLES III

(1) A simply supported beam has an overall size 250 mm wide × 550 mm depth. The beam is reinforced with 5 no. 16 mm diameter bars. The beam is subjected to 42 kN shear force at support. Check the shear stresses and design the shear reinforcement at support. The materials are M15 grade concrete and tor steel reinforcement of grade Fe 415. (*Note:* For secondary reinforcement like stirrups, use of lower grade steel is permitted.)

(2) If the shear of the section of Example (1) is increased to 85 kN and 2 no. 16 mm dia. bars are bent at 675 mm from face of the support, design the shear reinforcement at support.

(3) A simply supported beam has section 230 mm wide × 460 mm effective depth. It is reinforced with 6 no. 16 mm diameter bars. It is subjected to a shear of 120 kN at support. Design the shear reinforcement at support if (a) all the bars are carried into the support (b) 2 no. 16 mm dia. bars are curtailed at 500 mm from support and (c) 2 no. 16 mm dia. bars are bent at 625 mm from face of the support. In all cases, the materials are M15 grade concrete and tor steel reinforcement of grade Fe 415.

(4) A simply supported tee beam of 6 m span is carrying a load of 28 kN/m. The section of beam is 230 mm wide × 460 mm effective depth. It is reinforced with 5 no. 25 mm dia. bars. Design the beam for shear using (a) only vertical stirrups (b) 2 no. 25 mm dia. bars are bent at 625 mm from the face of the support. The materials are M15 grade concrete and mild steel reinforcement.

(5) Design shear reinforcement in Example (4) using available design tables.

(6) A doubly reinforced rectangular beam of size 230 mm × 460 mm effective depth requires as tension reinforcement 818 mm² area and 213 mm² area as compressive steel. It is reinforced with 2 no. 12 mm dia. bars in compression and 3 no. 20 mm dia. bars in tension. Calculate the development length of bars. The materials are grade M15 concrete and mild steel reinforcement.

(7) Calculate the anchorage length of a group of four bundled bars in contact and of equal diameter in tension. The materials are grade M20 concrete and tor steel reinforcement of grade Fe 415.

(8) Check the anchorage length of bent bar of Example (2) above.

(9) A beam fixed to the column requires 1026 mm² tor steel reinforcement of grade Fe 415 for negative moment at face of column. It is reinforced with 4 no. 20 mm dia. bars. Design and detail suitable anchorage for the reinforcement. Size of the column is 300 mm × 400 mm and size of beam is 300 mm × 600 mm overall depth (refer fig. 3-24). Concrete is of M20 grade.

(10) A simply supported rectangular beam of 5 m span carries a U.D.L. of 16 kN/m inclusive of self weight. Width of beam is 230 mm. Design the beam for flexure and shear. The materials are M15 grade concrete and mild steel reinforcement.

(11) A simply supported tee beam of 5 m span carries a U.D.L. of 20 kN/m inclusive of self weight. Width of web is 230 mm. Design the beam for flexure and shear. The materials are M15 grade concrete and tor steel reinforcement of grade Fe 415.

(12) A cantilever rectangular beam of span 2 m is loaded with uniformly distributed load of 16 kN/m inclusive of its self weight. The width of beam is 230 mm and depth is restricted to 450 mm. Design the beam for flexure and shear. The materials are M15 grade concrete and mild steel reinforcement.

4

Deflection, Cracking and Torsion

DEFLECTION

4-1. Introductory: When a member carries the load, it also deflects. According to IS : 456, the deflection of a structure or part thereof shall not adversely affect the appearance or efficiency of the structure or finishes or partitions apart from the structural considerations. The deflections therefore shall be limited to the following:

(a) The final deflection due to all loads including the effects of temperature, creep and shrinkage and measured from the as-cast level of the supports of floors, roofs and all other horizontal members, should not normally exceed span/250.

(b) The deflection including the effects of temperature, creep and shrinkage occurring after erection of partitions and the application of finishes (i.e. all dead loads) should not normally exceed span/350 or 20 mm whichever is less.

Two methods are suggested in the clause 22.2.1 of IS : 456 for checking that deflection is not excessive. These are:

(1) Control of span/effective depth ratio and is discussed in clause 22.2.1 and shall be used for all normal cases.

(2) For special cases, the deflections may be actually calculated as given in appendix B of IS : 456.

4-2. Span/effective depth ratio: This method is based on the calculations of deflection and tests on practical beams. This is a semi-empirical method and is covered in clause 22.2.1 of IS : 456. Accordingly the basic span/effective depth ratio for different support conditions are given and shall be modified for (a) amount of tension steel (b) amount of compression steel and (c) type of beam. This is discussed below.

For beams and slabs, the vertical deflection limits may generally be assumed to be satisfied provided that the span to depth ratios are not greater than the values obtained as below.

(a) Basic values of span to effective depth ratios for spans upto 10 m:

cantilever	7
simply supported	20
continuous	26

These values are based on a rectangular beam with one per cent tension reinforcement of 410 N/mm^2 characteristic strength and they limit the deflection to span/250.

(b) For spans above 10 m, the values in (a) may be multiplied by 10/span in metres, except for cantilever in which case deflection calculations should be made.

(c) Depending on the area and the type of steel for tension reinforcement, the values in (a) or (b) shall be modified as per fig. 4-1.

A close study of fig. 4-1 shows that for a given section, if amount of tension reinforcement increases, the permissible span/depth ratio decreases. This is because,

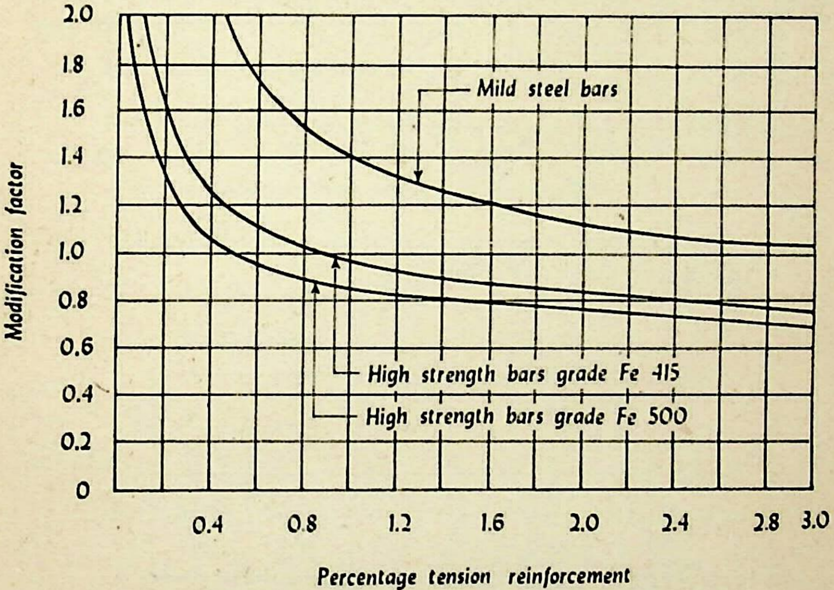
(1) When the area of steel reinforcement increases, the neutral axis shifts towards the tension steel. Thus, the area of concrete in compression zone increases which leads to a larger deflection due to creep.

(2) The smaller area of concrete in tension zone reduces the stiffness of the beam.

(d) Depending on the area of compression reinforcement, the values of span to depth ratio shall be further modified as per fig. 4-2.

A close study of fig. 4-2 shows that when compression steel increases, the permissible span/depth ratio increases. The reinforcement in compression zone reduces the shrinkage and increases the stiffness of the beam. Thus larger area of compression steel reduces the deflection.

(e) For flanged beams, the values in (a) or (b) shall be modified as per fig. 4-3 and the reinforcement percentages for use in fig. 4-1 and fig. 4-2 should be based on area of section equal to $b_f\,d$ where b_f is the width of flange.



Modification factor for tension reinforcement

FIG. 4-1

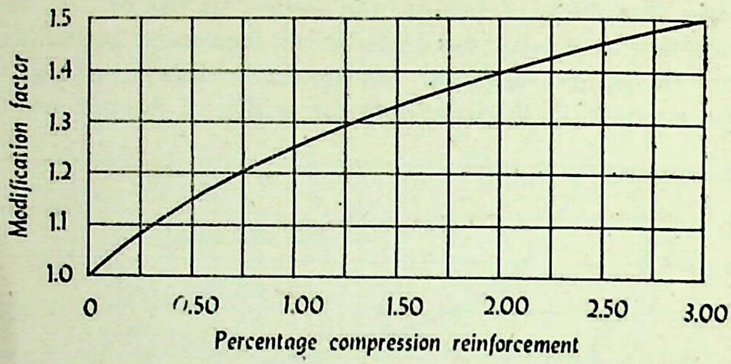
As the flange width increases, the concrete in tension zone is reduced. This reduces the stiffness of the beam. Thus permissible span/depth ratio is reduced by the reduction factor.

Now the allowable span/depth ratio is given by,

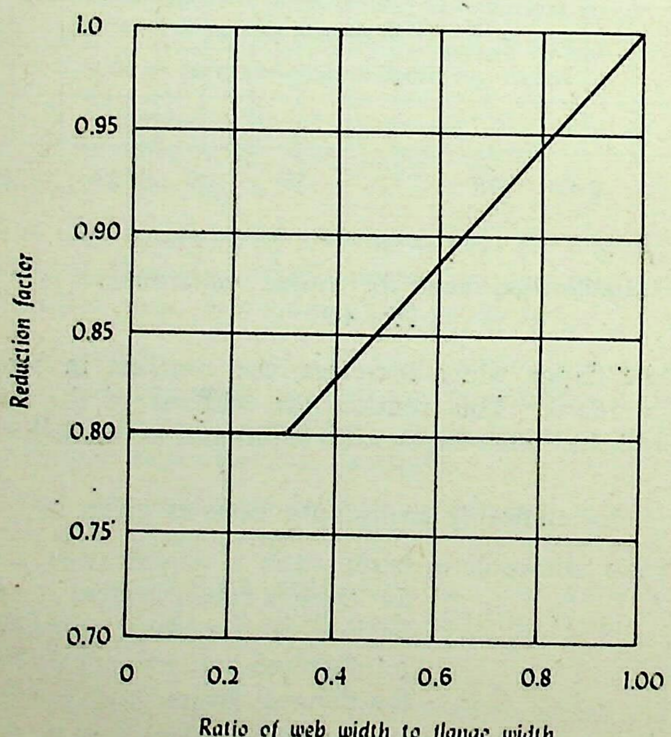
span/d ratio allowable = basic ratio × modification factor for tension reinforcement × modification factor for compression reinforcement × reduction factor for flanged beam..........(4-1)

The actual span/depth ratio shall be less than or equal to the allowable span/d ratio.

For solid slabs the control of deflection is discussed in clause 23.1 of IS : 456 and is discussed in chapter 6.



Modification factor for compression reinforcement
FIG. 4-2



Reduction factors for ratios of span to effective depth for flanged beams
FIG. 4-3

Example 4-1.

The beam of Ex. 2-2 is shown in fig. 4-4. If the span of this beam is 6 m, check the deflection of the beam using rules discussed above. The materials are M15 grade concrete and mild steel reinforcement.

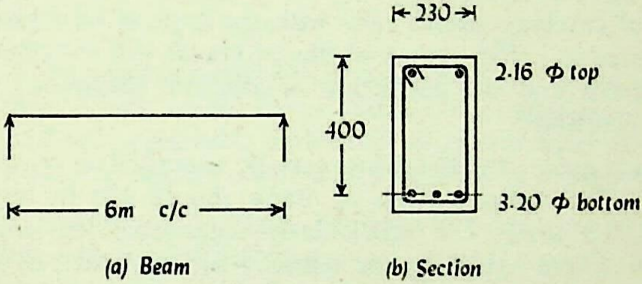


FIG. 4-4

Solution:

For simply supported beam, basic span/d ratio = 20

$$\text{per cent tension steel} = \frac{3 \times 314 \times 100}{230 \times 400} = 1{\cdot}02.$$

$$\text{Modification factor} = 1{\cdot}4$$

$$\text{per cent compression steel} = \frac{2 \times 201 \times 100}{230 \times 400} = 0{\cdot}44.$$

$$\text{Modification factor} = 1{\cdot}13$$

$$\text{then permissible span/d ratio} = 20 \times 1{\cdot}4 \times 1{\cdot}13 = 31{\cdot}64.$$

$$\text{Actual span/d ratio} = \frac{6000}{400} = 15 < 31{\cdot}64.$$

The beam is satisfactory for deflection.

4-3. Deflection calculations: If the calculations for deflection are to be made, they shall be done in accordance with appendix B of IS : 456. This is considered outside the scope of this book. However, for further studies IS : 456 and SP-16 may be referred.

CRACKING

4-4. Introductory: The limit state of serviceability is discussed in clause 34.3.2 of IS : 456. For working stress method also the same rules are adopted. It is stated,

"Cracking of concrete should not adversely affect the appearance or durability of the structure, the acceptable limits of cracking would vary with the type of structure and environment. The actual widths of cracks will vary between wide limits and the prediction of absolute maximum width is not possible".

As a guide, the following may be regarded as reasonable limits. The surface width of crack should not in general, exceed 0·3 mm. For particularly aggressive environment such as severe exposure, the assessed surface width of cracks at points nearest to the main reinforcement should not in general, exceed 0·004 times the nominal cover to the main reinforcement.

The possibility of some cracks being wider may be taken into account, if necessary.

To ensure that the crack width is not excessive, any of the two methods given below may be considered.

(a) *Bar spacing controls:* The rules of maximum spacing for bars in tension is given in clause 25.3 of IS : 456. This shall be used in all normal cases.

(b) *Crack width calculations:* Where it is required to calculate the crack width, the crack width calculations may be made by any available methods. The calculation of crack width is considered outside the scope of this book and reference may be made to CP : 110 (The structural use of concrete, The British Standards Institution.)

4-5. Bar spacing controls: This is covered in clause 25.3 of IS : 456 and is discussed below.

The diameter of a round bar shall be its nominal diameter, and in the case of bars which are not round or in case of deformed bars, the diameter shall be taken as the diameter of a circle giving an equivalent effective area. Where spacing limitations and minimum concrete cover are based on bar

diameter, a group of bars bundled in contact shall be treated as a single bar of diameter derived from the total equivalent area. For example, the effective diameter of a group of 4 no. 20 mm diameter bars can be given as:

$$\frac{\pi\phi^2}{4} = 4 \times \frac{\pi}{4}(20)^2$$

or $$\phi = 40 \text{ mm.}$$

For bar spacing controls it is stated, that unless the calculation of crack widths shows that a greater spacing is acceptable, the following rules shall be applied to flexural members in normal internal or external conditions of exposure.

(*a*) *Beams:* (1) The horizontal distance between parallel main reinforcement bars, or groups, near the tension face of a beam shall not be greater than the values given in table 10 of IS : 456 and reproduced in table 4-1 depending on the amount of redistribution carried out in analysis and the characteristic strength of the reinforcement.

TABLE 4-1

CLEAR DISTANCE BETWEEN BARS

	Percentage redistribution To or From section considered				
f_y	− 30	− 15	0	+ 15	+ 30
	CLEAR DISTANCE BETWEEN BARS				
N/mm²	mm	mm	mm	mm	mm
250	215	260	300	300	300
415	125	155	180	210	235
500	105	130	150	175	195

A close study of the above table shows that

(a) Smaller clear distance are required for high grade steel because the stresses and hence, the strains are higher.

(b) The analysis of cracking is based on elastic analysis i.e. with zero per cent redistribution of moment. Therefore, the permissible clear distances are decreased or increased if the percentage redistribution of moment at the section is increased or decreased.

(c) The table is applicable for redistribution of moments upto 30 per cent. The allowable redistribution of moments in elastic theory is 15 per cent and in limit state theory it is 30 per cent subjected to some checks. To understand redistribution of moments refer art. 5-15 of this book.

(2) Where the depth of the web in a beam exceeds 750 mm, side face reinforcement shall be provided along the two faces. The total area of such reinforcement shall not be less than 0·1 per cent of web area and shall be distributed equally on two faces at a spacing not exceeding 300 mm or web thickness whichever is less.

(*b*) *Slabs:* (1) The horizontal distance between parallel main reinforcement bars shall not be more than three times the effective depth of a solid slab or 450 mm whichever is smaller.

(2) The horizontal distance between parallel reinforcement bars provided against shrinkage and temperature shall not be more than five times the effective depth of a solid slab or 450 mm whichever is smaller.

In above discussion the main reinforcement refers to the reinforcement provided for flexure due to loads. The secondary or distribution bars are provided perpendicular to the main bars and are not resisting the flexure. These are provided:

(1) To distribute uniformly the concentrated load on the slab.

(2) To keep the main reinforcement in position.

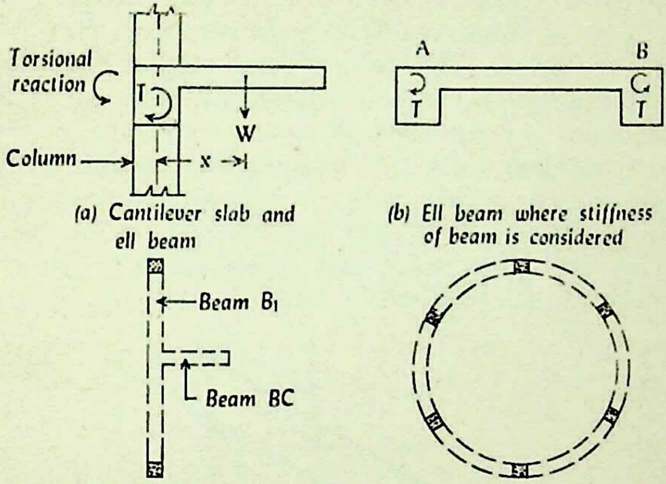
(3) To resist stresses due to shrinkage and temperature.

The above rules are observed in the examples considered in the chapters to follow.

TORSION

4-6. General: In addition to flexure and shear, the reinforced concrete elements are also sometimes subjected to the torsion. A section subjected to only torsion is rarely found. In most cases, bending and shear are predominant. Some examples of torsion are illustrated in fig. 4-5.

Fig. 4-5(a) shows a slab cantilevered from a beam which is assumed to be fixed at supporting columns. Slab load W induces a torsional moment of Wx in beam.



(a) Cantilever slab and ell beam

(b) Ell beam where stiffness of beam is considered

(c) Cantilever BC supported on B_1

(d) Circular beam

Torsion in structural systems

FIG. 4-5

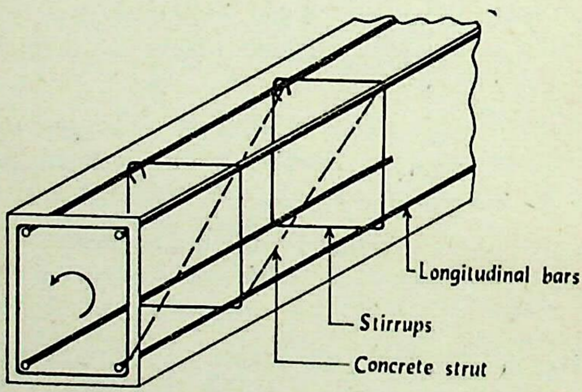
Fig. 4-5(b) shows ell beams. If the stiffness of beam is not taken into account, which is a usual case, there is no torsion in ell beams. However, if the stiffness of beams is taken into account, the negative moment of slab at A and B is a torsional moment for ell beams.

In fig. 4-5(c), a small beam BC is cantilevered from beam B_1. Beam B_1 is considered fixed at columns. Here, negative moment of beam BC will be the torsional moment of beam B_1.

Fig. 4-5(d) shows a circular ring beam supported on columns. In this case, because of the shape of the beam, torsional moment is induced.

If the beam in case (a) is supported throughout its length by a load bearing masonry wall, the case is a "pure torsion".

4-7. Effect of torsional moment: Provision of reinforcement: The plane of torsional moment is perpendicular to the plane of bending moment. Torsional moment induces shear stresses in the beam. Because of the torsion, a beam fails in diagonal tension forming spiral cracks around the beam. The behaviour of concrete structures subjected to torsion is complicated and not clearly understood. As a simplification, the effect of torsional moment is split up into (a) equivalent shear and (b) equivalent bending moment. The provision of reinforcement is then simplified to vertical stirrups in addition to stirrups for diagonal tension induced due to vertical shear force and longitudinal reinforcement in addition to that required for bending moment.



Space truss analogy
FIG. 4-6

A space truss analogy for the behaviour of torsion is shown in fig 4-6. In this analogy, the longitudinal reinforcements, the stirrups and struts of concrete in compression together form a space truss to resist the torsion. The longitudinal reinforcement helps in reducing the crack width through dowel action and stirrups crossing the cracks resist shear due to vertical loads and torsion.

4-8. Code provisions: According to IS : 456 design for torsion shall be made as follows:

General: In general, where the torsional resistance or stiffness of members has not been taken into account in the analysis of structure, no specific calculations for torsion will be necessary; adequate control of any torsional cracking being provided by the required nominal shear reinforcement. Where the torsional resistance or stiffness of members is taken into account in the analysis, the members shall be designed for torsion.

The approach to design for torsion is as follows:

Torsional reinforcement is not calculated separately from that required for bending and shear. Instead, the total longitudinal reinforcement is determined for a fictitious bending moment which is a function of actual bending moment and torsion. Similarly, web reinforcement is determined for a fictitious shear which is a function of actual shear and torsion.

The design rules given below shall apply to beams of solid rectangular cross-section. However, these may also be applied to flanged beams by substituting b_w for b, in which case they are generally conservative; therefore specialist literature may be referred to.

For design of torsion, sections located less than a distance d, from the face of the support may be designed for the same torsion as computed at a distance d, where d is the effective depth.

Design rules: The design rules for torsion as indicated above are based on equivalent shear and equivalent moment. They are explained below:

(*a*) *Shear and Torsion — Equivalent shear:* Equivalent shear, V_e shall be calculated from the formula:

$$V_e = V + 1{\cdot}6\,\frac{T}{b} \qquad \text{(4-2)}$$

where

V_e = equivalent shear

V = shear

T = torsional moment and

b = breadth of beam.

The equivalent nominal shear stress, τ_{ve} is given by,

$$\tau_{ve} = \frac{V_e}{bd} \qquad \text{(4-3)}$$

The equivalent nominal shear stress τ_{ve} shall not exceed the values of $\tau_{c\ max}$ as given in table 3-2.

If τ_{ve} does not exceed τ_c as given in table 3-1, only minimum shear reinforcement shall be provided as explained in art. 3-7. However, if τ_{ve} exceeds τ_c, both longitudinal and transverse reinforcement shall be provided as explained below.

(*b*) *Longitudinal reinforcement:* The longitudinal reinforcement shall be designed to resist an equivalent bending moment M_{e1}, given by,

$$M_{e1} = M + M_t \qquad \text{(4-4a)}$$

where

M = bending moment at the cross-section and

$$M_t = T\,\frac{\left(1 + \frac{D}{b}\right)}{1 \cdot 7} \qquad \text{(4-4b)}$$

where

T = torsional moment
D = overall depth of the beam and
b = breadth of the beam.

If the numerical value of M_t as defined by equation (4-4*b*), exceeds the numerical value of the moment M, longitudinal reinforcement shall be provided on the flexural compression face, such that the beam can also withstand an equivalent moment M_{e2} given by $M_{e2} = M_t - M$, the moment M_{e2} being taken as acting in the opposite sense to the moment M.

(*c*) *Transverse reinforcement:* Two legged closed hoops enclosing the corner longitudinal bars shall have an area of cross-section A_{sv}, given by,

$$A_{sv} = \frac{T \cdot s_v}{b_1 d_1 \sigma_{sv}} + \frac{V \cdot s_v}{2 \cdot 5\ d_1 \sigma_{sv}} \qquad \text{(4-5a)}$$

but the total transverse reinforcement shall not be less than

$$\frac{(\tau_{ve} - \tau_c)\, b \cdot s_v}{\sigma_{sv}} \qquad \text{(4-5b)}$$

where

T = torsional moment

V = shear force

s_v = spacing of the stirrup reinforcement

b_1 = centre to centre distance between corner bars in the direction of the width

d_1 = centre to centre distance between corner bars in the direction of the depth

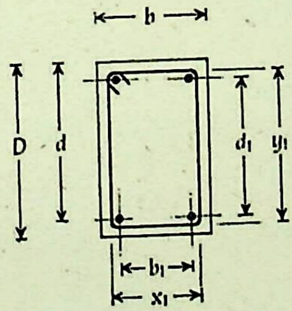
b = breadth of the member

σ_{sv} = permissible tensile stress in shear reinforcement

τ_{ve} = equivalent shear stress as defined in eq. 4-3 and

τ_c = shear strength of the concrete as specified in table 3-1.

(*d*) *Distribution of torsion reinforcement:* When a member is designed for torsion, torsion reinforcement shall be provided as below:



Details of torsion reinforcement
FIG. 4-7

(1) The transverse reinforcement for torsion shall be rectangular closed stirrups placed perpendicular to the axis of the member. The spacing of the stirrups shall not exceed the least of x_1, $\frac{x_1 + y_1}{4}$ and 300 mm, where x_1 and y_1 are respectively the short and long dimensions of the stirrups. Refer fig. 4-7.

(2) Longitudinal reinforcement shall be placed as close as practicable to the corners of the cross-section and in all

cases, there shall be at least one longitudinal bar in each corner of the ties. When the cross-sectional dimension of the member exceeds 450 mm, additional longitudinal bars shall be provided along the two faces. The total area of such reinforcement shall be not less than 0·1 per cent of the web area and shall be distributed equally on two faces at a spacing not exceeding 300 mm or web thickness whichever is less.

Example 4-2.

A rectangular beam of size 230 mm wide × 400 mm overall depth, is reinforced with 2 no. 10 mm dia. bars at top and 3 no. 16 mm dia. bars at bottom being tension reinforcement. It is subjected to a shear force of 18 kN, a torsional moment of 1·2 kNm and a bending moment of 18 kNm. Check for the torsion reinforcement. The materials are M15 grade concrete and mild steel reinforcement.

Solution:

$$M = 18 \text{ kNm}$$
$$T = 1{\cdot}2 \text{ kNm}$$
$$V = 18 \text{ kN}$$
$$b = 230 \text{ mm}$$
$$d = 400 - 25 - 8 = 367 \text{ mm}$$

Equivalent shear force

$$V_e = V + 1{\cdot}6\,\frac{T}{b}$$
$$= 18 + 1{\cdot}6\,\frac{1{\cdot}2 \times 10^6}{230} \times 10^{-3}$$
$$= 26{\cdot}35 \text{ kN.}$$

Equivalent nominal shear stress

$$\tau_{ve} = \frac{26{\cdot}35 \times 10^3}{230 \times 367} = 0{\cdot}31 \text{ N/mm}^2.$$

$$\frac{100\,A_s}{bd} = \frac{100 \times 3 \times 201}{230 \times 367} = 0{\cdot}71.$$

From table 3-1, $\tau_c = 0{\cdot}332 \text{ N/mm}^2 > \tau_{ve}$.

There is no need of torsion reinforcement. However, minimum stirrups shall be provided.

Using 6 mm dia. bars $A_{sv} = 2 \times 28 = 56$ mm².

Now, $$\frac{A_{sv}}{bs_v} \geqslant \frac{0{\cdot}4}{f_y}$$

$$\frac{56}{230 \times s_v} \geqslant \frac{0{\cdot}4}{250} \text{ which gives}$$

$$s_v \leqslant 152.$$

Provide 6 mm ϕ two-legged stirrups about 150 mm c/c.

Example 4-3.

A rectangular beam of size 230 mm wide × 600 mm overall depth is subjected to a sagging bending moment of 32 kNm, shear force of 32 kN and a torsional moment of 12 kNm. Design the reinforcement at section. The materials are M15 grade concrete and mild steel reinforcement.

Solution:

$M = 32$ kNm $b = 230$ mm

$V = 32$ kN $D = 600$ mm

$T = 12$ kNm.

Assuming 25 mm cover and 20 mm dia. bars in one layer

$$d = 600 - 25 - 10 = 565 \text{ mm.}$$

Equivalent shear

$$V_e = V + 1{\cdot}6\left(\frac{T}{b}\right)$$

$$= 32 + 1{\cdot}6\left(\frac{12}{0{\cdot}23}\right)$$

$$= 32 + 83{\cdot}5 = 115{\cdot}5 \text{ kN.}$$

Equivalent shear stress

$$\tau_{ve} = \frac{V_e}{bd} = \frac{115{\cdot}5 \times 10^3}{230 \times 565} = 0{\cdot}89 \text{ N/mm}^2.$$

For M15 mix from table 3-2, $\tau_{c\,max} = 1{\cdot}6$ N/mm²

$\tau_{ve} < \tau_{c\,max}$ ....(O.K.)

Assuming tension reinforcement = 0·5%

$$\tau_c = 0{\cdot}29 \text{ N/mm}^2 < \tau_{ve}.$$

Thus, design of torsion is necessary.

Longitudinal reinforcements:

Equivalent bending moment $M_{e1} = M + M_t$

$$= M + \frac{T\left(1 + \frac{D}{b}\right)}{1 \cdot 7}$$

$$= 32 + \frac{12\left(1 + \frac{60}{23}\right)}{1 \cdot 7} = 32 + 25 \cdot 5$$

$$= 57 \cdot 5 \text{ kNm.}$$

Since $M > M_t$, no reversal of moment is considered and steel on compression side is not required.

Now, $M_{e1} = 57 \cdot 5$ kNm

$$d_{required} = \sqrt{\frac{57 \cdot 5 \times 10^6}{0 \cdot 87 \times 230}}$$

$$= 536 \text{ mm} < 565 \text{ mm} \ldots\ldots\ldots\ldots\ldots\ldots \text{(O.K.)}$$

$$A_{st} = \frac{57 \cdot 5 \times 10^6}{140 \times 0 \cdot 87 \times 565} = 835 \text{ mm}^2.$$

Provide 3 no. 20 mm dia. bars = 942 mm².

At top provide 2 no. 12 mm dia. anchor bars.

As the depth of beam is more than 450 mm, side reinforcement has to be provided.

Minimum area on each face $= \frac{1}{2} \times \frac{0 \cdot 1}{100} \times 230 \times 600$

$= 69$ mm².

However use 1-12 ϕ on each face at centre of web.

Spacing of bars then $\frac{530}{2} = 265$ mm.

Spacing should not exceed

(1) 300 mm

(2) web thickness = 230 mm.

Second criteria is not satisfied. Therefore provide 4-12 ϕ side face reinforcement as shown in fig. 4-8.

Now spacing $= \frac{530}{3} = 176 \cdot 6 < 230$ mm............(O.K.)

Transverse reinforcements: Assuming 10 mm ϕ two-legged stirrups,

$$A_{sv} = 2 \times 78{\cdot}5 = 157 \text{ mm}^2.$$

$$A_{sv} = \frac{T \,.\, s_v}{b_1 d_1 \,\sigma_{sv}} + \frac{V \,.\, s_v}{2{\cdot}5 d_1 \,\sigma_{sv}}$$

Substituting, referring fig. 4-8,

$$157 = \frac{12 \times 10^6 \; s_v}{168 \times 534 \times 140} + \frac{32 \times 10^3 \; s_v}{2{\cdot}5 \times 534 \times 140}$$

$$= (0{\cdot}955 + 0{\cdot}171) \; s_v$$

$$= 1{\cdot}126 \; s_v$$

which gives $s_v = 139$ mm.................................(1)

Also $\quad A_{sv} \not< \dfrac{(\tau_{ve} - \tau_c) \; b \,.\, s_v}{\sigma_{sv}}$

Substituting,

$$157 \not< \frac{(0{\cdot}89 - 0{\cdot}29) \times 230 s_v}{140}$$

which gives $s_v < 159$ mm.................................(2)

Now spacing should not exceed

(a) $\quad x_1 = 190$ mm

(b) $\quad \dfrac{x_1 + y_1}{4} = \dfrac{190 + 560}{4} = 187{\cdot}5$ mm

(c) $\quad 300$ mm

i.e. $\quad s_v \not> 187{\cdot}5$ mm.................................(3)

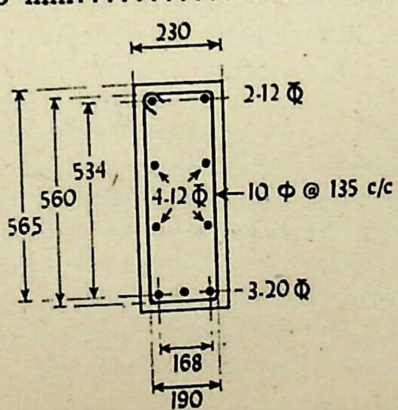


FIG. 4-8

From (1), (2) and (3) provide 10 mm ϕ two-legged stirrups about 135 mm c/c. The designed section is shown in fig. 4-8.

4-9. General cases of torsion: The analysis for torsion in reinforced concrete beam is done using elastic theory. Two most general cases of torsion are discussed below:

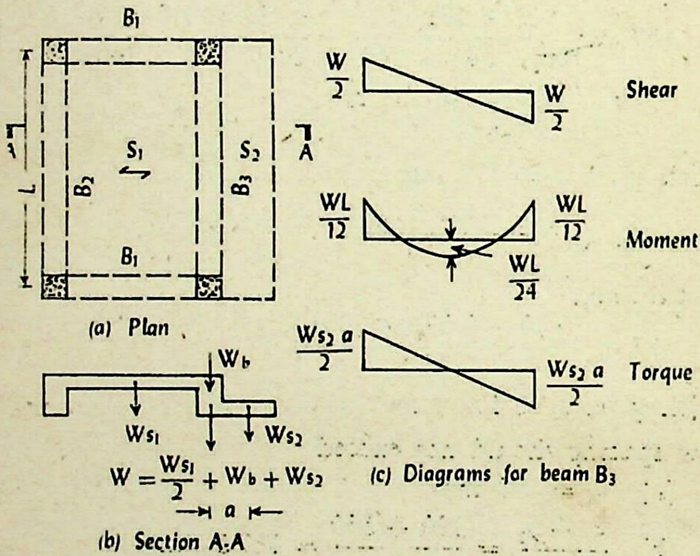


FIG. 4-9

(a) *Cantilever slab inducing torsion in supporting beam:* A beam B_3 fixed into columns and subjected to torsion due to cantilever slab S_2 is shown in plan in fig. 4-9(a). Note that if level of slab S_1 and that of S_2 were same, S_2 could be cantilevered from S_1 not inducing any torsion in beam B_3. Here it is assumed that reinforcement of slab S_2 are anchored in beam. If the main reinforcement of slab S_2 are anchored in slab S_1 through the beam B_3, upto $12\,\phi$ beyond the point of contraflexure, S_1 and S_2 can be made continuous not inducing torsion in beam B_3. Let total load on slab S_1 be W_{s1}, on slab S_2 be W_{s2} and load acting directly on beam (e.g. self weight of beam and masonry wall if any) be W_b. Then total load on beam B_3 is $W = \dfrac{W_{s1}}{2} + W_b + W_{s2}$. The shear,

moment and torque diagrams for beam B_3 are shown in fig. 4-9(c).

Note that beam B_3 is considered fixed at columns. It is also possible to develop connection of beam and column such that beam is simply supported for vertical loads at supports and restrained against torsion. In that case, bending moment at support will be zero and $\frac{Wl}{8}$ at centre.

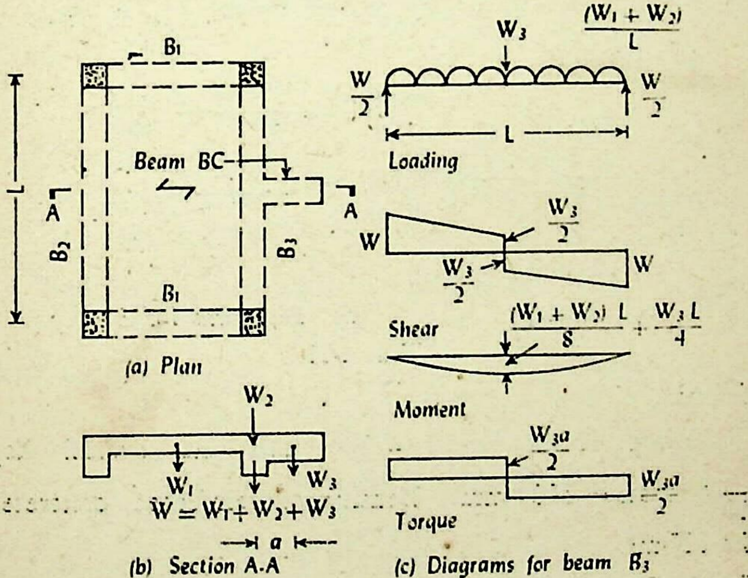


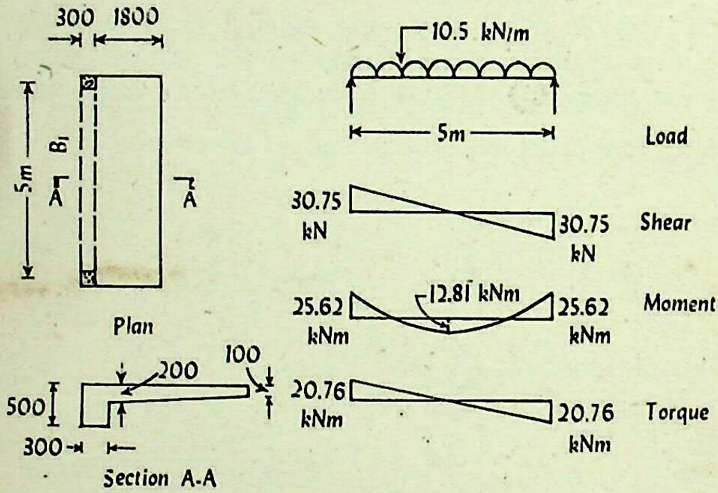
FIG. 4-10

(b) *Cantilever beam inducing torsion in supporting beam:* A beam B_3 simply supported at columns for vertical loads and restrained against torsion is shown in fig. 4-10(a) and (b). W_1 and W_2 are uniformly distributed loads while W_3 is central point load on beam B_3 inducing a torsional moment. The loading shear, moment and torque diagram for beam B_3 are shown in fig. 4-10(c).

Example 4-4.

A canopy beam of 5 m span fixed at support for vertical loads

and torsion, is shown in fig. 4-11. Live load on slab is 1 kN/m². Design beam for torsion at support. The materials are M15 grade concrete and mild steel reinforcement.



(a) Canopy (b) Diagrams for B_1

FIG. 4-11

Solution:

Adopt beam width = 300 mm and overall depth $\frac{1}{10} \times$ span = 500 mm. Density of reinforced concrete is 25 kN/m³.

Load on beam:

(i) From rectangular slab section,
$1{\cdot}8 \times 0{\cdot}1 \times 25 = 4{\cdot}5$ kN/m

(ii) From triangular slab section
$= 1{\cdot}8 \times 0{\cdot}1 \times \frac{25}{2} = 2{\cdot}25$ kN/m

(iii) Self wt. of beam $= 0{\cdot}3 \times 0{\cdot}5 \times 25 = 3{\cdot}75$ kN/m

(iv) Live load from slab $= 1 \times 1{\cdot}8 = 1{\cdot}8$ kN/m
total U.D.L. on beam $= 12{\cdot}3$ kN/m.

Maximum shear force at support $= 12{\cdot}3 \times \frac{5}{2} = 30{\cdot}75$ kN.

Maximum negative bending moment at support

$$= 12 \cdot 3 \times \frac{5^2}{12} = 25 \cdot 62 \text{ kNm.}$$

Maximum positive bending moment at centre

$$= 12 \cdot 3 \times \frac{5^2}{24} = 12 \cdot 81 \text{ kNm.}$$

Maximum torque at support

$$= \frac{5}{2} (4 \cdot 5 \times 1 \cdot 05 + 2 \cdot 25 \times 0 \cdot 75 + 1 \cdot 8 \times 1 \cdot 05)$$

$$= 20 \cdot 76 \text{ kNm.}$$

At support,

$M = 25 \cdot 62$ kNm $\qquad b = 300$ mm

$V = 30 \cdot 75$ kNm $\qquad D = 500$ mm

$T = 20 \cdot 76$ kNm.

Assuming 25 mm cover and 20 mm dia. bars in one layer $d = 500 - 25 - 10 = 465$ mm.

Equivalent shear:

$$V_e = V + 1 \cdot 6 \frac{T}{b}$$

$$= 30 \cdot 75 + 1 \cdot 6 \times \frac{20 \cdot 76}{0 \cdot 3}$$

$$= 30 \cdot 75 + 110 \cdot 72 = 141 \cdot 47 \text{ kN.}$$

Equivalent shear stress

$$\tau_{ve} = \frac{V_e}{bd} = \frac{141 \cdot 47 \times 10^3}{300 \times 465} = 1 \cdot 01 \text{ N/mm}^2.$$

For M15 mix from table 3-2 $\tau_{c\,max} = 1 \cdot 6$ N/mm²

$\tau_{ve} < \tau_{c\,max}$ .... (O.K.)

Assuming tension reinforcement $= 0 \cdot 5\%$

$\tau_c = 0 \cdot 29$ N/mm² $< \tau_{ve}$.

Thus design for torsion is necessary.

Longitudinal reinforcements:

Equivalent bending moment

$$M_{e1} = M + M_t$$

$$= M + \frac{T\left(1 + \frac{D}{b}\right)}{1{\cdot}7}$$

$$= 25{\cdot}62 + \frac{20{\cdot}76\left(1 + \frac{500}{300}\right)}{1{\cdot}7}$$

$$= 25{\cdot}62 + 32{\cdot}56$$

$$= 58{\cdot}18 \text{ kNm.}$$

Here $M < M_t$ and reversal of moment shall be considered.

$$M_{e2} = M_t - M$$

$$= 32{\cdot}56 - 25{\cdot}62 = 6{\cdot}94 \text{ kNm.}$$

This is a smaller value than maximum positive moment at centre. Therefore this will be taken care of by the positive reinforcement carried into the support.

Now $M_{e1} = 58{\cdot}18$ kNm.

At support the beam behaves as rectangular beam and at centre, in this case, it is an isolated L beam.

$$d_{required} = \sqrt{\frac{58{\cdot}18 \times 10^6}{0{\cdot}87 \times 300}}$$

$$= 462 \text{ mm} < 465 \text{ mm (provided)} \ldots\ldots \text{(O.K.)}$$

$$A_{st} = \frac{58{\cdot}18 \times 10^6}{140 \times 0{\cdot}87 \times 465} = 1027 \text{ mm}^2.$$

Provide 4 no. 20 mm dia. bars = 1256 mm^2.

It is seen from the bending moment and torque diagrams that at centre, the torque is zero and increases towards the support, while positive bending moment is maximum at centre and decreases towards the support. Design at any section for longitudinal bars shall be done using the summation $M + M_t$ as done for the support. In this case, if design for top bars at support and bottom bars at centre is done, it will be sufficiently accurate.

At centre, longitudinal steel required

$$= \frac{12{\cdot}81 \times 10^6}{140 \times 0{\cdot}87 \times 465} = 226 \text{ mm}^2.$$

Minimum steel $= \frac{0\cdot 34}{100} \times 300 \times 465$ (refer art. 5-5)

$= 474 \text{ mm}^2$.

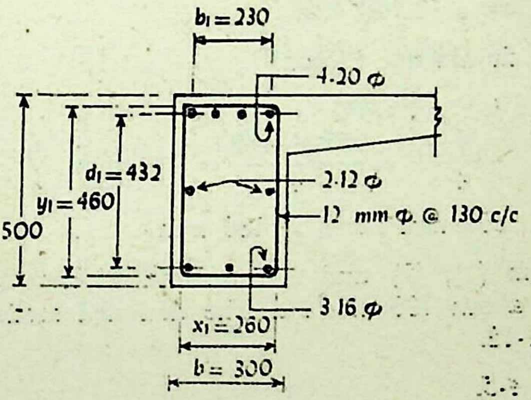
Provide 3 no. 16 mm dia. bottom bars = 603 mm².

These bars are utilised at support to resist the reversal moment due to torsion M_{e2} equal 6·94 kNm.

As the depth of beam is more than 450 mm, side face reinforcement has to be provided.

Area of one bar required $= \frac{0\cdot 1}{100} \times \frac{1}{2} \times 300 \times 500$

$= 75 \text{ mm}^2$ on each side.



Section at support

Fig. 4-12

Use 1-12 mm diameter bar on each face at centre.

Spacing $= \frac{432}{2} = 216$ mm.

Spacing should not exceed

(1) 300 mm

(2) web thickness = 300 mm..................(O.K.)

Transverse reinforcements:

Assuming 12 mm dia. two-legged stirrups,

$$A_{sv} = 2 \times 113 = 226 \text{ mm}^2.$$

$$A_{sv} = \frac{T \cdot s_v}{b_1 d_1 \sigma_{sv}} + \frac{V \cdot s_v}{2{\cdot}5\ d_1 \sigma_{sv}}$$

$$226 = \frac{20{\cdot}76 \times 10^6\ s_v}{230 \times 432 \times 140} + \frac{30{\cdot}75 \times 10^3\ s_v}{2{\cdot}5 \times 432 \times 140}$$

$$= 1{\cdot}49\ s_v + 0{\cdot}2\ s_v = 1{\cdot}69\ s_v$$

$\therefore \quad s_v = 133{\cdot}7$ mm.................................(1)

Also $A_{sv} \nless \dfrac{(\tau_{ve} - \tau_c)\ b\ s_v}{\sigma_{sv}}$

$$226 \nless \frac{(1{\cdot}01 - 0{\cdot}29) \times 300 \times s_v}{140}$$

which gives $s_v < 146$ mm..................................(2)

Spacing should not exceed

(1) $x_1 = 260$ mm

(2) $\dfrac{x_1 + y_1}{4} = \dfrac{260 + 460}{4} = 180$ mm

(3) 300 mm i.e. $s_v \ngtr 180$ mm..................(3)

From (1), (2) and (3) provide 12 mm ϕ two-legged stirrups about 130 mm c/c. The designed section is shown in fig. 4-12.

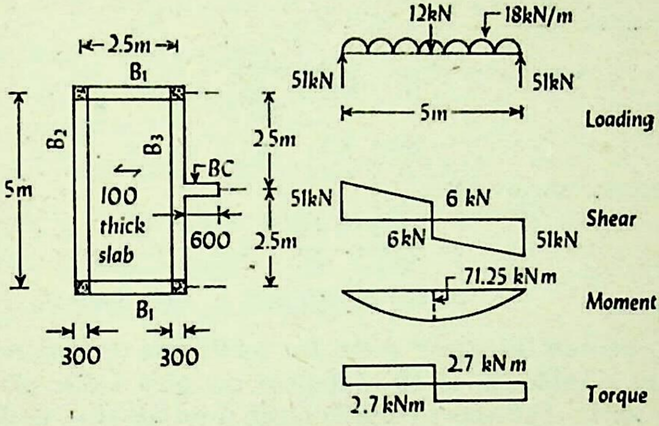
Example 4-5.

For the beam B_3, as shown in plan in fig. 4-13, loading from slab is 8 kN/m, self weight of beam and masonry wall above the beam B_3 transfers a U.D.L. of 10 kN/m on beam. Beam BC at centre of beam B_3 is cantilevered from beam B_3 and carries a U.D.L. of 20 kN/m inclusive of self weight. Beam B_3 is simply supported for vertical loads and restrained against torsion. Design the reinforcement at the centre and support of beam B_3. The materials are grade M15 concrete and tor steel reinforcenent of grade Fe 415.

Solution:

Load on B_3 from slab	8 kN/m
direct load on beam	10 kN/m
Total	18 kN/m.

A central point load from $BC = 0{\cdot}6 \times 20 = 12$ kN. Torque at centre of beam = 0.



(a) Plan (b) Beam diagrams

FIG. 4-13

Torque just at right or left of the centre of beam

$$= \frac{12 \times (0{\cdot}3 + 0{\cdot}15)}{2} = 2{\cdot}7 \text{ kNm}.$$

Shear force at support

$$= \frac{1}{2}(18 \times 5 + 12) = 51 \text{ kN}.$$

Maximum B.M. at centre

$$= 51 \times 2{\cdot}5 - \frac{2{\cdot}5^2}{2} \times 18$$

$$= 127{\cdot}5 - 56{\cdot}25 = 71{\cdot}25 \text{ kNm}.$$

The S.F., B.M. and torque diagrams are shown in fig. 4-13(*b*).

At centre of the beam,

$$M = 71{\cdot}25 \text{ kNm}$$
$$V = 6 \text{ kN}$$
$$T = 2{\cdot}7 \text{ kNm}$$
adopt $b = 300$ mm.
$$D = \frac{1}{10} \times 5000 = 500 \text{ mm}.$$

Assuming 25 mm cover and 20 mm dia. bars in one layer,

$$d = 500 - 25 - 10 = 465 \text{ mm.}$$

Equivalent shear $V_e = V + 1{\cdot}6\ \dfrac{T}{b}$

$$= 6 + 1{\cdot}6 \times \frac{2{\cdot}7}{0{\cdot}35}$$

$$= 20{\cdot}4 \text{ kN.}$$

Equivalent shear stress

$$= \frac{20{\cdot}4 \times 10^3}{300 \times 465}$$

$$= 0{\cdot}146 \text{ N/mm}^2 < 0{\cdot}2 \text{ N/mm}^2,$$

being permissible shear stress for minimum tension reinforcement. (Table 17 of IS : 456 does not give value of τ_c for 0·2% steel. The above value is taken from SP-16—table 80.)

There is no need of designing torsion reinforcement at centre.

For positive B.M. = 71·25 kNm

approximate lever arm $= d - \dfrac{D_f}{2} = 465 - \dfrac{100}{2} = 415$ mm.

$$A_{st} = \frac{71{\cdot}25 \times 10^6}{230 \times 415} = 746{\cdot}4 \text{ mm}^2.$$

Use 4 no. 16 mm Φ = 804 mm².

Checking of stresses in flexure is left to the reader. Provide minimum shear reinforcement from table 3-4 as 8 mm Φ about 345 c/c.

At support,

$$M = O$$
$$V = 51 \text{ kN}$$
$$T = 2{\cdot}7 \text{ kNm.}$$

Equivalent shear

$$V_e = V + 1{\cdot}6\ \frac{T}{b}$$

$$= 51 + 1{\cdot}6 \times \frac{2{\cdot}7}{0{\cdot}30}$$

$$= 51 + 14{\cdot}4 = 65{\cdot}4 \text{ kN.}$$

Equivalent shear stress

$$\tau_{ve} = \frac{65 \cdot 4 \times 10^3}{300 \times 465} = 0 \cdot 469 \ \text{N/mm}^2.$$

$$\frac{100 A_s}{bd} = \frac{100 \times 804}{300 \times 465} = 0 \cdot 58$$ assuming all bars are carried into support.

τ_c from table 3-1 $= 0 \cdot 306$ N/mm².

As $\tau_{ve} > \tau_c$, design for torsion is necessary.

Longitudinal reinforcement:

Equivalent bending moment

$$M_{e1} = M + M_t$$

$$= M + \frac{T\left(1 + \frac{D}{b}\right)}{1 \cdot 7}$$

$$= 0 + \frac{2 \cdot 7 \left(1 + \frac{500}{300}\right)}{1 \cdot 7}$$

$$= 4 \cdot 24 \text{ kNm.}$$

$M < M_t$, reversal of moment shall be considered.

$$M_{e2} = M_t - M$$

$$= 4 \cdot 24 - 0 = 4 \cdot 24 \text{ kNm.}$$

$$A_{st} = \frac{4 \cdot 24 \times 10^6}{230 \times 0 \cdot 90 \times 465} = 44 \cdot 0 \text{ mm}^2$$

2-12 Φ will be sufficient = 226 mm²

provide 2-12 Φ at top. These bars will be used as anchor bars.

As the depth of beam is more than 450 mm, side face reinforcement has to be provided.

Area of one bar required on each side

$$= \frac{1}{2} \times \frac{0 \cdot 1}{100} \times 350 \times 500 = 87 \cdot 5 \text{ mm}^2.$$

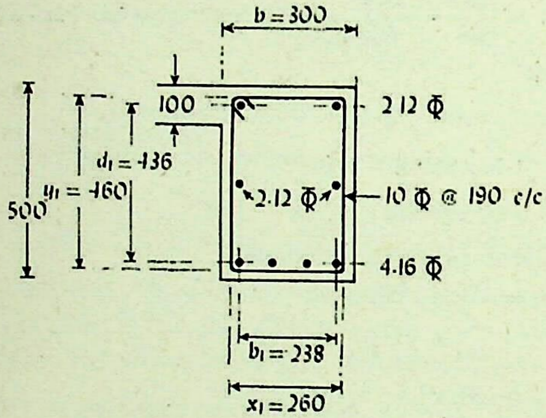
Use 1-12 ϕ bars as side face reinforcement on each side.

$$\text{Spacing} = \frac{436}{2} = 218 \text{ mm.}$$

Spacing should not exceed

(1) 300 mm

(2) web thickness = 300 mm................ (O.K.)



Section at support

FIG. 4-14

Transverse reinforcement:

Assuming 10 mm Φ two-legged stirrups,

$A_{sv} = 2 \times 78{\cdot}5 = 157 \text{ mm}^2.$

$$A_{sv} = \frac{T \cdot s_v}{b_1 d_1\, \sigma_{sv}} + \frac{V \cdot s_v}{2{\cdot}5\, d_1 \sigma_{sv}}$$

$$157 = \frac{2{\cdot}7 \times 10^6\, s_v}{238 \times 436 \times 230} + \frac{51 \times 10^3\, s_v}{2{\cdot}5 \times 436 \times 230}$$

$$= 0{\cdot}113\, s_v + 0{\cdot}203\, s_v = 0{\cdot}316\, s_v$$

$\therefore \quad s_v = 496$ mm..................................(1)

Also

$$A_{sv} \nless \frac{(\tau_{ve} - \tau_c)\, b\, s_v}{\sigma_{sv}}$$

$$157 \nless \frac{(0{\cdot}469 - 0{\cdot}29) \times 300 \times s_v}{230}$$

or $\quad s_v < 672$......................................(2)

Spacing should not exceed

(1) $x_1 = 260$ mm

(2) $\dfrac{x_1 + y_1}{4} = \dfrac{260 + 460}{4} = 180$ mm

(3) 300 mm

i.e. $s_v \not> 180$ mm.................................(3)

From (1), (2) and (3) provide 10 mm Φ two-legged stirrups about 180 c/c.

Note that use of 8 mm Φ stirrups here, would be economical. The section at support is shown in fig. 4-14. The section at centre is same as section at support, except for stirrups.

EXAMPLES IV

(1) A doubly reinforced rectangular beam of size 250 mm × 350 mm effective depth is reinforced with 2 no. 20 mm dia. bars at top and 3 no. 20 mm dia. bars at bottom tension reinforcement. If the span of the beam is 6 m and simply supported, check the deflection of beam. The materials are M15 grade concrete and tor steel reinforcements of grade Fe 415.

(2) A tee beam of flange width 1750 mm, effective depth 460 mm and rib width of 200 mm is reinforced with 4 no. 16 mm dia. bars in tension. If the span of beam is 6 m and simply supported, check the deflection of beam. The materials are M15 grade concrete and mild steel reinforcements.

(3) For the beam B_3 as shown in plan in fig. 4-13, loading from slab is 10 kN/m. Self weight of beam and masonry wall above the beam B_3 transfers a U.D.L. of 12 kN/m on beam. Beam BC at centre of beam B_3 is cantilevered from beam B_3 and carries a U.D.L. of 16 kN/m inclusive of self weight and a point load of 6 kN at the end of a cantilever. Beam B_3 is fixed at supports. Design the reinforcement at the centre and support of beam B_3. The materials are grade M15 concrete and tor steel reinforcement of grade Fe 415.

(4) A tee beam of size 300 mm × 450 mm overall depth is subjected to a shear force of 20 kN, a torsional moment of 2 kNm and a bending moment of 28 kNm at centre. Design the reinforcements. Thickness of slab is 120 mm. The materials are M15 grade concrete and tor steel reinforcement of grade Fe 415.

(5) A rectangular beam of size 300 mm × 500 mm overall depth is subjected to a hogging bending moment of 30 kNm, shear force of 30 kN and a torsional moment of 16 kNm at support. Design the reinforcement at the given section. The materials are M20 grade concrete and tor steel reinforcement of grade Fe 415.

(6) The projection of a canopy in fig. 4-11 is now increased from 1800 mm to 2 m. The thickness of slab at support is 200 mm and at edge 100 mm. The span of beam is now 6 m and size of the beam is 300 mm × 600 mm overall. If the other data remain unchanged, design the reinforcement for the canopy beam.

(7) Check the criteria of deflection and prepare a sketch to satisfy cracking requirements for the beam sections of Examples (10), (11) and (12) of chapter 3.

(8) A slab of 3 m span and thickness of 120 mm is reinforced with 10 mm Φ @ 150 mm c/c as main reinforcement. If the slab is simply supported, check for deflection of the slab.

(9) A rectangular beam of span 3 m is subjected to a load of 10 kN/m inclusive of self weight. It is also subjected to a torsional moment of 3 kNm. Design the beam for flexure, shear and torsion. The width of beam is 300 mm. The materials are M15 grade concrete and tor steel reinforcement of grade Fe 415.

5

Design of Beams

This chapter is intended to give complete design of the beams using the principles developed in previous chapters. A beam primarily is a flexural member and resists load in vertical bending. However, sometimes it resists lateral loads also. It resists the load by bending and shear. The checks for development length, deflection and cracking are required in design, Some typical designs are treated in this chapter.

SIMPLY SUPPORTED BEAMS

5-1. Introductory: Simply supported beams may be supported on masonry walls or R.C.C. columns. When supported on masonry walls, bed block or template below the beam at support is necessary to transfer the concentrated load from beam to the masonry walls. The simply supported beams may be rectangular beams or flanged beams.

The *effective span* of a beam that is not built integrally with its supports shall be taken as clear span plus the effective depth of beam or centre to centre of supports, whichever is less. For a beam built integrally with supports it is centre to centre of supports.

5-2. Design procedure: The design of any member is always followed by the analysis of forces, it has to withstand. The procedure for design of a beam may be summarized as follows:

Estimation of loads.: The correct estimation of loads, a beam has to bear, leads to an economical and safe design of the beams. A designer should not forget to account for any possible load acting on the structure, as this leads to an underdesign of the member and subsequently the failure of the beam. The dead loads on the beam may be self weight from slabs and beams, floor finish, partitions, false ceiling and some special fixed loads if specified. The live loads shall

be diffierent for different structures, depending on the functional use of the building. This shall be taken from IS : 875, the loading standards. The unit weights of plain concrete and reinforced concrete made with sand and gravel or crushed natural stone aggregate may be taken as 24 kN/m^3 and 25 kN/m^3 respectively.

Analysis: Using the above determined loads, the shear forces and bending moments are found out and diagrams drawn.

Design: After analysis, design the beam as follows:

(1) Using maximum moment, calculate the depth of beam required for balanced section. If the size of beam is specified, check whether it is singly reinforced or doubly reinforced.

(2) Find out steel area required for design moment.

(3) Check the shear stresses and development length of bars.

(4) If some bars are curtailed, check for curtailment using curtailment rules.

(5) Check the deflection and cracking using rules for control of deflection and cracking.

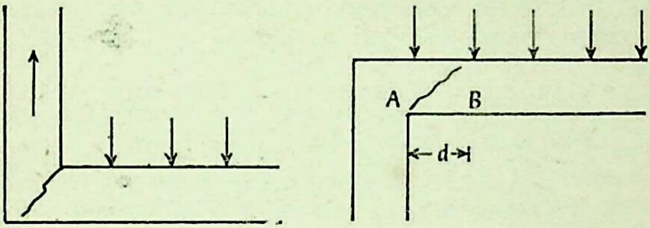
(6) Draw complete sketches of designed beam with elevations and sections.

5-3. Critical sections for moment and shear: These are summarized as follows:

(*a*) *Moment:* For a simply supported beam maximum moment occurs at point of zero shear in the span and shall be considered in design.

(*b*) *Shear:* The shears computed at the face of the support shall be used in design of the member at that section except, when the reaction in the direction of the applied shear introduces compression into the end regions of the member, sections located at a distance less than *d* from the face of the support may be designed for the same shear as that computed at distance *d* as shown in fig. 5-1.

Fig. 5-1(*a*) shows that loads introduce tension at the end region of the member, e.g. the beam at first floor level is supported on suspenders which are suspended from the beam

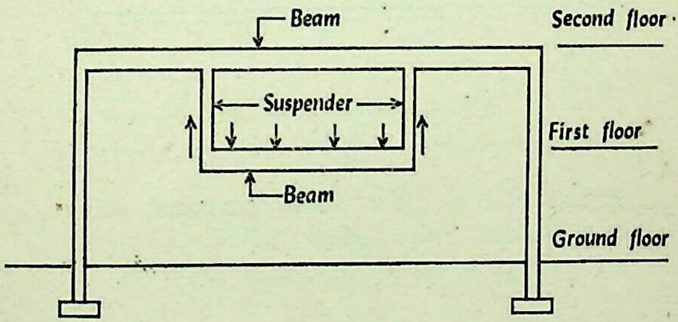


(a) Critical section at the face of support (b) Critical section at distance *d* from the face of the support

Critical sections for shear

FIG. 5-1

at second floor level as shown in fig. 5-2. In this case the loads will introduce tension at the end region and shear force at the face of the support shall be considered in design.



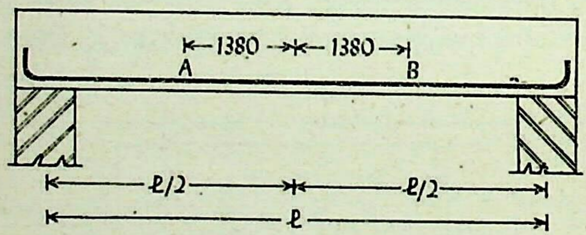
Suspended beam

FIG. 5-2

Fig. 5-1(*b*) shows that load introduces a compression at the end region of a member. This is a usual case and the sections located at a distance less than *d* from the face of the support i.e. *AB* region shall be designed for shear at *B*. This provision gives relaxation in design of shear reinforcements as the shear at *B* is lower than shear at *A*. However, it is recommended by the author that it may not be used if a beam under consideration receives a heavy point load in region *AB* of fig. 5-1(*b*).

5-4. Anchorage of bars: check for development length: According to clause 25.2 of IS : 456, the calculated tension or compression in any bar at any section shall be developed on each side of the section by an appropriate development length or end anchorage or by a combination thereof.

The above requirements put first restriction on bent bars or curtailed bars that no bar can be bent up or curtailed upto a distance of development length from the point of maximum moment, e.g. for tor steel reinforcement of grade Fe 415, the development length in concrete of grade M15 in tension is 69 ϕ. If 20 mm diameter bars are used, the bars cannot be bent or curtailed upto a distance of $69 \times 20 = 1380$ mm from the point of maximum bending moment. After this point, if a bar has to be curtailed, it shall comply with bar curtailment rules. This is illustrated in fig. 5-3.



A bar can be bent up or curtailed using curtailment rules after point *A* or *B*.

Check for development length

FIG. 5-3

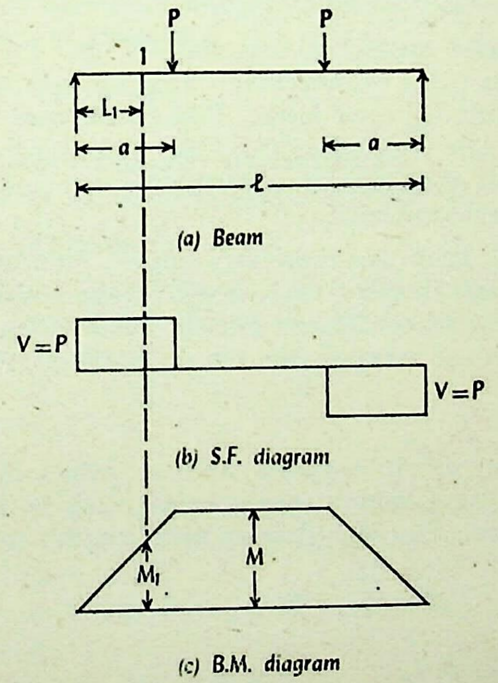
Suppose that the first restriction as mentioned above is over-come and a designer can bent or curtail some bars and he decides the number of bars to be bent or curtailed. Now for the remaining bars also the above requirement has to be checked.

Let M_1 be the moment of resistance of remaining bars assuming all the bars stressed upto the permissible stress σ_{st} i.e. $M_1 = \sigma_{st}.\ A_{st1}\ jd$ where A_{st1} is the area of remaining bars.

Find out from the bending moment diagram a point where the moment is equal to M_1. According to above

requirement, from this point the remaining bars have to be anchored upto L_d. This anchorage can be a sum of distance of centre of support from this point and anchorage beyond the centre of support. Let us formulate this requirement.

When a formula is to be prepared, the worst combination of the loads shall be considered. The point loads will serve this purpose. Consider a simply supported beam with two point loads and draw S.F. and B.M. diagrams as shown in fig. 5-4. Let some bars be bent or curtailed.



Check for development length
FIG. 5-4

Let M_1 be the moment of resistance of remaining bars assuming all the bars stressed to the permissible stress σ_{st}. In the B.M. diagram find out the point 1 where the B.M. is equal to M_1. Let the distance of point 1 from the centre of support be L_1. Then from above discussion it is clear that the total anchorage $L_1 + L_0$ (where L_0 is the anchorage of

bars beyond the centre of the support) shall be greater than or equal to L_d.

$\therefore \quad L_1 + L_0 \geqslant L_d.$

From fig. 5-4, it is clear that

$$L_1 = \frac{M_1}{P} = \frac{M_1}{V}.$$

Therefore, we conclude that

$$\frac{M_1}{V} + L_o \geqslant L_d \qquad \text{(5-1}a\text{)}$$

The same expression can also be used for continuous beam at the point of inflection. This formula may be used for any combination of loads. This is the same formula as given in clause 25.2.3.3 of IS : 456. According to the code the following are the anchoragement requirements for positive moment reinforcements:

(a) At least one-third the positive moment reinforcement in simple members and one-fourth the positive moment reinforcement in continuous members shall extend along the same face of the member into the support, to a length equal to $\frac{L_d}{3}$.

(b) At simple supports and at points of inflection, positive moment tension reinforcement shall be limited to a diameter such that L_d (development length) computed for f_d does not exceed $\frac{M_1}{V} + L_o$

or $\quad \frac{M_1}{V} + L_o \geqslant L_d$

where

M_1 = Moment of resistance of the section assuming all reinforcement at the section to be stressed to f_d.

f_d = Permissible stress σ_{st} for working stress design.

V = Shear force at the section (support for simply supported beam and point of inflection for continuous beam) due to design loads.

L_o = Sum of the anchorage beyond the centre of the support and the equivalent anchorage value of any hook or mechanical anchorages at simple support and a point of inflection. L_o is limited to the effective depth of the member or 12 ϕ whichever is greater, and

ϕ = diameter of bar.

The value of $\frac{M_1}{V}$ in the above expression may be increased by 30 per cent when the ends of the reinforcement are confined by a compressive reaction.

i.e. $1{\cdot}3\,\frac{M_1}{V} + L_o \geqslant L_d$ ....(5-1*b*)

5-5. Reinforcement requirements: Beam reinforcement shall comply the following requirements:

(*a*) *Tension reinforcement:*

(*1*) *Minimum reinforcement:* The minimum area of tension reinforcement shall not be less than that given by the following expression:

$$\frac{A_s}{bd} = \frac{0{\cdot}85}{f_y}$$

where

A_s = Minimum area of tension reinforcement

b = breadth of the beam or breadth of web of T beam

d = effective depth

f_y = characteristic strength of reinforcement in N/mm^2.

For mild steel,

$$\frac{100A_s}{bd} = \frac{100 \times 0{\cdot}85}{250} = 0{\cdot}34.$$

For tor steel, Fe 415 grade,

$$\frac{100A_s}{bd} = \frac{100 \times 0{\cdot}85}{415} = 0{\cdot}205.$$

For tor steel, Fe 500 grade

$$\frac{100A_s}{bd} = \frac{100 \times 0{\cdot}85}{500} = 0{\cdot}17.$$

(2) *Maximum reinforcement:* The maximum area of tension reinforcement shall not exceed 0·04 bD.

(*b*) *Compression reinforcement:*

The maximum area of compression reinforcement shall not exceed 0·04 bD. Compression reinforcement in beams shall be enclosed by stirrups for effective lateral restraint.

5-6. Slenderness limits for beams to ensure lateral stability: A beam is usually vertical load carrying member. However, for long span, the beam may bend laterally. To ensure the lateral stability of a beam, IS : 456 states,

"A simply supported or continuous beam shall be so proportioned that the clear distance between the lateral restraints does not exceed $60\,b$ or $\dfrac{250\,b^2}{d}$ whichever is less where d is the effective depth and b is the breadth of the compression face midway between the lateral restraints.

For a cantilever, the clear distance from the free end of the cantilever to the lateral restraint shall not exceed $25\,b$ or $\dfrac{100\,b^2}{d}$ whichever is less".

Example 5-1.

A simply supported rectangular beam of 4 m span carries a uniformly distributed load including self weight of 20 kN/m. The beam section is 230 mm × 500 mm overall. Design the beam. The materials are grade M15 concrete and mild steel reinforcements. The beam is suspended from the upper floor level.

Solution:

$$M_{max} = 20 \times \frac{4^2}{8} = 40 \text{ kNm}$$

$$V_{max} = 20 \times \frac{4}{2} = 40 \text{ kN.}$$

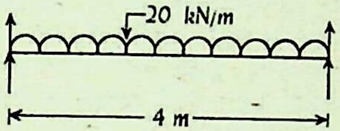
(a) Moment steel:

The section is 230 mm × 500 mm overall. Assuming one layer of 16 mm dia. bars, effective depth shall be

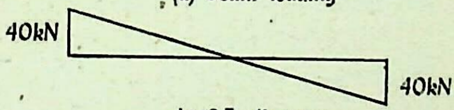
$d = 500 - 25$ (cover) $- 8$ (centre of reinforcement)
$= 467$ mm.

Depth required for singly reinforced section

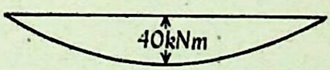
$$= \sqrt{\frac{40 \times 10^6}{0.87 \times 230}} = 447 < 467.$$



(a) Beam loading



(b) S.F. diagram



(c) B.M. diagram

FIG. 5-5

$\therefore$ Design as singly reinforced section.

$$A_{st} = \frac{40 \times 10^6}{140 \times 0.87 \times 467} = 703 \text{ mm}^2.$$

Minimum steel required

$$A_s = \frac{0.34}{100} \times 230 \times 467 = 365 \text{ mm}^2.$$

Provide 4 no. 16 mm $\phi = 4 \times 201 = 804$ mm^2.

Let 2 bars are bent at $1.25\ D$

$= 1.25 \times 500 = 625$ mm, say 600 mm, from the face of the support.

(b) Check for development length:

(1) A bar can be bent up at a distance greater than $L_d = 58\ \phi$ from centre of support i.e. $58 \times 16 = 928$ mm.

In this case, this distance is $(2000 - 600) = 1400$ mm.
............................(O.K.)

(2) For the remaining bars,

$M_1 = 2 \times 201 \times 140 \times 0{\cdot}87 \times 467 \times 10^{-6} = 22{\cdot}87$ kNm

$V = 40$ kN.

If the bars are hooked at 10 cm beyond the centre of the support, the anchorage value $L_o = 16\,\phi + 100$. L_o is limited to $12\,\phi = 192$ mm or effective depth $= 447$ whichever is greater i.e. 447 mm. However, provided $L_o = 16\,\phi + 100 = 356$ mm.

$\therefore$ Use $L_o = 356$ mm.

Now, $\dfrac{M_1}{V} + L_o \geqslant L_d$

$$\therefore \quad \frac{22{\cdot}87 \times 10^6}{40 \times 10^3} + 356 \geqslant 58\,\phi$$

$928 \geqslant 58\,\phi$ or $\phi \leqslant 16$ mm.

Provided diameter is 16 mm. Therefore 2-16 ϕ can be bent up.

(c) Check of shear:

At support, $V = 40$ kN.

$$\tau_v = \frac{40 \times 10^3}{230 \times 467} = 0{\cdot}372 \text{ N/mm}^2.$$

$$\frac{100\,A_s}{bd} = \frac{2 \times 201 \times 100}{230 \times 467} = 0{\cdot}374$$

$$\tau_c = 0{\cdot}255 \text{ N/mm}^2 < 0{\cdot}372 \text{ N/mm}^2.$$

$\therefore$ Shear design is necessary.

Note that the critical section for checking the shear stress in this case is the face of the support (and not at distance *d* from the face of the support) because the reaction at support induces tension in end region.

At support 2 bent bars can be used to carry shear stress. These give a shear of,

$0{\cdot}707 \times 2 \times 201 \times 140 \times 10^{-3} = 39{\cdot}78$ kN.

Shear resistance of concrete,

$\therefore \quad \tau_c\, bd = 0{\cdot}255 \times 230 \times 467 \times 10^{-3} = 27{\cdot}39$ kN.

$V_s = V - \tau_c\, bd = 40 - 27{\cdot}39 = 12{\cdot}61$ kN.

Bent bars share $50\% = 6{\cdot}3$ kN

Stirrups provide $50\% = 6{\cdot}3$ kN.

Using 6 mm ϕ two-legged stirrups, spacing can be given by

$\dfrac{\sigma_{sv} . A_{sv} . d}{V_s}$ where $A_{sv} = 2 \times 28 = 56$ mm²

$$\therefore \quad s_v = \frac{140 \times 56 \times 467}{6{\cdot}3 \times 10^3} = 581 \text{ mm} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

At a distance 600 mm from support,

$V = 40 - 0{\cdot}6 \times 20 = 28$ kN

$V_s = 28 - 27{\cdot}39 = 1{\cdot}39$ kN.

This will give larger spacing than above.

For 230 mm wide beam minimum shear reinforcement from table 3-4 is 6 ϕ about 150 c/c......................(2)

From (1) and (2) minimum shear reinforcement shall be provided i.e. 6 mm ϕ about 150 c/c.

(d) Check for deflection:

Basic span/d ratio = 20

$$p_t = \frac{100 \times 4 \times 201}{230 \times 467} = 0{\cdot}75.$$

Modification factor = 1·58.

$\therefore$ Span/d ratio permissible $= 20 \times 1{\cdot}58 = 31{\cdot}6$.

Actual span/d ratio $= \dfrac{4000}{467} = 8{\cdot}56 < 31{\cdot}6$....... (O.K.)

(e) Check for cracking:

Clear distance between bars

$$= \frac{230 - 50 - 4 \times 16}{3} = 38{\cdot}66 \text{ mm.}$$

Maximum distance permitted = 300 mm (zero per cent redistribution).

(f) Practical requirement:

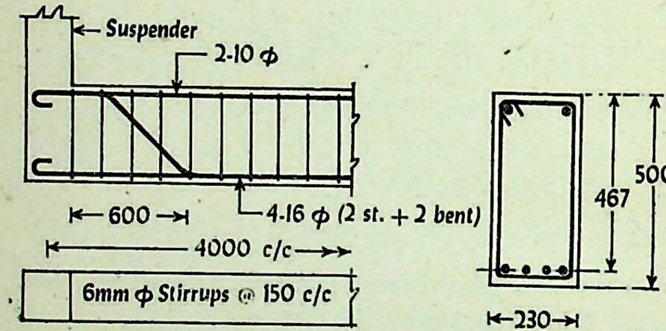
Minimum distance between bars permitted $= h_{agg} + 5$ mm $= 20 + 5 = 25$ mm (using 20 mm aggregate size)

$= \phi$ of bar i.e. 16 mm.

i.e. clear distance should be more than 25 mm.

Actual clear distance $= 38{\cdot}66$ mm $>$ 25mm.... (O.K.)

The beam as designed above is shown in fig. 5-6. Provide 2-10 ϕ anchor bars.



(a) Beam section (b) Section

FIG. 5-6

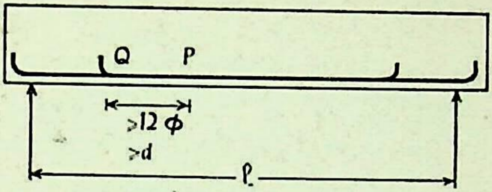
(g) Supplementary details:

In this example even if the bars are not bent, one leads to the minimum shear reinforcement. To save the reinforcement in this case, bars may be curtailed or taken straight into support.

5-7. Curtailment of bars: When the tension bars are not required to resist the moment, they can be curtailed. For curtailment, reinforcement shall extend beyond the point at which it is no longer required to resist flexure for a distance equal to the effective depth of the member or 12 times the diameter whichever is greater except at simple support or end of cantilever. These requirements are explained in fig. 5-7.

In addition to above, the following requirements shall also be satisfied. These are given in clause 25.2.3.2 of IS : 456.

It states, "Flexural reinforcement shall not be terminated in a tension zone, unless any one of the following conditions is satisfied.



P — Theoretical cut off point
Q – Actual cut off point
Requirements for bar curtailment
FIG. 5-7

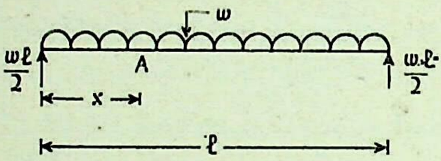
(a) The shear at cut off point does not exceed two-thirds that permitted including the shear strength of web reinforcement provided.

(b) Stirrup area in excess of that required for shear and torsion is provided along each terminated bar over a distance from the cut off point equal to three-fourth the effective depth of the member. The excess stirrup area shall not be less than $0{\cdot}4\ bs/f_y$, where b is the breadth of beam, s is the spacing and f_y is the characteristic strength of reinforcement in N/mm^2. The resulting spacing shall not exceed $\dfrac{d}{8\,\beta_b}$, where β_b is the ratio of the area of bars cut off to the total area of bars at the section and d is the effective depth.

(c) For 36 mm and smaller bars, the continuing bars provide double the area required for flexure at the cut off point and the shear does not exceed three-fourths that permitted".

A close study of these requirements shows that shear is playing an important role. In fact, when a curtailment is done, the complicated shear stresses are induced and must be resisted by suitable reinforcement. A designer can understand after solving some problems that curtailment induces elaborate calculations. Then choice is left to the designer, whether he curtails the bars or not. After illustrating one example, curtailment will not be done in this book.

Consider a simply supported beam loaded with uniformly distributed loads as shown in fig. 5-8. If 50% bars are to be curtailed, theoretical cut off point can be found out as below.



Curtailment of 50% bars
FIG. 5-8

Referring fig. 5-8,
if A is the theoretical point of cut off,

$$M_A = \frac{wlx}{2} - \frac{wx^2}{2} = \frac{wl^2}{16}.$$

This gives $x = 0{\cdot}146l$ (or $\frac{l}{7}$ approximately). From this point, bars are to be extended *at least* 12 ϕ or effective depth whichever is greater. Then curtailment rules are checked.

Example 5-2.

If 2-16 ϕ bars of Example 5-1 are required to be curtailed. Check the curtailment of bars.

Solution:

Theoretical cut off point $= 0{\cdot}146 \times 4000 = 584$ mm

$$12\,\phi = 12 \times 16 = 192$$
$$d = 467.$$

Curtailment can be done at $584 - 467 = 117$ mm from the centre of the support.

It can be seen that there is no meaning of curtailment in this case.

Example 5-3.

Design a simply supported tee beam of span 8 m and spaced at 3 m centres. The thickness of slab is 120 mm and total load

including self weight of beam is 28 kN/m. Use curtailment rules. The materials are grade M15 concrete and tor steel reinforcement of grade Fe 415.

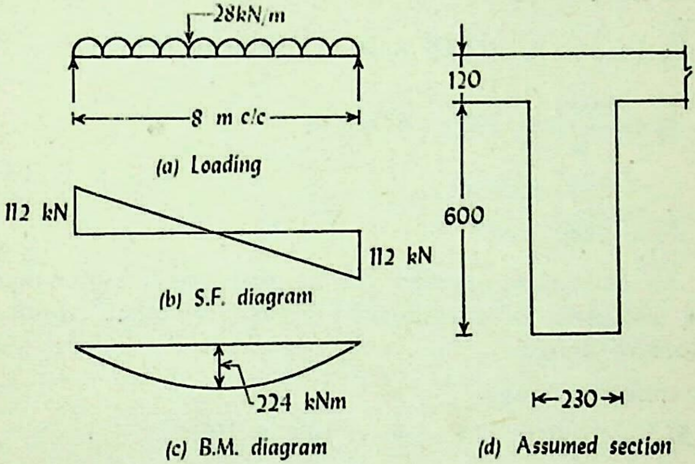


FIG. 5-9

Solution:

Moment design:

Assume depth $\frac{1}{12}$ to $\frac{1}{10}$ of the span i.e. 666 mm to 800 mm.

Consider rib depth = 600 mm giving $D = 120 + 600$
$= 720$ mm.

Adopt $b_w = 230$ mm.

Assuming 2 layers of 20 mm Φ bars,

$d = 720 - 25 - 20 - 10 = 665$ mm.

As a priliminary design,

assume lever arm $= d - \frac{D_f}{2}$

$$= 665 - \frac{120}{2} = 605 \text{ mm.}$$

$$M_{max} = 28 \times \frac{8^2}{8} = 224 \text{ kNm.}$$

$$A_{st} = \frac{224 \times 10^6}{230 \times 605} = 1610 \text{ mm}^2.$$

Provide 6-20 Φ giving $A_{st} = 1884 \text{ mm}^2$.

Check for M.R. of section:

For a tee beam,

$$b_f = \frac{l_0}{6} + b_w + 6D_f \text{ and } b_f \not> \text{actual width of flange.}$$

$$\therefore \quad b_f = \frac{8000}{6} + 230 + 6 \times 120$$

$$= 2283 < 3000 \ldots\ldots\ldots\ldots \text{(O.K.)}$$

Use $b_f = 2280$ mm.

To determine whether the neutral axis lies in flange or web, moments of transformed areas are taken about the bottom of flange.

For concrete flange,

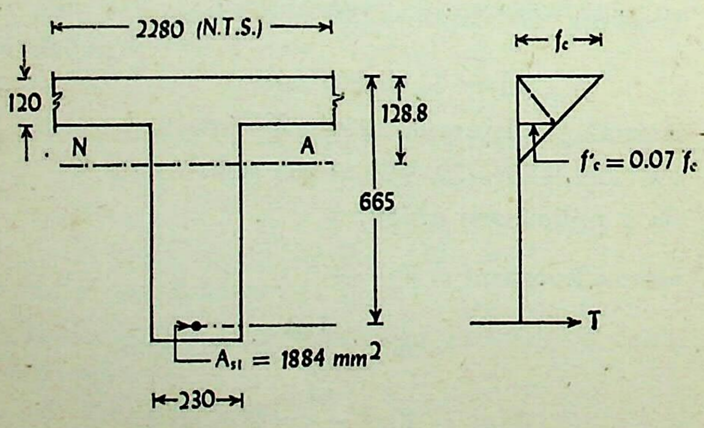
$$M_{tc} = 2280 \times 120 \times 60 = 1{\cdot}64 \times 10^7.$$

For reinforcement,

$$M_{ts} = 18{\cdot}66 \times 1884 \times (665 - 120)$$
$$= 1{\cdot}92 \times 10^7$$

$$M_{ts} > M_{tc}$$

$\therefore$ N.A. lies in web.



(a) Section (b) Stress diagram

FIG. 5-10

Taking moments about N.A. and neglecting the concrete in web portion,

$$2280 \times 120\ (x - 60) = 18{\cdot}66 \times 1884\ (665 - x)$$
$$273600x - 16416000 = 23378368 - 35155x$$
$$\therefore\ 308755x = 39794368$$
$$\therefore\ x = 128{\cdot}8 \text{ mm.}$$

Depth of critical N.A. $= 0{\cdot}29 \times 665 = 192{\cdot}85 > 128{\cdot}8$

$\therefore$ The beam is under-reinforced.

To find c.g. of compressive forces, take moments about top flange.

$$\bar{y} = \frac{f_{cb} \times \dfrac{120}{2} \times \dfrac{120}{3} + 0{\cdot}07 f_{cb} \times \dfrac{120}{2} \times \dfrac{2}{3} \times 120}{f_{cb} \times \dfrac{120}{2} + 0{\cdot}07\ f_{cb} \times \dfrac{120}{2}}$$

$$= \frac{2400 f_{cb} + 336 f_{cb}}{60 f_{cb} + 4{\cdot}2 f_{cb}} = 42{\cdot}6 \text{ mm.}$$

Lever arm $= d - \bar{y} = 665 - 42{\cdot}6 = 622{\cdot}4$ mm.

$$\text{M.R.} = A_{st} \,.\, \sigma_{st}(d - \bar{y})$$
$$= 1884 \times 230 \times 622{\cdot}4 \times 10^{-6}$$
$$= 269{\cdot}7 \text{ kNm} > 224 \text{ kNm} \ldots\ldots\ldots\ldots\ldots \text{(O.K.)}$$

Thus section is verified.

Curtailment of bars:

50% of bars to be curtailed. Theoretical point of cut off from support $= 0{\cdot}146 \times 8000 = 1168$ mm.

$$12\,\phi = 12 \times 20 = 240$$
$$d_{eff} = 665.$$

Curtail 3-20 ϕ bars at a distance $1168 - 665 = 503$ say 500 mm from centre of support.

Check for development length:

At support $A_s = 3 \times 314 = 942$ mm²

$$M_1 = 942 \times 230 \times 0{\cdot}9 \times 665 \times 10^{-6}$$
$$= 129{\cdot}67 \text{ kNm (assume } j = 0{\cdot}9)$$
$$V = 112 \text{ kN.}$$

As the ends of reinforcements are confined with compressive reactions,

$$1{\cdot}3\,\frac{M_1}{V} + L_o \geqslant L_d$$

let $\quad L_o = 12\,\phi$

then $\quad 1{\cdot}3 \times \dfrac{129{\cdot}67 \times 10^6}{112 \times 10^3} + 12\,\phi \geqslant 69\,\phi$

which gives $\quad 1505 \geqslant 57\,\phi$

or $\quad \phi \leqslant 26{\cdot}4.$

ϕ provided is 20 mm........................ (O.K.)

At support, as the ends of reinforcements are confined with compressive reaction, shear at distance '*d*' will be used.

Then $\quad V = 112 - 0{\cdot}665 \times 28 = 93{\cdot}38$ kN.

$$\frac{100A_s}{bd} = \frac{100 \times 3 \times 314}{230 \times 665} = 0{\cdot}62$$

$$\tau_c = 0{\cdot}314 \text{ N/mm}^2.$$

Shear resistance of concrete

$$= \tau_c\, bd = 0{\cdot}314 \times 230 \times 665 \times 10^{-3} = 48{\cdot}03 \text{ kN}.$$

$\therefore \quad V_s = 93{\cdot}38 - 48{\cdot}03 = 45{\cdot}35$ kN.

Using 8 mm Φ two-legged stirrups,

$$A_{sv} = 100 \text{ mm}^2, \qquad \sigma_{sv} = 230 \text{ N/mm}^2$$

$$s_v = \frac{A_{sv}\,.\,\sigma_{sv}\,.\,d}{V_s} = \frac{100 \times 230 \times 665}{45{\cdot}35 \times 10^3} = 337 \text{ mm}.$$

For a 230 wide beam from table 3-4, minimum shear reinforcement = 8 Φ about 450 c/c or 6 ϕ about 150 c/c.

$\therefore$ Provide 8 Φ about 300 c/c.

For minimum shear reinforcement 8 Φ about 450 c/c,

$$V_s = \frac{2 \times 50 \times 230 \times 665}{450} \times 10^{-3} = 33{\cdot}99 \text{ kN}.$$

Total shear capacity of section with minimum shear reinforcement = $48{\cdot}03 + 33{\cdot}99 = 82{\cdot}02$ kN.

Stirrups more than minimum are required upto

$$\frac{112 - 82{\cdot}02}{28} = 1{\cdot}07 \text{ m from support.}$$

No. of stirrups required

$$= \frac{1070}{300} + 1 \simeq 5.$$

Provide 8 mm Φ about 300 c/c upto 5 no. then 8 mm Φ about 450 c/c.

Check for curtailment:

Three checks are given in IS : 456 out of which one is to be satisfied.

(a) Shear at cut off point

$$= 112 - 0{\cdot}5 \times 28 = 98 \text{ kN.}$$

Shear resistance of section

= shear resistance of concrete + shear resistance of reinforcement

$$= \tau_c\, bd + A_{sv} \,.\, \sigma_{sv} \,.\, \frac{d}{s_v}$$

$$= 48{\cdot}03 + \frac{100 \times 230 \times 665}{300} \times 10^{-3}$$

$$= 48{\cdot}03 + 50{\cdot}98 = 99 \text{ kN.}$$

Now shear at cut off point $\leqslant \frac{2}{3} \times$ shear resistance of section

or $98 \leqslant \frac{2}{3} \times 99$, not satisfied.

(b) Moment at cut off point

$$= 112 \times 0{\cdot}5 - \frac{0{\cdot}5^2}{2} \times 28$$

$$= 56 - 3{\cdot}5 = 52{\cdot}5 \text{ kNm.}$$

Steel required for this moment

$$= \frac{52{\cdot}5 \times 10^6}{230 \times 0{\cdot}9 \times 665} = 381 \text{ mm}^2.$$

The continuing bars shall provide double this area i.e. $2 \times 381 = 762$ mm². In fact continuing bars provide $3 \times 314 = 942$ mm²..............................(O.K.)

However, shear at cut off point shall not exceed three-fourth that permitted.

i.e. $98 \leqslant \frac{3}{4} \times 99$ or $98 \leqslant 74{\cdot}25$.

This is not satisfied.

The above two calculations show that curtailment of 50% bars cannot be done. Also it predicts that shear capacity of the section has to be increased. Check (b) requires that shear capacity of section shall be increased by $98 - 74{\cdot}25 = 23{\cdot}75$ kN.

Let us now try to provide excess stirrup area as per third check.

(c) Excess stirrup area to be provided $= \dfrac{0{\cdot}4\ b\ s}{f_y}$

The shear resisted by this area $V_s = \dfrac{A_{sv} \cdot \sigma_{sv} \cdot d}{s_v}$

Substitute $A_{sv} = \dfrac{0{\cdot}4\ b\ s}{f_y}$ and $s_v = s$

$$V_s = \frac{0{\cdot}4\ b\ s}{f_y} \times \frac{\sigma_{sv} \cdot d}{s}$$

$$= 0{\cdot}4\ bd\ \frac{(\sigma_{sv})}{f_y}.$$

Now $\dfrac{\sigma_{sv}}{f_y}$ for any steel is constant as $\dfrac{f_y}{\sigma_{sv}}$ is a factor of safety. This factor of safety is equal to $1{\cdot}8$.

Substituting, $V_s = \dfrac{0{\cdot}4\ bd}{1{\cdot}8} = \dfrac{bd}{4{\cdot}5}$

This means shear capacity of section is to be increased by $\dfrac{bd}{4{\cdot}5}$.

In present case $\dfrac{bd}{4{\cdot}5} = \dfrac{230 \times 665}{4{\cdot}5} \times 10^{-3} = 34$ kN.

Thus now stirrups to be provided for,

$$45{\cdot}35 + 34 = 79{\cdot}35 \text{ kN}.$$

∴ Using 8 mm Φ two-legged stirrups

$$s_v = \frac{A_{sv} \cdot \sigma_{sv} \cdot d}{V_s} = \frac{100 \times 230 \times 665}{79{\cdot}35 \times 10^3}$$
$$= 192{\cdot}75 \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

Now resulting spacing shall not exceed $\dfrac{d}{8\beta_b}$

where $\beta_b = \dfrac{\text{area of cut off bars}}{\text{total area of bars at section}} = 0{\cdot}5$ in this case.

Now spacing $\dfrac{665}{8 \times 0{\cdot}5} = 166$ mm$\ldots\ldots\ldots\ldots\ldots\ldots$(2)

From (1) and (2), provide 8 mm Φ stirrups about 160 mm c/c.

This excess area to be provided upto $\frac{3}{4}$ the effective depth of member i.e. $\frac{3}{4} \times 665 = 498$ mm from cut off point. From support this is done upto $500 + 498 = 998$ mm. Now minimum shear reinforcements are required upto a point, 1·07 m from support. Therefore provide 8 mm Φ about 160 mm c/c upto 1·07 m, i.e. 7 no.

Finally with 3-20 Φ bars cut off at 500 mm from the centre of support, stirrups provided are 8 mm Φ about 160 mm c/c upto 7 no. and then 8 mm Φ about 450 c/c.

Check for deflection:

basic span/d ratio = 20 $\qquad b_f = 2280$

$$\frac{100\,A_{st}}{b_f d} = \frac{100 \times 6 \times 314}{2280 \times 665} = 0{\cdot}125.$$

Modification factor = 1·9

$$\frac{b_w}{b_f} = \frac{230}{2280} = 0{\cdot}1.$$

Reduction factor = 0·8

span/d permissible = $20 \times 1{\cdot}9 \times 0{\cdot}8 = 30{\cdot}4$

actual span/d ratio $= \dfrac{8000}{665} = 12{\cdot}03 < 30{\cdot}4 \ldots$ (O.K.)

Check for cracking:

$$\text{Clear distance between bars} = \frac{230 - 50 - 3 \times 20}{2}$$

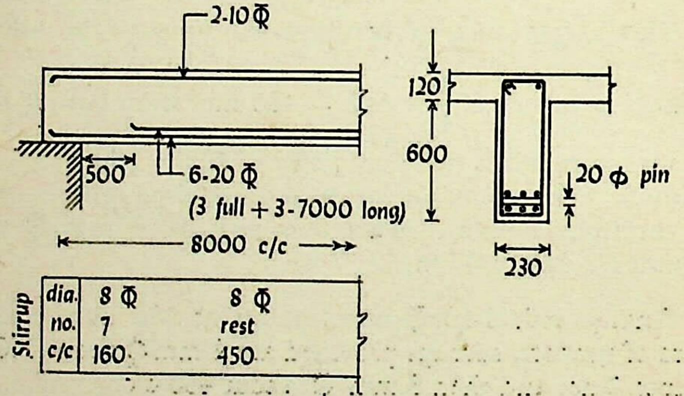
$$= 60 \text{ mm} < 180 \text{ mm}.$$

Also clear distance between bars is more than minimum required for concreting.

The check for cracking is usually not critical for beams.

For beams, the check for minimum clear distance between bars as explained in art. 2-4 is more important; e.g. in this beam all six bars cannot be placed in one layer.

Use 2-10 Φ bars as top anchor. The beam section and elevation are shown in fig. 5-11.



(a) Beam elevation (b) Section

Fig. 5-11

Supplementary details:

(1) The beam is resting on masonry walls and a bed block or template has to be designed to distribute a heavy load on walls.

(2) IS : 456 does not give any recommendation for top anchor bars in beam. It is recommended by the author to use 8 mm φ bars for a depth upto 450 mm and 10 mm φ bars for a beam of depth above 450 mm as anchor bars for normal use.

(3) For the normal cases, the curtailment of positive bars in beam leads to an elaborate calculations and economy

tending to zero e.g. in above example 3-20 Φ bars of (500 mm + 300 mm beyond centre of support) 800 mm length are saved. This is equal to a saving of $3 \times 0{\cdot}8 \times 2{\cdot}46$ kg/m = 5·9 kg mass of steel. However, excess stirrups (7 − 5 = 2) 2 no. are provided. Additional mass of stirrups will be $\frac{2}{1000} \times 2\ (230 + 720) \times 0{\cdot}395$ kg/m = 1·5 kg and a net saving of $5{\cdot}9 - 1{\cdot}5 = 4{\cdot}4$ kg is achieved on one side. For one beam this saving would be 8·8 kg total. From this data it reveals that curtailment of the bars in the beam may be left to designer's choice.

Practical notes:

(a) Width of beam: It is a modern practice to use flush type construction i.e. when a beam is running in a line of masonry wall, it should not be projected out. This restricts the width of beam equal to the unplastered thickness of wall. Usually the walls are 115 mm, 230 mm or 350 mm thick.

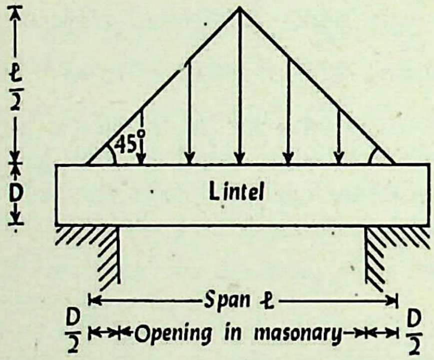
(b) Concrete shall be as far as possible, machine mixed and if hand mixed, 10% additional cement shall be used.

(c) Use of vibrator to have good compaction is always necessary.

5-8. Design of a lintel: The lintel is a beam which supports bricks or other masonry over a door or window opening.

Loads: The brick masonry transfers its load by arch action. Hence, the load on a lintel from masonry shall be of triangular shape. This fact may be verified by studying the settlement of lintel of an old building where a perfect triangle over a lintel may be observed. For good masonry work the height of the triangle is taken as one-half the base i.e. base angle of a triangle is considered as 45°. For poor masonry this angle may be considered as 60°. If a slab transfers the load within the height of a triangle, the loads from slab also shall be considered in the design of lintel. An example of lintel loading is illustrated in fig. 5-12. To get the perfect arch action, the height of masonry above lintel

shall be at least $1\frac{1}{4}$ times the height of triangle considered above. Otherwise a rectangular load of masonry above lintel shall be considered.



Example of lintel loading

FIG. 5-12

Size: The width of lintel is equal to the width of wall. The depth usually is taken as span/12 rounded to the brick layer size, e.g. one brick layer of traditional brick measures 76 mm + 12 mm mortar. Then for a lintel, for a width of opening upto 900 mm, one layer thickness can be adopted. This is done to facilitate the masonry work in a level.

Example 5-4.

Design a lintel to support a 230 mm thick brick masonry wall over a 2·0 m opening. The materials are grade M15 concrete and mild steel reinforcement.

Solution:

Width = 230 mm

$$\text{Depth} = \frac{200}{12} = 16{\cdot}66 \text{ mm}$$

or 2 brick layer + 1 mortar layer

$2 \times 76 + 13 = 165$ mm.

Use overall $D = 165$ mm.

Span of lintel $= 2 \cdot 0 + 0 \cdot 165 = 2 \cdot 165$ m say $2 \cdot 17$ m. The loading is shown in fig. 5-13.

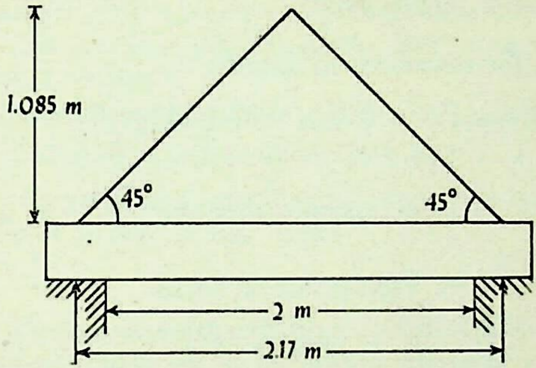


FIG. 5-13

Density of masonry work with plaster may be considered as 20 kN/m³.

$$\text{Self wt. of lintel} = 0 \cdot 165 \times 0 \cdot 23 \times 25 = 0 \cdot 95 \text{ kN/m.}$$

$$\text{Weight of masonry} = \tfrac{1}{2} \times 2 \cdot 17 \times 1 \cdot 085 \times 0 \cdot 23 \times 20$$
$$= 5 \cdot 42 \text{ kN.}$$

$$\text{Shear force} = 0 \cdot 95 \times \frac{2 \cdot 17}{2} + \frac{5 \cdot 42}{2}$$
$$= 1 \cdot 03 + 2 \cdot 71 = 3 \cdot 74 \text{ kN.}$$

$$\text{B.M.} = 0 \cdot 95 \times \frac{2 \cdot 17^2}{8} + 2 \cdot 71 \times 1 \cdot 085 - 2 \cdot 71 \times \frac{1 \cdot 085}{3}$$
$$= 0 \cdot 56 + 2 \cdot 94 - 0 \cdot 98 = 2 \cdot 52 \text{ kNm.}$$

$d_{required}$ for singly reinforced section

$$= \sqrt{\frac{2 \cdot 52 \times 10^6}{0 \cdot 87 \times 230}} = 112$$

$$d_{provided} = 165 - 15 \text{ (cover)} - 5 \text{ (use 10 } \phi \text{ bars)}$$
$$= 145 \text{ mm.} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(O.K.)}$$

$$A_{st} = \frac{2 \cdot 52 \times 10^6}{140 \times 0 \cdot 87 \times 145} = 143 \text{ mm}^2.$$

Provide 2 no. 10 mm ϕ = 157 mm².

$$p_t = \frac{157 \times 100}{230 \times 145} = 0{\cdot}47 > 0{\cdot}34$$

(minimum requirement)........................(O.K.)

Check for development length:

$$M_1 = 157 \times 140 \times 0{\cdot}87 \times 145 \times 10^{-6} = 2{\cdot}77 \text{ kNm.}$$

$$V = 3{\cdot}74 \text{ kN.}$$

$$L_o = 12\ \phi \text{ (assume) then } 1{\cdot}3\ \frac{M_1}{V} + L_o \geqslant L_d.$$

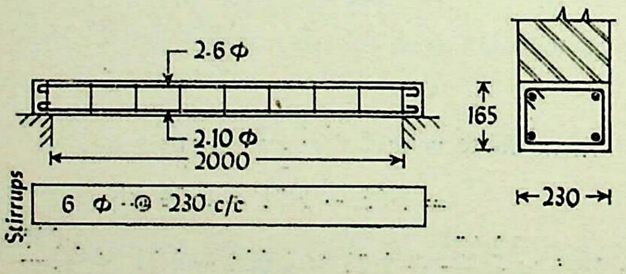
For M.S. $L_d = 58\ \phi$

$$\therefore\quad 1{\cdot}3 \times \frac{2{\cdot}77 \times 10^6}{3{\cdot}74 \times 10^3} + 12\ \phi \geqslant 58\ \phi$$

$$\therefore\quad 962 \geqslant 46\phi$$

$$\therefore\quad \phi \leqslant 20{\cdot}9.$$

Provided diameter = 10 mm..................(O.K.)



(a) Elevation (b) Section

FIG. 5-14

Check for shear:

S.F. at distance d, neglecting small triangular load $= 3{\cdot}74 - 0{\cdot}145 \times 0{\cdot}95 = 3{\cdot}6$ kN.

$$\frac{100\ A_s}{bd} = 0{\cdot}47, \qquad \tau_c = 0{\cdot}28 \text{ N/mm}^2.$$

$$\tau_v = \frac{3{\cdot}6 \times 10^3}{230 \times 145} = 0{\cdot}108 \text{ N/mm}^2 < \tau_c.$$

This is a lintel and also $\tau_v < 0{\cdot}5\ \tau_c$. There is no need of providing shear reinforcement and like a slab only distribution steel may be provided. However, for a lintel of depth greater than 76 mm, it is usual to provide some shear reinforcement. The spacing can be more than minimum shear reinforcement required.

Provide 2-8 ϕ top anchor bars, 2-10 ϕ bottom bars and 6 mm ϕ two-legged stirrups about 230 c/c.

Check for deflection, cracking etc. can be made as usual. The section is shown in fig. 5-14.

CANTILEVER BEAMS

5-9. Design considerations: The design principles of the cantilever beams are same as simply supported beams. However, some points of consideration are:

(a) Even though the deflection check is satisfied, usually a camber is provided under a beam. *Camber* is a term applied to the slight upward curve of a beam made in construction such that on loading it will straighten out and attain its correct shape. For the long span simply supported beams, continuous beams, cantilever slabs etc. also this can be done.

(b) Designer shall be satisfied in providing sufficient anchoragement of bars.

(c) Stability of a structure with respect to overturning is an important thing when an overhang is designed. According to clause 19.1 of IS : 456,

"The stability of a structure as a whole against overturning shall be ensured so that the restoring moment shall be not less than the sum of 1·2 times the maximum overturning moment due to the characteristic imposed loads. In cases, where dead load provides the restoring moment, only 0·9 times the characteristic dead load shall be considered. Restoring moment due to imposed loads shall be ignored.

The anchorages of counter-weights provided for overhanging members (during constructon and service) should be such that static equilibrium should remain, even when overturning moment is doubled".

Example 5-5.

An overhanging beam BC is shown in fig. 5-15. Find out minimum counter-weight w if it has to be safe against overturning.

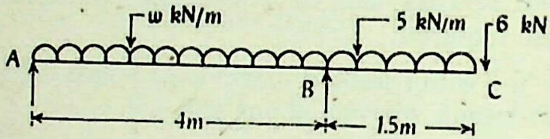


Fig. 5-15

Solution:

$$\text{Overturning moment} = 6 \times 1{\cdot}5 + 5 \times \frac{1{\cdot}5^2}{2}$$

$$= 9 + 5{\cdot}625 = 14{\cdot}625 \text{ kNm.}$$

Static equilibrium shall be there even if this moment is doubled i.e. $M = 2 \times 14{\cdot}625 = 29{\cdot}25$ kNm.

To find minimum counter-weight w means reaction at A shall be zero (shall not be negative).

$$\text{Reaction at } A = \frac{w \times 4}{2} - \frac{29{\cdot}25}{4} = 0$$

OR $$w = 3{\cdot}66 \text{ kN/m.}$$

The counter-weight in portion AB shall be more than 3·66 kN/m.

Example 5-6.

Design a cantilever rectangular beam of span 3 m and carrying a U.D.L. of 8 kN/m. Assume that sufficient safety against overturning is there and reinforcement anchorages are also available. Use M15 grade concrete and mild steel reinforcement.

Solution:

Assume an initial trial section of 230 mm × 600 mm overall depth to consider self weight.

Then self wt. $= 0{\cdot}23 \times 0{\cdot}6 \times 25 = 3{\cdot}45$ kN/m.

Total load $= 8 + 3{\cdot}45 = 11{\cdot}45$ kN/m.

Loading S.F. and B.M. diagrams are shown in fig. 5-16.

S.F. $= 3 \times 11{\cdot}45 = 34{\cdot}35$ kN.

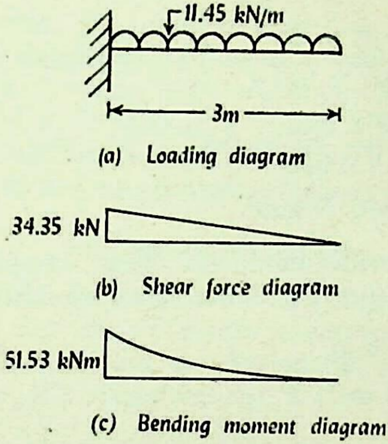


FIG. 5-16

$$\text{B.M.} = 11{\cdot}45 \times \frac{3^2}{2} = 51{\cdot}53 \text{ kNm.}$$

$$\text{Depth required} = \sqrt{\frac{51{\cdot}53 \times 10^6}{0{\cdot}87 \times 230}} = 507 \text{ mm.}$$

Using one layer of 20 mm ϕ bars and overall depth of 550 mm.

$$d = 550 - 25 - 10 = 515 \text{ mm.}$$

$$A_{st} = \frac{51{\cdot}53 \times 10^6}{140 \times 0{\cdot}87 \times 515} = 822 \text{ mm}^2.$$

Provide 3-20 ϕ giving $A_{st} = 3 \times 314 = 942$ mm².

Check for development length:

$$\text{Stress in bar} = 140 \times \frac{822}{942} = 122{\cdot}2 \text{ N/mm}^2.$$

$$L_d = \frac{\phi \times 122{\cdot}2}{4 \times 0{\cdot}6} = 51\,\phi = 51 \times 20 = 1020 \text{ mm.}$$

The bars shall extend into the support for a straight length of 1020 mm. Provide anchorage of 1200 mm. If for some case the bars are to be bent; e.g. anchored in column,

the bearing stress around the bend has to be checked as discussed in chapter 3.

Check for shear:

$$\tau_v = \frac{34.35 \times 10^3}{230 \times 515} = 0.29 \text{ N/mm}^2.$$

$$\frac{100\,A_s}{bd} = \frac{100 \times 942}{230 \times 515} = 0.8$$

$$\tau_c = 0.346 \text{ N/mm}^2.$$

$\tau_v < \tau_c$, provide minimum shear reinforcement. For 230 wide beam, provide 6 mm ϕ about 150 c/c.

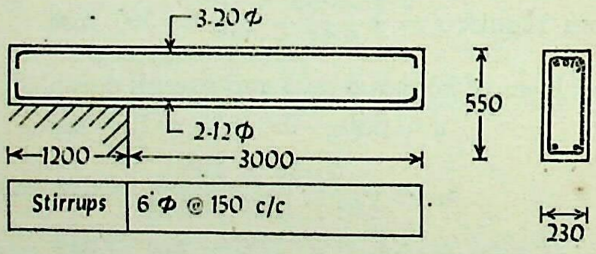
Check for deflection:

basic span/d = 7

$$100\,\frac{A_{st}}{bd} = 0.8 \qquad \therefore \quad \text{modification factor} = 1.54$$

span/d permissible = 7 × 1·54 = 10·78

$$\text{actual span/d ratio} = \frac{3000}{515} = 5.82 < 10.79 \ldots\ldots \text{(O.K.)}$$



(a) Elevation (b) Section

Fig. 5-17

Check for cracking:

$$\text{Clear distance between bars} = \frac{230 - 50 - 3 \times 20}{2}$$

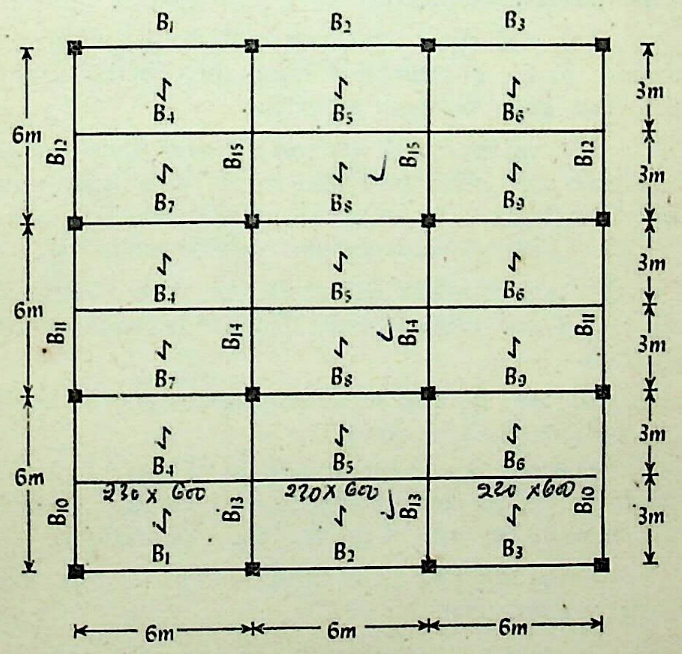
$$= 60 \text{ mm} < 300 \text{ mm} \ldots\ldots \text{(O.K.)}$$

The beam elevation and section are shown in fig. 5-17.

Note: The anchorage shown is minimum and assumed that sufficient counter-weight is available. However, unless a concentrated load is acting in this region of counter-balance, it is very difficult to achieve equilibrium with a double over-turning moment. In practice, as a thumb rule anchorages, of 1·5 times the cantilever length are given to such beams (provided that it satisfies the minimum anchorage require-ments) i.e. 4·5 m for this case. Even after the concrete has attained its sufficient strength, the centering of beam shall not be removed unless, the counter-weight is achieved.

CONTINUOUS BEAMS

5-10. Introductory: The continuous beams can be analysed by the elastic theory applicable to homogeneous material. These beams are frequently occurring in cast-in situ construction. Consider the R.C.C. floor as shown in fig. 5-18.



Typical cast-in-situ R.C.C. floor
FIG. 5-18

Span of the slabs is one way. The beams B_4-B_5-B_6 are the floor beams and may be designed as continuous beams capable of free rotations at the supports like B_{12} and B_{15}. The beams B_1-B_2-B_3; B_7-B_8-B_9; B_{10}-B_{11}-B_{12} and B_{13}-B_{14}-B_{15} are supported on columns. These beams together with columns shall be designed as continuous frames.

5-11. Analysis parameters: Important analysis parameters are summarized as follows:

(a) *Effective span:* In case of continuous beam or slab if the width of the support is less than $\frac{1}{12}$ of the clear span, the effective span shall be taken as clear span plus effective depth of slab or beam or centre to centre of supports, whichever is less. However if the supports are wider than $\frac{1}{12}$ the clear span or 600 mm whichever is less, the effective span shall be taken as under:

(1) For end span with one end fixed and the other continuous or for intermediate span, the effective span shall be the clear span between supports.

(2) For end span with one end free and other continuous, the effective span shall be equal to the clear span plus half the effective depth of beam or slab or the clear span plus half the width of discontinuous support, whichever is less.

In the case of spans with roller or rocker bearing, the effective span shall always be the distance between the centres of bearings.

(3) In the analysis of a continuous frame, centre to centre distance shall be used.

(b) *Stiffness:* The relative stiffness of the members may be based on the moment of inertia of the section determined on the basis of any one of the following definitions.

(1) Gross section: The gross section of the member ignoring reinforcement.

(2) Transformed section: The concrete cross-section plus the area of reinforcement transformed on the basis of modular ratio.

(3) Cracked section: The area of concrete in compression plus the area of reinforcement transformed on the basis of modular ratio.

The assumptions made shall be consistent for all the members of the structure throughout the analysis. However, for deflection calculations, appropriate values of moment of inertia as specified in appendix B of IS : 456 should be used.

The first definition is generally used in analysis of continuous beams. For a rectangular section, the moment of inertia is $\frac{1}{12}\, bD^3$, while for a flanged beam this shall be separately worked out. After arriving the width and thickness of flange, the centroidal axis is found out and about this axis the moment of inertia is found out. For ready reference, graphs of ratio $\frac{b_f}{b_w}$ verses coefficient K_t are given in chart 88 of SP : 16. Then moment of inertia of a flanged beam will be

$$K_t \times \frac{1}{12}\, b_w D^3.$$

Example 5-7.

Find the moment of inertia of a flanged beam shown in fig. 5-19.

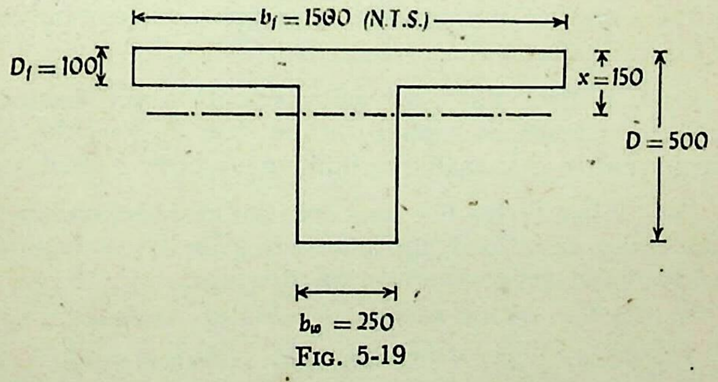


FIG. 5-19

Solution :

Taking moment about top flange,

$$\bar{y} = \frac{1500 \times 100 \times 50 + 250 \times 400 \times 300}{1500 \times 100 + 250 \times 400} = 150$$

$$I_{xx} = \tfrac{1}{3} \times 250 \times 500^3 + \tfrac{1}{3} \times 1250 \times 100^3 - (1500 \times 100 + 250 \times 400) \times 150^2$$
$$= 10{\cdot}4 \times 10^9 + 0{\cdot}42 \times 10^9 - 5{\cdot}62 \times 10^9$$
$$= 5{\cdot}2 \; 10^9 \text{ mm}^4.$$

Or using chart 88 of SP : 16,

$$\frac{D_f}{D} = \frac{100}{500} = 0{\cdot}2 \qquad \frac{b_f}{b_w} = \frac{1500}{250} = 6$$
$$K_t = 2$$
$$\therefore \quad I = 2 \times \frac{1}{12} \times 250 \times (500)^3$$
$$= 5{\cdot}2 \times 10^9 \text{ mm}^4.$$

5-12. Structural frames: The following simplifying assumptions may be used in the analysis of frames.

Arrangement of live load:

(a) Consideration may be limited to combinations of:

(1) Design dead load on all spans with full design live load on two adjacent spans.

(2) Design dead load on all spans with full design live load on alternate spans.

For a three span continuous beam the above requirements are shown in fig. 5-20.

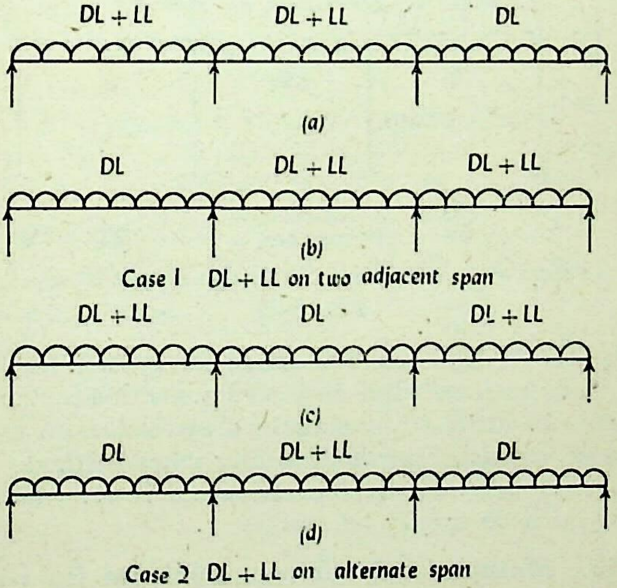
Case 1 gives the load arrangement to get maximum negative moments at supports while case 2 gives the load arrangement to get maximum positive moments at mid-span.

(b) When design live load does not exceed three-fourths of the design dead load, the load arrangement may be design dead load and design live load on all spans.

Note: For beams and slabs continuous over support (a) may be used.

Considering fig. 5-18, the load arrangement in (a) is applied to continuous beams like B_1-B_2-B_3 and B_4-B_5-B_6. If the design live load does not exceed three-fourths of the design dead load, then, for beams like B_1-B_2-B_3-(b) may be used. However for beams like B_4-B_5-B_6, supported on main beams, discussion in (a) shall be used.

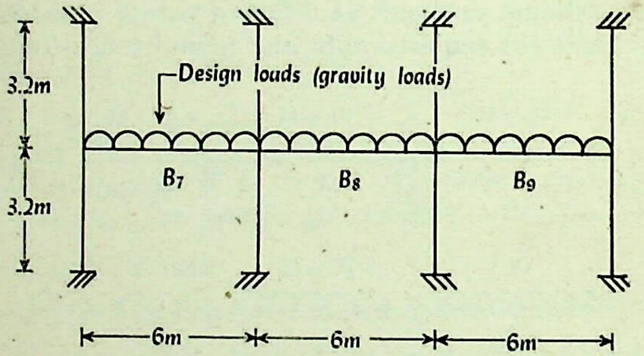
The close observations of fig. 5-18 shows that beams like B_1-B_2-B_3 are framed with column which restricts the rotation of the beam at the supports. Therefore, load arrangement as described in (b) is permitted. For the beams like B_4-B_5-B_6, the rotation is not restricted and the beams are capable of free rotation at supports. Hence, for these beams load arrangement (a) shall be used. Note that load arrangement (a) requires four moment distributions to get maximum moments at different points while the load arrangement (b) requires only one moment distribution.



Arrangement of live load
FIG. 5-20

Substitute frame: For a multi-storeyed frame, the analysis of moments and shears due to gravity loads can be done at a time for a whole frame using computers. However, this is clumsy and may involve serious mistakes if done manually. Therefore, to analyse a particular floor, a substitute frame is used. In this, for determining the moments

and shears at any floor or roof level due to gravity loads, the beams at that level together with columns above with their far ends fixed may be considered to constitute the substitute frame e.g. for a multistoreyed building shown in plan such as in fig. 5-18, if the storey height is 3·2 m, the substitute frame for beams B_7-B_8-B_9 of intermediate floor is shown in fig. 5-21.



Typical floor substitute frame for beams B_7-B_8-B_9

FIG. 5-21

The above discussion was made for gravity loads. For lateral loads such as wind and earthquake loads, the whole frame may be analysed by simplified methods such as portal method or cantilever method or any other method. When the frame is unsymmetrical or structure is very tall, more rigorous methods should be used.

5-13. Moment and shear coefficients for continuous beams: Unless more exact estimates are made, for beams of uniform cross-section which support substantially uniformly distributed loads over three or more spans which do not differ by more than 15 per cent of the longest, the bending moments and shear forces used in design may be obtained using the coefficients given in table 5-1 and table 5-2 respectively.

For moments at supports where two unequal spans meet or in case where the spans are not equally loaded, the average of the two values for the negative moment at the support may be taken for design.

Where coefficients given in table 5-1 are used for calculation of bending moments, redistribution of moment shall not be permitted.

Where a member is built into a masonry wall which develops only partial restraint, the member shall be designed to resist a negative moment at the face of the support of $\frac{Wl}{24}$, where W is the total design load and l is the effective span or such other restraining moment as may be shown applicable. For such a condition shear coefficients given in table 5-2 at the end support may be increased by 0·05.

TABLE 5-1

BENDING MOMENT COEFFICIENTS

Type of load	Span moments		Support moments	
	Near middle of end span	At middle of interior span	At support next to the end support	At other interior supports
Dead load and imposed load (fixed)	$+\frac{1}{12}$	$+\frac{1}{24}$	$-\frac{1}{10}$	$-\frac{1}{12}$
Imposed load (not fixed)	$+\frac{1}{10}$	$+\frac{1}{12}$	$-\frac{1}{9}$	$-\frac{1}{9}$

Note: For obtaining bending moment the coefficient shall be multiplied by the total design load and effective span.

TABLE 5-2

SHEAR FORCE COEFFICIENTS

Type of load	At end support	At support next to the end support		At all other interior supports
		Outer side	Inner side	
Dead load and imposed load (fixed)	0·4	0·6	0·55	0·5
Imposed load (not fixed)	0·6	0·6	0·6	0·6

Note: For obtaining the shear force, the coefficient shall be multiplied by the total design load.

5-14. Critical sections for moments and shear: These are covered in clause 21.6 of IS : 456 and are given below:

(a) *Moment:* For monolithic construction, the moments computed at the face of the supports shall be used in design of the members at those sections. For non-monolithic construction the design of the member shall be done considering the effective span as previously defined for simply supported beam.

The above statement suggests for the frames that when moment distribution of the frame is done on the basis of centre to centre distance, there can be a reduction in negative moment. This implies automatically that there can be a reduction to the positive moment also. The reduction to the negative moment is $\frac{VL_a}{3}$ and to the positive moment it shall be $\frac{VL_a}{6}$, where V is the shear force at the support and L_a is the width of the support in the direction of span of the beam. In the examples that follow, this correction is not introduced. The reader may refer, "Continuity in Concrete Building Frames" by Portland Cement Association, U.S.A. for further studies.

(b) *Shear:* The critical section for checking shear shall be as explained for simply supported beams in art. 5-3.

5-15. Redistribution of moment: For a continuous beam ABC as shown in fig. 5-22, prior to failure, a hinge is formed at the point of maximum moment at support B. This

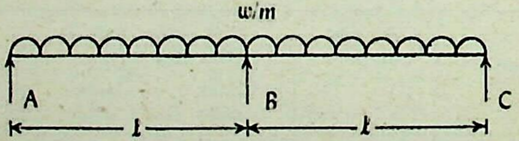


Fig. 5-22

will increase the positive moment in the span AB and BC. In other words, the moment redistribution takes place. This behaviour of materials can be utilised in design where a hogging moment as obtained by elastic analysis is reduced to a

desired value and span moments are calculated using this modified moment at support.

Theoretically speaking a hinge can be formed at point *B*. This kind of the analysis is developed for the design of steel sections. For concrete, however the following points shall have to be considered:

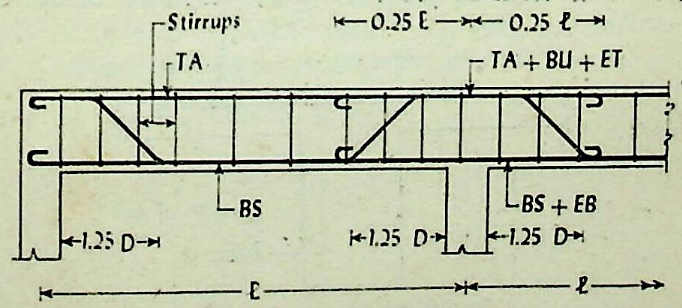
(1) There should not be serious cracking in the concrete.

(2) There should be adequate ductility at the hinge point.

To ensure above limitations, code permits the moment redistribution to 15 per cent for elastic theory of design and 30 per cent for limit state method of design subject to some requirements discussed in clause 36.1.1 of IS : 456.

5-16. Negative moment reinforcement: At least one-third of the total reinforcement provided for negative moment at the support shall extend beyond the point of inflection for a distance not less than the effective depth of the member or one-sixteenth of the clear span whichever is greater.

In normal cases, the point of inflection lies at 0·15 *l* from the continuous support. In most cases if the bar is extended for 0·1 *l* from this point, the above requirements are satisfied. Therefore, the bars are extended from centre of the support upto a distance equal to 0·25 *l* where *l* is the span of the beam. A typical continuous beam reinforcement details are illustrated in fig. 5-23.



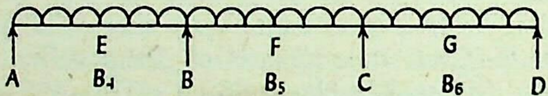
TA = Top anchor bars *ET* = Extra top bars

BS = Bottom straight bars *EB* = Extra bottom bars

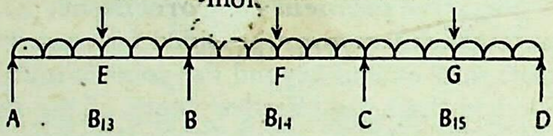
Typical reinforcement in continuous beam

FIG. 5-23

5-17. Further discussion of Tee beam: Consider a beam B_4-B_5-B_6 in fig. 5-18. A line diagram of this beam is shown in fig. 5-24(*a*). Consider points *E*, *F* or *G*. At this section, the moment is positive which induces compression at top. At top, the concrete of the flange area is available and the beam acts as a tee beam. Now consider a section near the support *B* or *C*. At this section the moment is negative inducing compression at bottom. The slab concrete is not available in compression zone of this section and the beam acts as a rectangular beam.



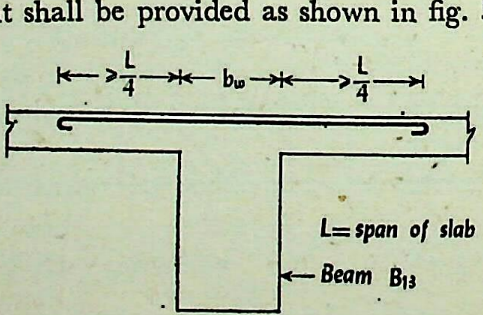
(a) Continuous beam B_4-B_5-B_6



(b) Continuous beam B_{13}-B_{14}-B_{15}

FIG. 5-24

Now consider beam B_{13}-B_{14}-B_{15} as shown in fig. 5-24(*b*). The beam acts as a rectangular beam at a section near the support *B* or *C* as discussed above. At point *E*, *F* or *G* the beam acts as a tee beam. However, in this case as the main reinforcement of slab is parallel to the beam, transverse reinforcement shall be provided as shown in fig. 5-25. Such



Section at E of beam B_{13}

Transverse reinforcement in flange of *T* beam when main reinforcement of slab is parallel to the beam

FIG. 5-25

reinforcement shall not be less than 60 per cent of the main reinforcement at mid span of the slab.

Example 5-8.

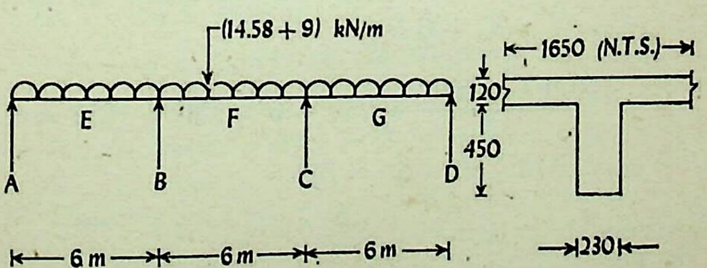
An R.C.C. floor is used as a banking hall and plan is shown in fig. 5-18. Design beams B_4-B_5-B_6. The materials are grade 15 concrete and tor steel reinforcement of grade Fe 415. Use live load 3 kN/m². Slab thickness is 120 mm and depth of rib is restricted to 450 mm.

Solution:

(a) Estimation of loads:

Slab 120 mm thick $0{\cdot}12 \times 25$	$= 3$	kN/m²
Floor finish	$= 1$	kN/m²
Live load	$= 3$	kN/m²
Total	$(4 + 3)$	kN/m²
Load on beam $= 3\ (4 + 3)$	$= 12 + 9$	kN/m
Self $0{\cdot}23 \times 0{\cdot}45 \times 25$	$= 2{\cdot}58 + 0$	kN/m
Total	$(14{\cdot}58 + 9)$ DL LL	kN/m

In this case one observes that the design live load is not exceeding three-fourth the design dead load ($0{\cdot}75 \times 14{\cdot}58 = 10{\cdot}95$ kN/m). However this is a continuous beam and not the part of a frame, the arrangement of loads shall be as per a(1) and a(2) of art. 5-12. Instead of doing analysis for all cases involving four moment distributions, the coefficients given in code shall be used. Also note that redistribution of the moment is not permitted when the coefficients are used.



(a) Beam loading (b) Assumed section

Fig. 5-26

(b) Analysis for shear and moments:

Using coefficients,

Moments:

$$M_E = M_G = \frac{1}{12} \times 14{\cdot}58 \times 6^2 + \frac{1}{10} \times 9 \times 6^2$$

$$= 43{\cdot}74 + 32{\cdot}4 = 76{\cdot}14 \text{ kNm.}$$

$$M_F = \frac{1}{24} \times 14{\cdot}58 \times 6^2 + \frac{1}{12} \times 9 \times 6^2$$

$$= 21{\cdot}87 + 27 = 48{\cdot}87 \text{ kNm.}$$

$$M_B = M_C = -\frac{1}{10} \times 14{\cdot}58 \times 6^2 - \frac{1}{9} \times 9 \times 6^2$$

$$= -52{\cdot}49 - 36 = -88{\cdot}49 \text{ kNm.}$$

Shears:

$$V_{AB} = V_{DC} = 0{\cdot}4 \times 14{\cdot}58 \times 6 + 0{\cdot}45 \times 9 \times 6$$

$$= 35 + 24{\cdot}3 = 59{\cdot}3 \text{ kN.}$$

$$V_{BA} = V_{CD} = 0{\cdot}6 \times 14{\cdot}58 \times 6 + 0{\cdot}6 \times 9 \times 6$$

$$= 52{\cdot}5 + 32{\cdot}4 = 84{\cdot}9 \text{ kN.}$$

$$V_{BC} = V_{CB} = 0{\cdot}55 \times 14{\cdot}58 \times 6 + 0{\cdot}6 \times 9 \times 6$$

$$= 48{\cdot}1 + 32{\cdot}4 = 80{\cdot}5 \text{ kN.}$$

(c) Design for moment:

Span AB:

Maximum positive moment $= 76{\cdot}14$ kNm. Beam is acting as tee beam.

$$b_f = \frac{l_o}{6} + b_w + 6\, D_f$$

$$= \frac{0{\cdot}7 \times 6000}{6} + 230 + 6 \times 120 = 1650 \text{ mm.}$$

$$d_{eff} = 570 - 25 - 10 = 535.$$

Assume lever arm $= d - \dfrac{D_f}{2}$

$$A_{st} = \frac{76{\cdot}14 \times 10^6}{230\ (535 - 60)} = 697 \text{ mm}^2.$$

Provide 4-16 mm Φ = 804 mm².

To find N.A. taking moment about N.A.

$$1650 \cdot x \cdot \frac{x}{2} = 18{\cdot}66 \times 804 \ (535 - x)$$

$$x^2 = 9732 - 18{\cdot}2\,x$$

which gives $x = 90$ mm.

Depth of critical N.A. $= 0{\cdot}29 \times 535 = 155{\cdot}15$ mm.

$\therefore$ The section is under-reinforced.

$$\text{M.R.} = A_{st}\,\sigma_{st}\left(d - \frac{x}{3}\right)$$

$$= 804 \times 230\ (535 - 30) \times 10^{-6}$$

$$= 93{\cdot}38 \text{ kNm} > 76{\cdot}14 \text{ kNm} \ldots\ldots\ldots\ldots\ldots \text{(O.K.)}$$

$\therefore$ Adopt 4-16 mm Φ.

Span BC:

$$A_{st} = \frac{48{\cdot}87 \times 10^6}{230\ (535 - 60)} = 447 \text{ mm}^2.$$

Provide 3-16 mm Φ = 603 mm²

$$\text{Minimum steel} = \frac{0{\cdot}205}{100} \times 230 \times 535 = 252 \text{ mm}^2$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(O.K.)}$$

Support B or C:

$M = 88{\cdot}49$ kNm.

The beam acts as a rectangular beam.

$$\frac{M}{bd^2} = \frac{88{\cdot}49 \times 10^6}{230 \times 535^2} = 1{\cdot}34 > 0{\cdot}66.$$

Design as a doubly reinforced beam.

$$M_1 = 0{\cdot}66 \times 230 \times 535^2 \times 10^{-6} = 43{\cdot}45 \text{ kNm}$$

$$M_2 = 88{\cdot}49 - 43{\cdot}45 = 45{\cdot}03 \text{ kNm}$$

$$A_{st1} = \frac{43{\cdot}45 \times 10^6}{230 \times 0{\cdot}9 \times 535} = 392 \text{ mm}^2.$$

Let the compression reinforcement be provided at $d' = 40$ mm from extreme compression fibre.

Then $A_{st2} = \dfrac{45{\cdot}03 \times 10^6}{230\,(535 - 40)} = 396\ \text{mm}^2.$

Corresponding compression steel.

$$A_{sc} = \frac{M_2}{(1{\cdot}5\,\text{m} - 1)\,\sigma_{cbc}\left(\dfrac{x - d'}{x}\right)\,(d - d')}$$

$$x = 0{\cdot}29 \times 535 = 155{\cdot}15\ \text{mm}.$$

$$A_{sc} = \frac{45{\cdot}03 \times 10^6}{(1{\cdot}5 \times 18{\cdot}66 - 1) \times 5 \times \left(\dfrac{155{\cdot}15 - 40)}{(155{\cdot}15}\right)(535 - 40)}$$

$$= 908\ \text{mm}^2$$

$$A_{st} = A_{st1} + A_{st2}$$

$$= 392 + 396 = 788\ \text{mm}^2.$$

Bottom bars of 4-16 mm ϕ will be used as compressive reinforcement plus 1-16 mm ϕ shall be used as extra bottom bar, totally giving $804 + 201 = 1005\ \text{mm}^2$. At top provide 4-16 mm $\phi = 804\ \text{mm}^2$ (tension).

Comment: When a doubly reinforced beam is designed using elastic theory, larger compression reinforcements are required as compared to the section designed by limit state theory, e.g. the section at support B, while designing with limit state method is a singly reinforced section having a tension steel of 850 mm² (refer Ex. 9-6) and does not require any compression steel. In practice, usually a limit state method is used which gives most economical section.

(d) Check for development length:

Support A,

$$V = 59{\cdot}3\ \text{kN}$$

$$M_1 = 4 \times 201 \times 230 \times 0{\cdot}9 \times 535 \times 10^{-6} = 89\ \text{kNm}.$$

$$\text{Service stress} = 230 \times \frac{697}{804} = 199{\cdot}4\ \text{N/mm}^2$$

$$L_d = \frac{\phi \times 199{\cdot}4}{1{\cdot}4 \times 4 \times 0{\cdot}6} = 59{\cdot}3\ \phi.$$

Now $1{\cdot}3\,\dfrac{M_1}{V} + L_o \geqslant L_d$

assuming $L_o = 12\ \phi$

$$1{\cdot}3 \times \frac{89 \times 10^6}{59{\cdot}3 \times 10^3} + 12\ \phi \geqslant 59{\cdot}3\ \phi$$

$1951 \geqslant 47{\cdot}3\ \phi$ or $\phi \leqslant 41$ .... (O.K.)

Span AB:

Point of contraflecture at $0{\cdot}15\ l = 0{\cdot}9$ m from support B,

$V = 84{\cdot}9 - 0{\cdot}9 \times 23{\cdot}58 = 63{\cdot}68$ kN.

L_o in this case is $12\ \phi = 12 \times 16 = 192$ mm

or $d_{eff} = 535$ whichever is greater.

$\therefore \quad L_o = 535$ mm.

$$1{\cdot}3\ \frac{M_1}{V} + L_o \geqslant L_d$$

$$1{\cdot}3 \times \frac{89 \times 10^6}{63{\cdot}68 \times 10^3} + 535 \geqslant 59.3\ \phi$$

$\phi \leqslant 39{\cdot}66$ .... (O.K.)

Span BC need not be checked.

(e) Check for shear:

Span AB, at support A: As the reinforcements are confined with compressive reaction, shear at distance d will be checked.

$V = 59{\cdot}3 - 0{\cdot}535 \times 23{\cdot}58 = 46{\cdot}68$ kN.

$$\frac{100\ A_s}{bd} = \frac{100 \times 4 \times 201}{230 \times 535} = 0{\cdot}65 \text{ and } \tau_c = 0{\cdot}32\ \text{N/mm}^2.$$

Shear resistance of concrete $= 0{\cdot}32 \times 230 \times 535 \times 10^{-3}$
$= 39{\cdot}38$ kN.

$$V_s = V - \tau_c\, bd = 46{\cdot}68 - 39{\cdot}38 = 7{\cdot}3 \text{ kN}.$$

For 230 wide beam, minimum shear reinforcement from table 3-4 is 6 mm ϕ about 150 c/c (mild steel) for which

$$V_s = \frac{140 \times 56 \times 535}{150} \times 10^{-3} = 27{\cdot}96 \text{ kN}.$$

Provide minimum shear reinforcement i.e. 6 mm ϕ about 150 c/c.

At support B, $V_{BA} = 84{\cdot}9$ kN.

At distance d, $V = 84{\cdot}9 - 0{\cdot}535 \times 23{\cdot}58 = 72{\cdot}28$ kN.

Shear resistance of concrete $= 39{\cdot}38$ kN.

Shear resistance provided by minimum shear reinforcement $= 27{\cdot}96$ kN.

Shear resistance of a section with minimum shear reinforcement $= 39{\cdot}38 + 27{\cdot}96 = 67{\cdot}34$ kN.

This occurs from support at $\dfrac{84{\cdot}9 - 67{\cdot}34}{23{\cdot}58} = 0{\cdot}74$ m.

At distance d from support

$$V = 72{\cdot}28 \text{ kN.}$$

$$\tau_c bd = 39{\cdot}38 \text{ kN.}$$

$$V_s = 72{\cdot}28 - 39{\cdot}38 = 32{\cdot}9 \text{ kN.}$$

$$s_v = \frac{140 \times 56 \times 535}{32{\cdot}9 \times 10^3}$$

$$= 127{\cdot}5 \text{ mm.}$$

Provide 6 mm ϕ about 120 c/c.

This is required upto 0·74 m, provide $\dfrac{0{\cdot}74}{0{\cdot}12} + 1 \simeq 8$ no.

Then provide 6 mm ϕ about 150 c/c minimum shear reinforcement.

Span BC, at B, $V = 80{\cdot}5$ kN.

At distance d,

$$V = 80{\cdot}5 - 0{\cdot}535 \times 23{\cdot}58 = 67{\cdot}88 \text{ kN.}$$

$$\frac{100\ A_s}{bd} = \frac{100 \times 3 \times 201}{230 \times 535} = 0{\cdot}49$$

$$\tau_c = 0{\cdot}29 \text{ N/mm}^2$$

$$\tau_c bd = 0{\cdot}29 \times 230 \times 535 \times 10^{-3} = 35{\cdot}68 \text{ kN.}$$

$$V_s = 67{\cdot}88 - 35{\cdot}68 = 32{\cdot}2 \text{ kN.}$$

Using 6 mm ϕ stirrups,

$$s_v = \frac{140 \times 56 \times 535}{32{\cdot}2 \times 10^3}$$

$$= 130{\cdot}2 \text{ mm}$$

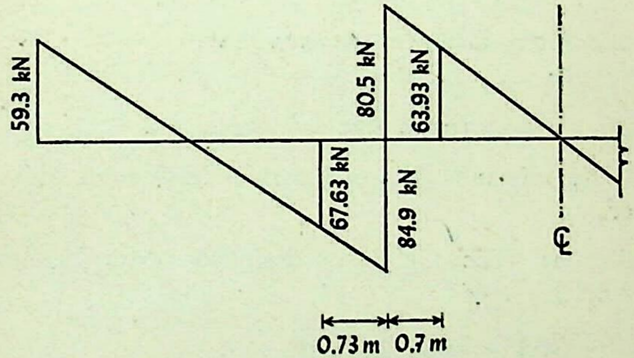
Provide 6 mm ϕ about 120 c/c.

With minimum shear reinforcements, the shear capacity of section $= 35 \cdot 68 + 27 \cdot 96 = 63 \cdot 64$ kN.

$$\text{This occurs at } \frac{80 \cdot 5 - 63 \cdot 64}{23 \cdot 58} = 0 \cdot 72 \text{ m from support.}$$

$$\text{Provide } \frac{0 \cdot 72}{0 \cdot 12} + 1 = 7 \text{ no.}$$

Provide 6 mm ϕ about 120 c/c 7 no. and then 6 mm ϕ about 150 c/c. The shear force diagram is drawn in fig. 5-27.



S.F. diagram

FIG. 5-27

(f) Check for deflection:

Span AB has larger reinforcement and area is critical.

$$\text{Basic } \frac{\text{span}}{\text{d}} \text{ ratio} = 26.$$

$$\frac{100 A_{st}}{bd} = \frac{4 \times 201 \times 100}{1650 \times 535} = 0 \cdot 09$$

Modification factor $= 2$

$$\frac{b_w}{b_f} = \frac{230}{1650} = 0 \cdot 4$$

Reduction factor $= 0 \cdot 8$

$$\frac{\text{span}}{\text{d}} \text{ permissible} = 26 \times 2 \times 0{\cdot}8 = 41{\cdot}6$$

$$\text{actual } \frac{\text{span}}{\text{d}} \text{ ratio} = \frac{6000}{535} = 11{\cdot}2 < 41{\cdot}6.$$

The beam is safe w.r.t. deflection.

(g) Check for cracking:

$$\text{Clear distance between bars} = \frac{230 - 50 - 4 \times 16}{3}$$

$$= 38{\cdot}66 < 180 \ldots\ldots \quad \text{(O.K.)}$$

Beam is satisfactory w.r.t. cracking.

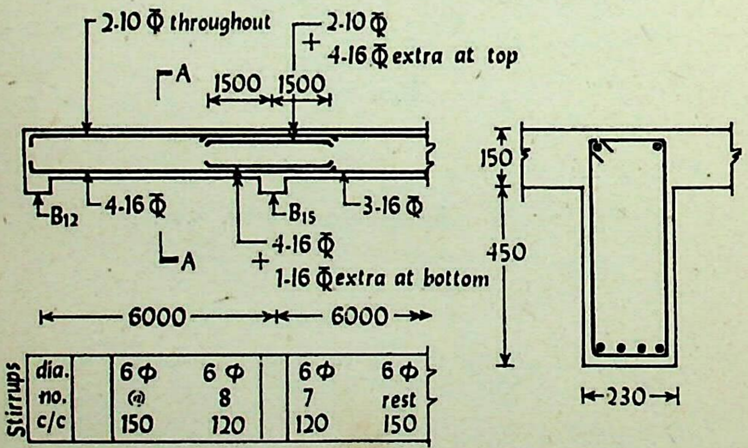
Also minimum distance between bars

(1) 16 mm (φ of bar)

(2) 20 mm (max. size of C.A.) + 5 = 25 mm.

As the distance between bars is 38·66 mm, this is also satisfied.

(h) Sketch: The sketch of designed beam is shown in fig. 5-28.



(a) Beam section (b) Section AA

FIG. 5-28

(i) Supplementary details:

(1) The beam is transferring the loads to the main beam and the reactions of this beam will be point loads for beams B_{12} and B_{15}.

The point load on beams B_{12}, B_{11} or B_{10} will be 59·3 kN.

The point load on beams B_{13}, B_{14} or B_{15} will be $84{\cdot}9 + 80{\cdot}5 = 165{\cdot}4$ kN.

(2) This beam is designed only for gravity loads. For the lateral loads like earthquake or wind loads this will not be designed. Even if it is asked to design the frame for the lateral loads, these beams will be considered ineffective for lateral loads. In fact for the lateral loads, the whole slab is acting as a wide beam and transfers the lateral loads to the main beams, which are framed with columns. Therefore the beam like B_7-B_8-B_9 shall be designed for lateral loads. In this book, the study is limited to gravity loads only.

Example 5-9.

Design the beams B_{13}-B_{14}-B_{15} of fig. 5-18 for the typical floor level. The building is a four storeyed building having a floor height of 4·2 m. Toilet and staircase block is a separate unit from this structure and is not considered for convenience. The size of rib may be taken as 350 mm × 600 mm. Size of the columns is 350 mm × 350 mm.

Solution:

Using the substitute frame, the beam is shown in fig. 5-29. The loads are point loads from floor beams and self uniformly distributed load $= 0{\cdot}35 \times 0{\cdot}6 \times 25 = 5{\cdot}25$ say 6 kN/m.

Point load from Ex. 5-8 = 165·4 kN say 166 kN.

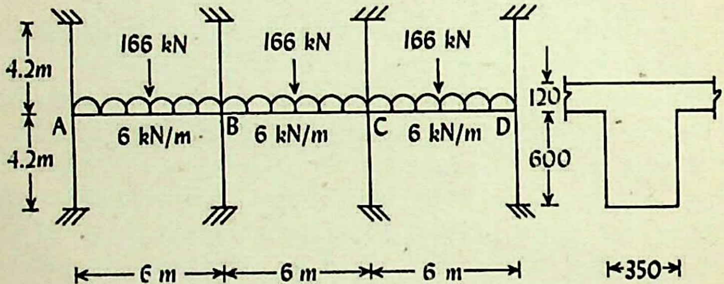
In this case, the beams are rectangular beams and hence, moment of inertia will be $\frac{1}{12}bD^3$. However if the beams B_7-B_8-B_9 were to be designed, moment of inertia of beam would be that of tee beam.

$$\text{M.I. of beam} = \frac{1}{12} \times 350 \times 720^3 = 1{\cdot}089 \times 10^{10}\ \text{mm}^4.$$

$$\frac{I}{L} = \frac{1{\cdot}089 \times 10^{10}}{6000} = 1{\cdot}81 \times 10^{6}\ \text{mm}^3.$$

$$\text{M.I. of column} = \frac{1}{12} \times 350 \times 350^3 = 1{\cdot}25 \times 10^9\ \text{mm}^4$$

$$\frac{I}{L} = \frac{1{\cdot}25 \times 10^9}{4200} = 2{\cdot}97 \times 10^5\ \text{mm}^3.$$



(a) Frame (b) Assumed section

Substitute frame for beam B_{13}-B_{14}-B_{15}

Fig. 5-29

At joint A,

Distribution factor for upper and lower column

$$= \frac{2{\cdot}97 \times 10^5}{2 \times 2{\cdot}97 \times 10^5 + 18{\cdot}1 \times 10^5} = 0{\cdot}12.$$

Upper and lower column 0·12

beam $(1 - 0{\cdot}12 - 0{\cdot}12) = 0{\cdot}76.$

At joint B,

$$\text{Upper and lower column} = \frac{2{\cdot}97}{2\,(2{\cdot}97 + 18{\cdot}1)} = 0{\cdot}07$$

$$\text{beam} \quad \frac{(1 - 0{\cdot}07 - 0{\cdot}07)}{2} = 0{\cdot}43.$$

Free shears:

Free shears are defined as the shears for separate spans considered as simply supported. The correction due to moments existing at the ends of a beam will be made after solving moment distribution.

Free shears:

$$AB = BA = BC = CB = CD = DC = \frac{1}{2}(6 \times 3 + 166)$$
$$= 92 \text{ kN.}$$

Fixed end moments:

$$AB = BA = BC = CB = CD = DC$$
$$= \frac{6 \times 6^2}{12} + 166 \times \frac{6}{8} = 18 + 124{\cdot}5 = 142{\cdot}5 \text{ kNm.}$$

Moment distribution:

0·76	0·43	0·43	0·43	0·43	0·76
– 142·5	142·5	– 142·5	142·5	– 142·5	142·5
108·3	0	0	0	0	– 108·3
	54·15	0	0	– 54·15	
	– 23·28	– 23·28	23·28	23·28	
– 11·64	0	11·64	– 11·64	0	11·64
8·84	– 5	– 5	5	5	– 8·84
– 37	168·37	– 159·14	159·14	– 168·37	37

Column moments:

0·12	0·07	0·07	0·12
17·1	– 3·79	3·79	– 17·1
1·4	– 0·82	0·82	– 1·4
18·5	– 4·61	4·61	– 18·5

Shear correction:

Free shear	92	92	92	92	92	92
Correction	– 21·9	21·9	0	0	21·9	– 21·9
Final shear	70·1	113·9	92	92	113·9	70·1
Point of zero shear	*A* 3 m		*B* 3 m		*C*	3 m *D*

Positive moments:

$$\text{Span } AB \text{ or } CD = 70{\cdot}1 \times 3 - \frac{3^2}{2} \times 6 - 37$$

$$= 210 - 27 - 37 = 146{\cdot}3 \text{ kNm.}$$

$$\text{Span } BC = 92 \times 3 - \frac{3^2}{2} \times 6 - 159{\cdot}14$$

$$= 276 - 27 - 159{\cdot}14 = 89{\cdot}86 \text{ kNm.}$$

Negative moments:

Support	A or D	37 kNm.
Support	B or C	168·37 kNm.

Note:

Total live load on span = 80·5 kN (refer Ex. 5-8).

Total dead load = 84·9 + 6 × 6 (self wt.)
= 120·9 kN.

As $LL < \frac{3}{4} DL$, all spans loaded with dead load plus live load are considered for analysis.

Design for moment steel:

All the beams are rectangular beams with $b = 350$ mm. Assuming two layers of 20 mm dia. reinforcement,

$$d = 720 - 25 - 20 - 10 = 665 \text{ mm.}$$

The calculations are tabulated in table 5-3.

Check for development length: Positive moment bars:

Span AB:

At A, $V = 70{\cdot}1$ kN.

$$M_1 = 1143 \times 230 \times 0{\cdot}9 \times 665 \times 10^{-6} = 157{\cdot}34 \text{ kNm.}$$

Consider $L_o = 12\ \phi$.

$1{\cdot}3 \dfrac{M_1}{V} + L_o \geqslant 69\ \phi$ (L_d may be taken for a permissible stress. This is conservative. However, considered to eliminate the calculation of service stress).

TABLE 5-3

Point	Moment M kNm	M/bd^2	$\frac{100A_{st}}{bd}$	$\frac{100A_{sc}}{bd}$	A_{st}	A_{sc}
A	37	$\frac{37 \times 10^6}{350 \times 665^2} = 0{\cdot}24$	Minimum 0·205	—	477 mm², provide 2—20 Φ = 628 mm²	—
B	168·37	$\frac{168{\cdot}37 \times 10^6}{350 \times 665^2} = 1{\cdot}09$	0·526	0·552	1224 mm², provide 4—20 Φ = 1256 mm²	1285 mm², provide 3—20 Φ +2—16 Φ = 1344 mm²
P	146·3	$\frac{146{\cdot}3 \times 10^6}{350 \times 665^2} = 0{\cdot}95$	0·458	0·375	1066 mm², provide 3—20 Φ +1-16 Φ = 1143 mm²	873 mm² provide 3—20 Φ = 942 mm²
Q	89·86	$\frac{89{\cdot}86 \times 10^6}{350 \times 665^2} = 0{\cdot}58$	0·28	—	652 mm², provide 2—16 Φ +1—20 Φ =716 mm²	—

Note: In calculations of this table, design tables from SP:16 are used.

$$1{\cdot}3 \times \frac{157{\cdot}34 \times 10^6}{70{\cdot}1 \times 10^3} + 12\,\phi \geqslant 69\,\phi$$

or $\phi \leqslant 51{\cdot}19$ actual $\phi = 20$ mm..........(O.K.)

Near B, at point of contraflecture at $0{\cdot}15 \times 6 = 0{\cdot}9$ m from B,

$$V = 113{\cdot}9 - 0{\cdot}9 \times 6 = 108{\cdot}5 \text{ kN.}$$

$$1{\cdot}3 \times \frac{157{\cdot}34 \times 10^6}{108{\cdot}5 \times 10^3} + 12\,\phi \geqslant 69\,\phi$$

or $\phi \leqslant 33$.......................................(O.K.)

Span BC:

$M_1 = 716 \times 230 \times 0{\cdot}9 \times 665 \times 10^{-6} = 98{\cdot}56$ kNm.

V at pt. of contraflecture $= 92 - 0{\cdot}9 \times 6 = 86{\cdot}6$ kN.

$$1{\cdot}3 \times \frac{98{\cdot}56 \times 10^6}{86{\cdot}6 \times 10^3} + 12\,\phi \geqslant 69\,\phi$$

or $\phi \leqslant 25{\cdot}96$.......................................(O.K.)

Check for development length: Negative moment bars:

At supports B and C the anchorage provided is 1500 mm i.e. using simplified rules, anchorage equal to $\frac{l}{4}$ is provided. At support A, the bar in a stressed condition has to be bend.

$$\text{Service stress} = 230 \times \frac{477}{628} = 174{\cdot}7 \text{ N/mm}^2$$

$$L_d = \frac{\phi \times 174{\cdot}7}{1{\cdot}4 \times 4 \times 0{\cdot}6} = 52\,\phi.$$

The arrangement is shown in fig. 5-30 and is equivalent to anchorage of $55\ \phi = 1100$ mm. The bearing stress inside the bend is now checked.

$$\phi = 20 \text{ mm}$$

$$a = 350 - 50 - 40 = 260 \text{ mm.}$$

$$\text{Allowable bearing stress} = \frac{f_{ck}}{1 + \frac{2\,\phi}{a}}$$

$$= \frac{15}{1 + \frac{2 \times 20}{260}} = 13 \text{ N/mm}^2.$$

At the centre of the bend, the anchorage available

$$= 120 + 149 = 269 \text{ mm.}$$

Stress in bar at centre of bend

$$= 174{\cdot}7 \times \frac{1100 - 269}{1100} = 132 \text{ N/mm}^2.$$

$$F_{bt} = 314 \times 132 \times 10^{-3} = 41{\cdot}45 \text{ kN.}$$

$$\text{Bearing stress} = \frac{F_{bt}}{r\,\phi}$$

$$= \frac{41{\cdot}45 \times 10^3}{180 \times 20} = 11{\cdot}51 \text{ N/mm}^2.$$

The arrangement is thus satisfactory.

Check for shear:

Span *AB:*

At A, $V = 70{\cdot}1$ kN.

$$\frac{100\,A_s}{bd} = \frac{100 \times 1143}{350 \times 665} = 0{\cdot}49, \quad \tau_c = 0{\cdot}29 \text{ N/mm}^2.$$

As the ends of reinforcements are confined with compressive reaction, shear at distance 'd' may be considered.

$$\therefore \quad V = 70{\cdot}1 - 0{\cdot}665 \times 6 = 66{\cdot}1 \text{ kN.}$$

$$\text{Shear stress} = \frac{66{\cdot}1 \times 10^3}{350 \times 665} = 0{\cdot}284 \text{ N/mm}^2.$$

Only minimum shear reinforcements are required.

From table 3-4 for 350 wide beam, use 8 mm Φ stirrups about 250 c/c (upto 295 c/c is possible) for which $\frac{V_s}{d} = 92$.

At *B:*

V at distance $d = 113{\cdot}9 - 0{\cdot}665 \times 6 = 109{\cdot}9$ kN

$$\tau_c = 0{\cdot}29 \text{ N/mm}^2$$

$\tau_c bd = 0{\cdot}29 \times 350 \times 665 \times 10^{-3} = 67{\cdot}5$ kN.
Shear resistance of minimum shear reinforcement
$= 92 \times 665 \times 10^{-3} = 61{\cdot}2$ kN.

Shear resistance of section with minimum shear reinforcement is $67{\cdot}5 + 61{\cdot}2 = 128{\cdot}7$ kN $>$ $109{\cdot}9$ kN.

Provide 8 Φ about 250 c/c throughout.

Span *BC:*

Shear force = 92 kN.

Provide minimum shear reinforcement 8 mm Φ about 250 c/c.

Check for deflection and cracking:

This can be done in the usual manner and is left to the reader. The designed beam is shown in fig. 5-30.

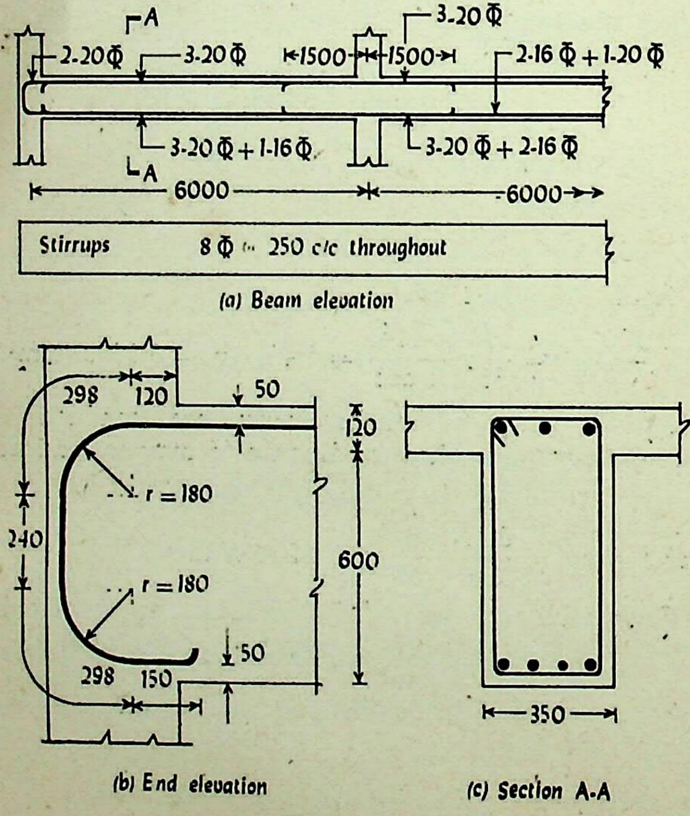


Fig. 5-30

EXAMPLES V

(1) A simply supported rectangular beam of 4·0 m clear span carries a uniformly distributed load (including self wt.) of 10 kN/m. It also carries a central point load of 25 kN. The beam rests on masonry walls of thickness 230 mm running parallel to the beam. The materials used are M15 grade concrete and tor steel reinforcement of grade Fe 415. Design the beam, if

(a) depth is not restricted and

(b) depth is restricted to 500 mm.

(2) Design a simply supported tee beam of 6 m effective span and spaced at 3·2 m centres. The thickness of slab is 130 mm and total load including self weight of beam is 35 kN/m. The materials are M15 grade concrete and mild steel reinforcement.

(3) Design the beam in Example (2) if concrete grade is M20 and tor steel reinforcement of grade Fe 415 are used.

(4) Design a cantilever beam of span 4 m and carrying a uniformly distributed load of 40 kN/m including self weight. Assume that sufficient safety against overturning moment is there and reinforcement anchorages are also available. Use M15 grade concrete and tor steel reinforcement of grade Fe 415.

(5) A beam *ABC* simply supported at *A* and *B* and *BC* is overhang. $AB = 6$ m and $BC = 3$ m. Load on *AB* portion inclusive of self weight is 25 kN/m and on overhang *BC* is 16 kN/m. Design the beam. Use M15 grade concrete and mild steel reinforcement.

(6) Design a two span continuous beam *ABC* where $AB = BC = 5$ m and supported on masonary walls at *A*, *B* and *C*. The beam is rectangular beam and carries a dead load inclusive of self weight equal to 12 kN/m. It also carries a live load of 10 kN/m. Design the beam using M15 grade concrete and tor steel reinforcement of grade Fe 415 if,

(a) the depth is not restricted and

(b) depth is restricted to 600 mm overall.

In both the cases width of beam is restricted to 230 mm.

(7) Design the beam B_4–B_5–B_6 of fig. 5-18 if it carries a dead load inclusive of self weight equal to 10 kN/m and live load of 10 kN/m. Slab thickness is 120 mm. Use M15 grade concrete and tor steel reinforcement of grade Fe 415.

(8) Design the beam B_1–B_2–B_3 of fig. 5-18 for intermediate floor, if it carries a dead load inclusive of self weight equal to 16 kN/m and live load of 10 kN/m. Thickness of slab is 120 mm and width of beam is 300 mm. Size of columns is 300 mm × 300 mm and storey height is 3·2 m.

6

Slabs

6-1. Introductory: Slabs are the plate elements having the depth D very small than its span and width. They usually carry a uniformly distributed load and form the floors or roof of the building. Like beams, slabs also may be simply supported, cantilever or continuous. They are classified according to the system of supports as under:

(a) One-way reinforced slabs

(b) Two-way reinforced slabs

(c) Flat slabs supported directly on columns without beams

(d) Circular and other shapes

(e) Grid slabs or waffle slabs.

A stair case is a special case of inclined slab and designed accordingly using available conditions of supports.

Slabs are primarily flexural members as beams and are analysed and designed in the same manner as the beams. Analysis may be carried out using:

(a) Elastic analysis: A strip of 1 m width of slab is considered and loads are found on this strip. Then strip is analysed as a beam of 1 m width. Redistribution of moments also may be carried out if necessary using the same rules as for the beams.

(b) Using coefficients: This is a semi-empirical method of analysis based on yield line theory. The coefficients given in code may be directly used to analyse the slabs. However, the redistribution of moments is not permitted in this case.

(c) Yield line method: This is a limit state design method or collapse load method, developed by Johansen.

According to clause 23.3 of IS : 456, "When the slabs are monolithic with supports, bending moments in slabs (except flat slabs) constructed monolithically with the supports, shall be calculated by taking such slabs either as continuous over supports and capable of free rotation or as members of a continuous framework with the supports, taking into account the stiffness of such supports. If such supports are formed due to beams which justify fixity at the support of slabs, then the effects on the supporting beam, such as the bending of the web in the transverse direction of the beam and the torsion in the longitudinal direction of the beam, wherever applicable, shall also be considered in the design of the beam".

In the usual case in this book, the slabs are considered as simply supported or continuous over support capable of free rotation, except where slabs give direct torsion in the beam.

6-2. One-way spanning slabs: The slab supported on two opposite supports is a one-way spanning slab. If the slab is supported on all four edges and if $\frac{l_y}{l_x} > 2$, where l_y is a longer span and l_x is the shorter span, then the slab is said to span one-way. The following are the design considerations for the one-way slab.

(*a*) *Effective span:* The effective span shall be as given for beams. For simply supported slab the effective span is clear span + effective depth or centre to centre of supports whichever is less. For continuous slabs art. 5-11 shall be referred.

(b) *General:* A simply supported one-way slab and a continuous one-way slab are shown in fig. 6-1(*a*) and (*b*). The reinforcement in the direction of span is the moment steel and is designed in the same way as beams. Design charts for singly reinforced beams can be used. Slabs are usually not designed as doubly reinforced except for some special cases. The moment steel is known as main reinforcement and is placed in the first layer near to the extreme fibre, keeping clear cover as per requirement to get maximum effective depth. The reinforcement perpendicular to the main reinforcement is known as distribution steel. These reinforcement

resist temperature stresses, keep the main reinforcement in position and distribute the concentrated or non-uniform loads throughout the slab. For a continuous slab at support, top reinforcement is provided as main steel to resist negative bending moment.

The spacing of slab bars is given by,

$$\text{spacing} = \frac{\text{area of one bar} \times 1000}{\text{required area per metre}}.$$

For convenience provided area of different bar diameters for different spacing is given in table 6-1.

TABLE 6-1

AREAS OF BARS IN SLABS (IN SQUARE MILLIMETRES PER METRE)

Spacing	Bar diameter in millimetres					
mm	6	8	10	12	16	20
100	283	503	785	1131	2011	3142
110	257	457	714	1028	1828	2856
120	236	419	654	942	1675	2618
130	217	387	604	870	1547	2417
140	202	359	561	808	1436	2244
150	188	335	524	754	1340	2094
160	177	314	491	707	1257	1963
170	166	296	462	665	1183	1848
180	157	279	436	628	1117	1745
190	149	265	413	595	1058	1653
200	141	251	393	565	1005	1571
210	135	239	374	539	957	1496
220	128	228	357	514	914	1428
230	123	218	341	492	874	1366
240	118	209	327	471	838	1309
250	113	201	314	452	804	1257

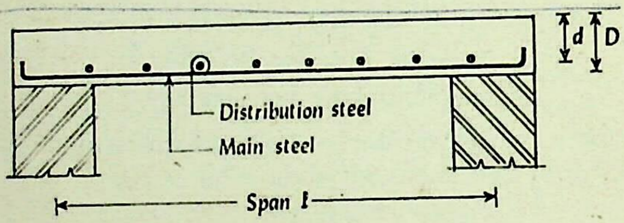
Note: These are the commonly used diameters for slabs (freely available in the market). For other diameters say 14 mm, one may actually find out the spacing.

(c) *Reinforcement requirements:*

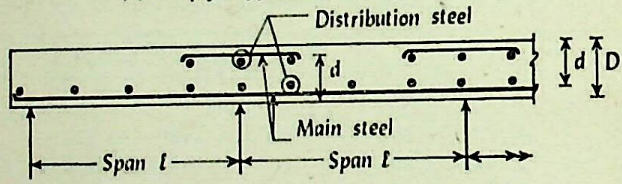
(1) Minimum reinforcement: The reinforcement in either direction in slabs shall not be less than 0·15 per cent of the total cross-sectional area. However, this value can be reduced to 0·12 per cent when high strength deformed bars or welded wire fabric are used.

(2) Maximum diameter: The diameter of reinforcing bars shall not exceed one-eighth of the total thickness of the slab.

These requirements shall apply to all kinds of slabs.



(a) Simply supported one-way slab



(b) Continuous one-way slab

FIG. 6-1

(*d*) *Shear stresses:* In normal cases the shear in slabs is not critical. However shear shall be checked in accordance with the code requirements of clause 47·2. This was discussed in art. 3-5 for beams. For solid slabs the permissible shear stress in concrete shall be $k\tau_c$ where value of τ_c is given in table 3-1 and *k* has the value given below:

Overall depth of slab mm	300 or more	275	250	225	200	175	150 or less
k	1·0	1·05	1·10	1·15	1·2	1·25	1·3

Note: This does not apply to flat slabs.

It can be seen from the above values that higher shear stresses are permitted for thinner slabs. This is due to the observations made that thin plates can resist more shear per unit area. If shear is critical in slabs, usually the shear rein-

forcement are not provided but the depth of slab is increased. This discussion shall apply to all kinds of slabs.

(*e*) *Deflection:* This shall be checked in the same manner as the beams. The slabs are thin elements and deflection may govern the thickness of the slab.

(*f*) *Cracking:* Unless the calculation of crack width shows that a greater spacing is acceptable, the following rules shall be applied to all kinds of slabs in normal internal or external conditions of exposure.

(1) The horizontal distance between parallel main reinforcement bars shall not be more than three times the effective depth of a solid slab or 450 mm whichever is smaller.

(2) The horizontal distance between parallel reinforcement bars provided against shrinkage and temperature shall not be more than five times the effective depth of a solid slab or 450 mm whichever is smaller.

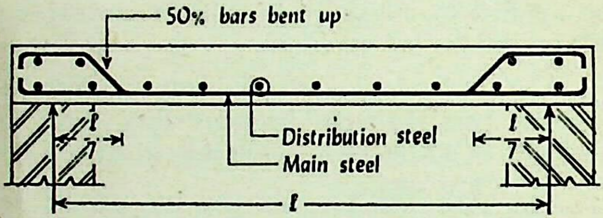
(*g*) *Cover:* For tensile, compressive, shear or any other reinforcement in slabs, minimum cover shall not be less than 15 mm, nor less than the diameter of such bar.

(*h*) *Bond:* The development length in the slabs shall be checked in the same manner as for beams.

For checking development length, L_0 may be assumed as 8 ϕ for tor steel (usually end anchorage is not provided) and 12 ϕ for mild steel (U hook is provided usually whose anchorage length is 16 ϕ. Now, L_0 is limited to 12 ϕ or effective depth whichever is greater. For slabs thickness cannot be greater than 12 ϕ as maximum diameter permitted is one-eighth of the total thickness of slab. Therefore assuming $L_0 = 12\ \phi$ is valid). However, one may find out the actual length L_0 by drawing the sketch of end anchorage.

6-3. Simply supported one-way slab: Typical details of reinforcement for a simply supported one-way slab is shown in fig. 6-2. It can be seen that 50% bars can be bent up where moment reduces to 50% i.e. at $\frac{l}{7}$ from support. The bars are bent and not curtailed and hence, curtailment rules shall not apply. The bent bars help in shear and also

resist some negative moment induced at support. In fact the slab is considered capable of free rotation at support and thus no negative moment is assumed to occur at support. Designer may choose to curtail the bars or let all the bars enter the support. Note that the bars can be bent up or curtailed only if continuing bars provide minimum reinforcement.



Typical details of simply supported slab
FIG. 6-2

Example 6-1.

A simply supported one-way slab of clear span 3·0 m is supported on masonry walls of thickness 350 mm. Slab is used for residential loads. Design the slab. The materials are grade M15 concrete and mild steel reinforcement. Live load shall be 2 kN/m².

Solution:

Depth of slab: The first trial of depth of slab can be arrived considering deflection criteria. Assuming percentage of steel reinforcement, find out modification factor as explained in art. 4-1. Percentage of steel depends on the loading on slab. A designer, after some practice will be able to find out his own thumb rules for the trial depth.

Assume 0·6 per cent steel as a first trial (for tor steel in residential loads one may assume 0·4 per cent steel for initial trial). Modification factor from fig. 4-1 is 1·75 i.e. $\frac{\text{span}}{\text{d}}$ ratio permissible $= 20 \times 1{\cdot}75 = 35$ and $d_{required}$ shall be $\frac{3100}{35} = 89$ mm. Overall depth using 12 mm dia. bars $= 89 + 6 + 15$ (cover) $= 110$ mm.

Assume 120 mm overall depth of slab.

$DL = 0{\cdot}12 \times 25 \quad = 3 \text{ kN/m}^2$

Floor finish $\quad = 1 \text{ kN/m}^2$

Live load $\quad = 2 \text{ kN/m}^2$

Total $\quad 6 \text{ kN/m}^2$

Effective span (1) $3000 + 350 = 3350$ mm c/c supports
(2) $3000 + 100$ (effective depth) $= 3100$ mm.

Use 3·1 m effective span.

Moment and shear:

Consider 1 m width of slab $\quad \therefore$ load $= 6$ kN/m.

$$\text{Maximum moment} = 6 \times \frac{3{\cdot}1^2}{8} = 7{\cdot}2 \text{ kNm.}$$

Maximum shear $= 6 \times \frac{3}{2} = 9$ kN (based on clear span).
Effective depth required for flexure

$$= \sqrt{\frac{7{\cdot}2 \times 10^6}{1000 \times 0{\cdot}87}} = 90{\cdot}97 \text{ mm.}$$

($k = 0{\cdot}87$ for M15 mix and mild steel)

$d_{provided} = 120 - 15$ (cover) $- 6$ (assume 12 ϕ bar)
$= 99$ mm.................................(O.K.)

Design for flexure:

$$A_{st} = \frac{7{\cdot}2 \times 10^6}{140 \times 0{\cdot}87 \times 99} = 597 \text{ mm}^2.$$

Alternatively from table 68 of SP : 16

$$\frac{M}{bd^2} = \frac{7{\cdot}2 \times 10^6}{1000 \times 99^2} = 0{\cdot}735$$

$$p_t = \frac{100\,A_{st}}{bd} = 0{\cdot}6$$

$$\therefore \quad A_{st} = \frac{0{\cdot}6 \times 1000 \times 99}{100} = 594 \text{ mm}^2.$$

Provide 10 mm ϕ about 130 c/c $= 604$ mm^2.

Note that use of tables give correct answer for steel required.

Half the bars are bent at $\frac{l}{7}$ and

remaining bars provide 302 mm² area.

$$\frac{100\ A_s}{bD} = \frac{100 \times 302}{1000 \times 120} = 0\cdot25 > 0\cdot15$$

i.e. remaining bars provide minimum steel. Thus half the bars can be bent up.

$$\text{Distribution steel} = \frac{0\cdot15}{100} \times 1000 \times 120 = 180 \text{ mm}^2,$$

maximum spacing $5 \times 99 = 495$ or 450 mm, i.e. 450 mm.

Provide 6 mm ϕ about 150 c/c = 188 mm².

Check for shear:

For bars at support, correct $d = 120 - 15 - 5 = 100$ mm.

$$\frac{100\ A_s}{bd} = \frac{100 \times 302}{1000 \times 100} = 0\cdot302$$

for slab upto 150 mm thickness, $k = 1\cdot3$

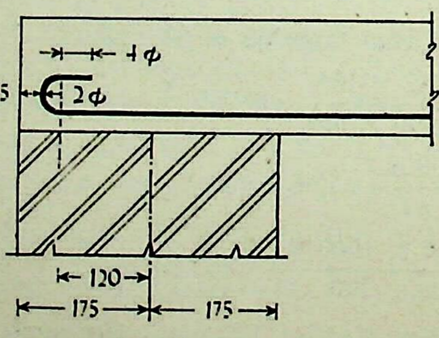
τ_c from table 3-1 $= 0\cdot24$ N/mm².

Permissible shear stress $= k\tau_c$

$$= 1\cdot3 \times 0\cdot24 = 0\cdot312 \text{ N/mm}^2.$$

$$\text{Actual shear stress} = \frac{9 \times 10^3}{1000 \times 100} = 0\cdot09 \text{ N/mm}^2$$

(too small).



Details at support

FIG. 6-3

Check for development length:

$L_o = 120 + 16\,\phi = 280$. L_o is limited to $12\,\phi = 120$ mm or $d = 100$ mm whichever is greater. Therefore $L_o = 120$ mm. For continuing bars, $A_s = 302$ mm^2.

$$M_1 = 302 \times 140 \times 0{\cdot}87 \times 100 \times 10^{-6} = 3{\cdot}68 \text{ kNm.}$$

$$V = 9{\cdot}0 \text{ kN}$$

$$1{\cdot}3\,\frac{M_1}{V} + L_o \geqslant L_d \qquad \text{where } L_d = \frac{\phi \times 140}{4 \times 0{\cdot}6} = 58{\cdot}3\,\phi$$

$$1{\cdot}3 \times \frac{3{\cdot}68 \times 10^6}{9 \times 10^3} + 120 \geqslant 58{\cdot}3\,\phi$$

or $11{\cdot}18 \geqslant \phi$.................................(O.K.)

Check for deflection:

$$\text{Basic } \frac{\text{span}}{\text{d}} \text{ ratio} = 20$$

$$p_t = \frac{100 \times 604}{1000 \times 100} = 0{\cdot}604$$

modification factor $= 1{\cdot}72$

$$\frac{\text{span}}{\text{d}} \text{ ratio permissible} = 20 \times 1{\cdot}72 = 34{\cdot}4$$

actual $\dfrac{\text{span}}{\text{d}}$ ratio $= \dfrac{3100}{100} = 31 < 34{\cdot}4$..........(O.K.)

Check for cracking:

Max. spacing permitted for main reinforcement

$= 3 \times 100 = 300$ mm

Actual spacing $= 130$ mm < 300 mm........ (O.K.)

For distribution steel, maximum spacing permitted

$= 5 \times 100 = 500$ or 450 mm i.e. 450 mm.

Spacing provided $= 150$ mm.................(O.K.)

For tieing the bent bars at top, 6 mm ϕ about 150 mm c/c distribution steel shall be provided.

Sketch: The cross-section of the slab is shown in fig. 6-4.

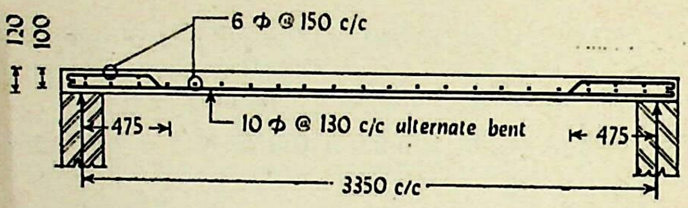
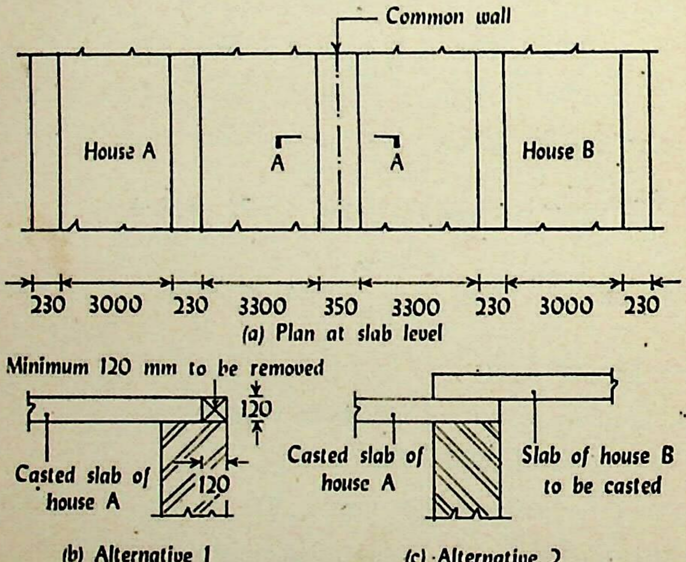


FIG. 6-4

From the figure, note that $\frac{l}{7}$ is considered from centre of the support for simplicity and rounded off to a figure in multiple of 5 mm on lower side. Never consider $\frac{l}{7}$ from the face of the support, as this will give a point where bending moment is more than 50 per cent (may be 51 or 52 per cent).



Bearing of slab on masonry wall
FIG. 6-5

Supplementary details:

(a) The slab requires a minimum bearing equal to its depth on masonry walls. The problem arises for a twin

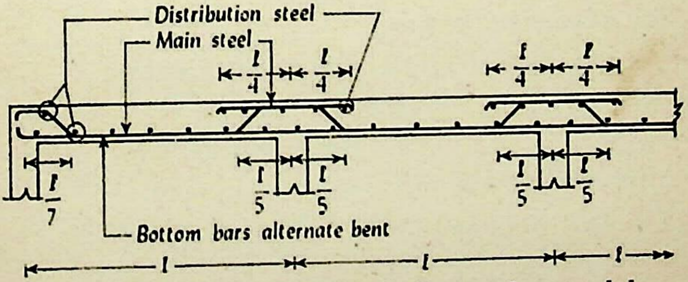
type house where the central wall is a common wall. If the slab of house A is casted having full bearing on the central common wall (refer fig. 6-5), for the casting of slab of house B, there can be two alternatives.

(1) 120 mm (thickness of slab for B block) casted slab shall be removed and slab for B block is casted.

(2) For slab B, the level is changed and full bearing may be given on walls.

(b) While providing 120 mm bearing, the development length of slab bars has to be checked once again.

6-4. Continuous one-way slab: For continuous slab, when loads are found, the arrangement of live load shall be as discussed in art. 5-12. This involves minimum four moment distributions and if required, the redistribution of moment is permitted. Alternatively, the moment and shear coefficients can be obtained from table 5-1 and table 5-2. In this case, the redistribution of moment is not permitted. Further, the slabs are considered simply resting on the supports and capable of free rotations at the supports. Therefore, this will not induce any kind of torsion in the supporting beams or walls.



Typical reinforcement details for continuous slab

FIG. 6-6

Like the continuous beams the bent bars of slab can be used as negative reinforcement at the support provided that bars can be bent up. The bars can be bent up only if the continuing bars provide minimum steel required and for continuing bars, check for development length is satisfied.

Assuming these requirements are satisfied, typical reinforcement details for continuous slab is shown in fig. 6-6.

Note that at simply supported end, the bars are bent at $\frac{l}{7}$ while at continuous support, the bars are bent at $\frac{l}{5}$. The contribution of bent bars for negative moment steel can be considered which results in economical design. However, for each design of slab, designer has to check whether the bars can be bent up or not.

Example 6-2.

A five span continuous one-way slab is to be used as an office floor. Separate facility for storage is not provided. The centre to centre distance of supporting beams is 2·75 m. Design the slab using M15 grade concrete and tor steel reinforcement of grade Fe 415.

Solution:

(a) Estimation of loads: According to table 1 of IS : 875, for office floor the live load shall be 2·5 kN/m² to 4·0 kN/m². Lower values shall be taken where separate facility for storage is provided and higher values shall be taken where such provisions are lacking. (The tables for live load are reproduced and given in appendix B of this book.) Consider a live load of 4 kN/m² as the facility for storage is not provided.

Try 12 cm thick slab.

$$\begin{aligned} \text{Dead load: self } 0{\cdot}12 \times 25 &= 3{\cdot}0 \text{ kN/m}^2 \\ \text{floor finish} &= 1{\cdot}0 \text{ kN/m}^2 \\ \hline \text{Total } & \quad 4{\cdot}0 \text{ kN/m}^2 \end{aligned}$$

Live load 4 kN/m².

The slab is to be designed for $\underset{DL}{(4} + \underset{LL}{4)}$ kN/m².

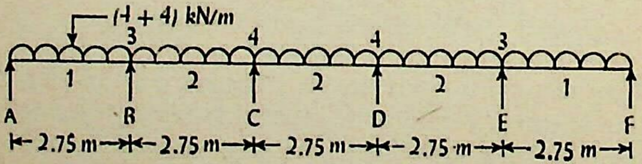


FIG. 6-7

(b) Analysis: The loads for 1 m wide strip are shown in fig. 6-7. The moment and shear coefficients from table 5-1 and table 5-2 are used.

Maximum shear is $V_{BA} = 0{\cdot}6 \times 4 \times 2{\cdot}75 + 0{\cdot}6 \times 4 \times 2{\cdot}75$
$= 6{\cdot}6 + 6{\cdot}6 = 13{\cdot}2$ kN.

The calculations are tabulated in table 6-2.

TABLE 6-2

Point	Moment	Steel required	Steel provided
1 (+)	$\frac{1}{12} \times 4 \times 2{\cdot}75^2 + \frac{1}{10} \times 4 \times 2{\cdot}75^2$ $= 2{\cdot}52 + 3{\cdot}03$ $= 5{\cdot}55$ kNm.	$\frac{5{\cdot}55 \times 10^6}{230 \times 0{\cdot}90 \times 100}$ $= 268$ mm²	8 mm Φ about 440 c/c+10 mm Φ about 440 c/c = 292 mm²
2 (+)	$\frac{1}{24} \times 4 \times 2{\cdot}75^2 + \frac{1}{12} \times 4 \times 2{\cdot}75^2$ $= 1{\cdot}26 + 2{\cdot}52$ $= 3{\cdot}78$ kNm.	$\frac{3{\cdot}78 \times 10^6}{230 \times 0{\cdot}90 \times 100}$ $= 183$ mm²	8 mm Φ about 220 c/c = 227 mm².
3 (−)	$\frac{1}{10} \times 4 \times 2{\cdot}75^2 + \frac{1}{9} \times 5 \times 2{\cdot}75^2$ $= 3{\cdot}03 + 3{\cdot}36$ $= 6{\cdot}39$ kNm.	$\frac{6{\cdot}39 \times 10^6}{230 \times 0{\cdot}9 \times 100}$ $= 309$ mm²	10 mm Φ about 220 c/c = 357 mm²
4 (−)	$\frac{1}{12} \times 4 \times 2{\cdot}75^2 + \frac{1}{9} \times 4 \times 2{\cdot}75^2$ $= 2{\cdot}52 + 3{\cdot}36$ $= 5{\cdot}88$ kNm.	$\frac{5{\cdot}88 \times 10^6}{230 \times 0{\cdot}9 \times 100}$ $= 284$ mm²	8 mm Φ about 440 c/c + 10 mm Φ about 440 c/c = 292 mm²

(c) Moment steel and arrangement: Points 1, 2, 3, 4 are showing the points of maximum moments. Maximum moment occurs at point 3.

$$M_3 = 6{\cdot}39 \text{ kNm}$$

$$d_{required} = \sqrt{\frac{6{\cdot}39 \times 10^6}{1000 \times 0{\cdot}65}} = 99{\cdot}2 \text{ mm}$$

$$d_{provided} = 120 - 15 \text{ (cover)} - 5 \text{ (using 10 } \phi \text{ bar)}$$
$$= 100 \text{ mm} \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(O.K.)}$$

For main steel

Minimum area $= \frac{0\cdot12}{100} \times 1000 \times 120 = 144 \text{ mm}^2.$

Maximum spacing $= 3 \times 100 = 300$ mm.

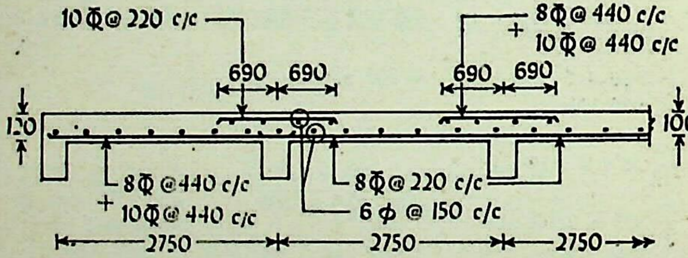
For distribution steel using mild steel

Minimum area $= \frac{0\cdot15}{100} \times 1000 \times 120 = 180 \text{ mm}^2.$

Maximum spacing $= 5 \times 100 = 500$ or 450 mm, whichever is less i.e. 450 mm.

Use 6 mm ϕ about 150 mm c/c $= 188 \text{ mm}^2$.

Note: Use of 6 mm ϕ mild steel for distribution steel is popular even when tor steel is used as main reinforcement. Moreover minimum diameter of tor steel freely available in market is 8 mm which gives larger spacing e.g. 340 mm in given case. Eventhough the cracking requirements are satisfied, the usual practice of providing the spacing of bars is 100 mm to 250 mm in slabs.



Details of continuous slab

Fig. 6-8

The above arrangement of bars is shown in fig. 6-8. The arrangement of bars can be done by many ways. If the bars are bent or curtailed, care must be taken that continuing bars provide (a) minimum reinforcement and (b) check for development length is satisfied. In this example, no bar is curtailed or bent. This is because (a) if the bars are curtailed, for remaining bars the development length is not satisfied and (b) for internal span the continuing bars provide 114 mm^2 area which is less than minimum (i.e. 144 mm^2).

While selecting the diameter and spacing of the reinforcement, care must be taken to see that spacing would result the same for all four moments and diameters may be changed. This would sometimes, for internal spans, result uneconomical. Some designers curtail the bars at the centre of the support and adopt different spacing in different spans. However, according to IS : 456 *at least* one-fourth bars at the continuous support shall extend along the same face of the support upto a distance equal to $\frac{L_d}{3}$. If the said method is to be adopted, the width of the support shall be equal to $\frac{2L_d}{3}$ i.e. $\frac{2}{3} \times 69\ \phi$ $= 368$ mm for this case. Alternatively the bars can be extended from both the adjoining spans upto 23 ϕ which would seem odd if different spacing is adopted. Therefore, the spacing shall be made same as far as possible sacrificing economy sometimes.

(d) Check for development length:

Span AB is critical for checking this requirement.

At support A

$$V = 0{\cdot}4 \times 4 \times 2{\cdot}75 + 0{\cdot}45 \times 4 \times 2{\cdot}75$$
$$= 4{\cdot}4 + 4{\cdot}95 = 9{\cdot}35 \text{ kN.}$$
$$M_1 = 292 \times 230 \times 0{\cdot}9 \times 100 \times 10^{-6} = 6{\cdot}04 \text{ kNm.}$$

Assuming $L_o = 8\ \phi$ (90° bend from centre of support)

$$1{\cdot}3 \times \frac{M_1}{V} + L_o \geqslant L_d$$

$$1{\cdot}3 \times \frac{6{\cdot}04 \times 10^6}{9{\cdot}35 \times 10^3} + 8\ \phi \geqslant 69\ \phi$$

$\therefore$ $13{\cdot}77 \geqslant \phi.$ $\phi_{provided} = 10$ mm................(O.K.)

Note: For slab bars, when tor steel is used, 90° bend is not necessary. The bar length used in standard bend is more than 8 ϕ and for such bend maximum anchorage value is 8 ϕ. Thus, the slab bars are used without end anchorage when tor steel is used. However, when mild steel is used, U bends are provided at end. U bend in tor steel is rarely used.

At support B

Point of contraflexure is assumed at $0{\cdot}15\,l$ from B.

$$\therefore \quad \text{S.F.} = (0{\cdot}6 \times 8 \times 2{\cdot}75) - 0{\cdot}15 \times 2{\cdot}75 \times 8$$
$$= 13{\cdot}2 - 3{\cdot}3 = 9{\cdot}9 \text{ kN.}$$

$$1{\cdot}3\,\frac{M_1}{V} + L_o \geqslant L_d.$$

Assume $L_o = 12\,\phi$ (actual anchorage is more than $12\,\phi$ but L_o is limited to $12\,\phi$ or d, i.e. 100 mm whichever is greater).

$$\therefore \quad 1{\cdot}3 \times \frac{6{\cdot}04 \times 10^6}{9{\cdot}9 \times 10^3} + 12\,\phi \geqslant 69\,\phi$$

or $13{\cdot}91 \geqslant \phi$..............................(O.K.)

(e) Check for shear:

Max. S.F. at B for BA

$$V = 13{\cdot}2 \text{ kN} \qquad \frac{100\,A_s}{bd} = \frac{100 \times 357}{1000 \times 100} = 0{\cdot}357$$

$$\tau_c = 0{\cdot}25 \text{ N/mm}^2.$$

For 120 mm thick slab, $k = 1{\cdot}3$.

$$\text{Permissible shear stress} = k\,\tau_c = 1{\cdot}3 \times 0{\cdot}25$$
$$= 0{\cdot}325 \text{ N/mm}^2.$$

$$\text{Actual shear stress} = \frac{13{\cdot}2 \times 10^3}{1000 \times 100} = 0{\cdot}132 \text{ N/mm}^2$$

..............(O.K.)

Note that at support B, the moment is negative and negative steel is used for the calculation of $\frac{100A_s}{bd}$.

(f) Check for deflection:

Maximum positive moment is in span AB. Therefore, this check is critical for span AB.

$$\frac{100A_{st}}{bd} = \frac{100 \times 292}{1000 \times 100} = 0{\cdot}292$$

Basic $\frac{\text{span}}{\text{d}}$ ratio $= 26$

Modification factor $= 1{\cdot}4$

$$\frac{\text{span}}{\text{d}} \text{ ratio permissible} = 1{\cdot}4 \times 26 = 36{\cdot}4$$

$$\therefore \text{ actual } \frac{\text{span}}{\text{d}} \text{ ratio} = \frac{2750}{100} = 27{\cdot}5 < 36{\cdot}4 \ldots\ldots\ldots (\text{O.K.})$$

(g) Check for cracking:

This was incorporated in design calculations.

(h) Sketch:

Designed section is shown in fig. 6-8.

Supplementary details:

(1) The designer conversant with IS : 456-1964 may have little difficulty in understanding—why the bars cannot be bent up? According to IS : 456-1978, the criteria for checking the deflection are made more conservative than previous code. For different materials different span-depth ratios are permitted. This is in fact a better and rational approach and keeping with international standards. However, this results in providing more depth and percentage steel decrease in comparison to previous code. Therefore, checks of minimum steel requirement is critical as compared to previous code.

Also the check for development length is made conservative as compared to previous code. This also may restrict the bending of bars.

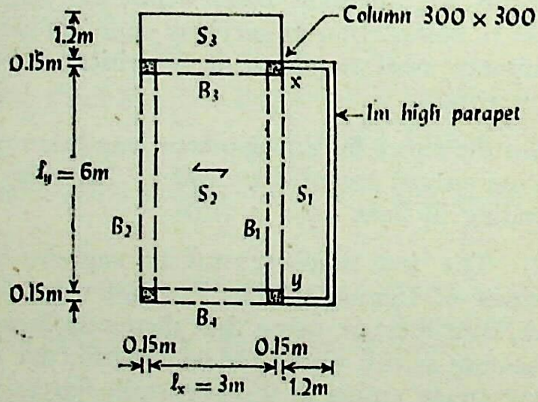
(2) The top reinforcement at support must have a clear cover of 15 mm, neither less, nor more. While casting the slab, care must be taken that these reinforcements do not sit to bottom as this would reduce the effective depth of slab and may create cracks at top perpendicular to main reinforcement. Chairs of reinforcement are provided to achieve this depth.

(3) When there is a change of steel such as positive reinforcement from span *AB* to *BC*, then usually the bars are curtailed on the other side of the support as shown in fig. 6-8. This will provide the anchoragement of $\frac{L_d}{3}$ to the positive reinforcement.

(4) Initial trial section of overall depth sometimes may need correction to achieve strength or economy, as this has to pass through many checks. Similarly the selection of bar diameter and spacing also will need some trials to achieve economy. However, after some practice, the designer will have little difficulty in achieving a safe and economical section.

6-5. Cantilever slab: The design criterias for cantilever slab are same as previous articles. An example of cantilever slab is a balcony slab in a building. While designing a cantilever balcony, one should check the counter-balancing loads. The cantilever slabs are indicated in fig. 6-9. If the internal slab is continuous with cantilever slab as S_2-S_1 of fig. 6-9, then slab S_1 can be designed as cantilever slab capable of free rotation at support B_1.

Usually the formwork of slab S_1 or S_3 when designed as cantilever, is kept tight to provide a little camber.



Cantilever balcony slab

FIG. 6-9

Example 6-3.

Design the slab S_2-S_1 of fig. 6-9, if it is to be used for residential purpose. At the free end of slab S_1 there is a concrete parapet of 75 mm thick and 1 m high. The materials are grade M15 concrete and mild steel reinforcement. Use IS : 875 for live loads.

Solution:

(a) Estimation of loads:

For S_2 live load shall be 2 kN/m². For slab S_1 which is a balcony slab, live load shall be 3 kN/m². Assume 12 cm thick slab.

$$\begin{array}{llrl} \text{For } S_2 \text{ self load} = & 0{\cdot}12 \times 25 = & 3 & \text{kN/m}^2 \\ & \text{floor finish} & 1 & \text{kN/m}^2 \\ & \text{live load} & 2 & \text{kN/m}^2 \\ \hline & \text{Total} & (4+2) & \text{kN/m}^2 \end{array}$$

$$\begin{array}{llrl} \text{For } S_1 & \text{self load} = & 3 & \text{kN/m}^2 \\ & \text{floor finish} & 1 & \text{kN/m}^2 \\ & \text{live load} & 3 & \text{kN/m}^2 \\ \hline & \text{Total} & (4+3) & \text{kN/m}^2 \end{array}$$

Weight of parapet $0{\cdot}075 \times 25 \times 1 = 1{\cdot}875$ kN/m.

(b) Analysis:

Consider 1 m wide strip.

(1) To get maximum positive moment in slab S_2 only dead loads on slab S_1 and full load on slab S_2 shall be considered. The parapet load is a dead load but will not be considered as sometimes the owner of the building or architect may change his mind and would provide simply a railing. Considering fig. 6-10(*a*),

$$\text{Cantilever moment} = \frac{1{\cdot}2^2}{2} \times 4 = 2{\cdot}88 \text{ kNm.}$$

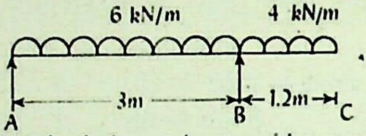
$$\text{Reaction at } A = \frac{6 \times 3}{2} - \frac{2{\cdot}88}{3} = 9 - 0{\cdot}96$$
$$= 8{\cdot}04 \text{ kN.}$$

$$\text{Point of zero shear from } A = \frac{8{\cdot}04}{6} = 1{\cdot}34 \text{ m.}$$

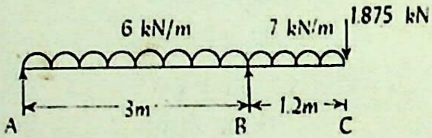
$$\text{Maximum positive moment} = 8{\cdot}04 \times 1{\cdot}34 - \frac{1{\cdot}34^2}{2} \times 6$$
$$= 10{\cdot}78 - 5{\cdot}38 = 5{\cdot}4 \text{ kNm.}$$

To check shear and development length at A, shear may be considered as 8·04 kN. Note that for the cantilever, clear span is considered.

(2) To get maximum negative moment and maximum shear at B, the slab is loaded with full loads as shown in fig. 6-10(b).



(a) Loads for maximum positive moment



(b) Loads for maximum negative moment, shear for cantilever span and reaction at supports

FIG. 6-10

$$\text{Maximum negative moment} = \frac{1{\cdot}2^2}{2} \times 7 + 1{\cdot}2 \times 1{\cdot}875$$
$$= 5{\cdot}04 + 2{\cdot}25 = 7{\cdot}29 \text{ kNm.}$$

Maximum shear at B

$$V_{BA} = \frac{6 \times 3}{2} + \frac{7{\cdot}29}{3} = 9 + 2{\cdot}43 = 11{\cdot}43 \text{ kN.}$$

$$V_{BC} = 7 \times 1{\cdot}2 + 1{\cdot}875 = 10{\cdot}275 \text{ kN.}$$

(c) Moment steel:

Maximum moment = 7·29 kNm

$$d_{required} = \sqrt{\frac{7{\cdot}29 \times 10^6}{1000 \times 0{\cdot}87}} = 91{\cdot}5 \text{ mm}$$

$$d_{provided} = 120 - 15 - 6 \text{ (assume } 12\,\phi) = 99 \text{ mm}$$

....(O.K.)

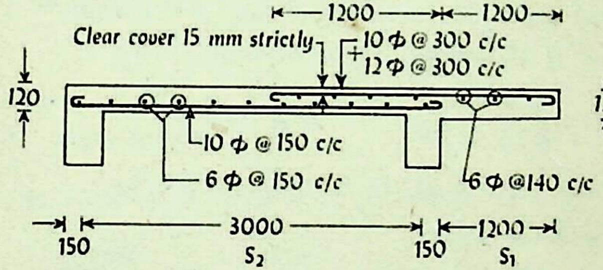
$$A_{st}\,(+) = \frac{5{\cdot}4 \times 10^6}{140 \times 0{\cdot}87 \times 99} = 448 \text{ mm}^2$$

$$A_{st}\,(-) = \frac{7{\cdot}29 \times 10^6}{140 \times 0{\cdot}87 \times 99} = 605 \text{ mm}^2.$$

For positive moment provide 10 mm ϕ about 150 c/c giving 523 mm² and for negative moment provide 10 mm ϕ about 300 c/c + 12 mm ϕ about 300 c/c giving total 638 mm² area. The arrangement of reinforcement is shown in fig. 6-11.

$$\text{Distribution steel} = \frac{0{\cdot}15}{100} \times 1000 \times 120 = 180 \text{ mm}^2.$$

Provide 6 mm ϕ about 150 c/c = 187 mm².



Longitudinal section through S_2-S_1
FIG. 6-11

For negative reinforcement,

$$\text{service stress} = \frac{604}{638} \times 140 = 132{\cdot}54 \text{ N/mm}^2.$$

$$L_d = \frac{132{\cdot}54\ \phi}{4 \times 0{\cdot}6} = 55{\cdot}2\ \phi.$$

$$\text{Average} \quad L_d = 55{\cdot}2 \times \frac{(10+12)}{2} = 608 \text{ mm}.$$

The bars must be anchored upto 608 mm. Also they should be extended upto 12 ϕ beyond the point of contraflexure, which may be found out. Alternatively as a thumb rule, a bar shall be given an anchorage equal to the length of the cantilever. Adopting this, carry the top bars upto 1200 mm in the internal span. This is shown in fig. 6-11.

(d) Check for development length:

At A, $M_1 = 523 \times 140 \times 0{\cdot}87 \times 99 \times 10^{-6} = 6{\cdot}3$ kNm.
$V = 8{\cdot}04$ kN.

Consider $L_o = 12\ \phi$

then $1{\cdot}3\ \dfrac{M_1}{V} + L_o \geqslant L_d$

$$1{\cdot}3 \times \frac{6{\cdot}3 \times 10^6}{8{\cdot}04 \times 10^3} + 12\ \phi \geqslant 58{\cdot}3\ \phi$$

which gives $\phi \leqslant 19{\cdot}53$mm. . . . . (O.K.)

At B, $M_1 = 6{\cdot}3$ kNm.

Near point of contraflexure i.e. $0{\cdot}15\,l$ from B

$V = 11{\cdot}43 - 0{\cdot}45 \times 6 = 8{\cdot}73$ kN.

$$1{\cdot}3 \times \frac{6{\cdot}3 \times 10^6}{8{\cdot}73 \times 10^3} + 12\ \phi \geqslant 58{\cdot}3\ \phi$$

which gives $\phi \leqslant 18{\cdot}15$ mm. . . . . (O.K.)

(e) Check for shear:

Span AB:

At A, $V_{AB} = 9{\cdot}00$ kN (for maximum loading).

At B, shear at point of contraflexure $= 8{\cdot}73$ kN.

Use $V_{AB} = 9$ kN.

$$\text{Shear stress } \tau_v = \frac{9 \times 10^3}{1000 \times 99} = 0{\cdot}09 \text{ N/mm}^2$$

$$\frac{100\ A_s}{bd} = \frac{100 \times 523}{1000 \times 99} = 0{\cdot}53$$

$$k\tau_c = 0{\cdot}29 \times 1{\cdot}3 = 0{\cdot}377 \text{ N/mm}^2 > \tau_v$$

. . . . (O.K.)

Span BC:

$$V = 11{\cdot}43 \text{ kN.}$$

$$\text{Shear stress } \tau_v = \frac{11{\cdot}43 \times 10^3}{1000 \times 99} = 0{\cdot}115 \text{ N/mm}^2.$$

$$\frac{100\ A_s}{bd} = \frac{100 \times 638}{1000 \times 99} = 0{\cdot}64.$$

$$k\tau_c = 0{\cdot}318 \times 1{\cdot}3 = 0{\cdot}413 \text{ N/mm}^2 > \tau_v$$

. . . . (O.K.)

(f) Check for deflection:

Slab S_2 shall not be treated as a continuous slab, as an overhang would not justify the fixity of slab at point B. If the minimum dead load moments of cantilever slab exceeds the maximum fixed end moments of span AB with full load, then and then it can be considered as continuous over support B.

For span AB

$$\text{Basic } \frac{\text{span}}{\text{d}} \text{ ratio} = 20$$

$$\frac{100\, A_{st}}{bd} = \frac{100 \times 523}{1000 \times 99} = 0{\cdot}53.$$

Modification factor $= 1{\cdot}85$.

$$\text{Permissible } \frac{\text{span}}{\text{d}} \text{ ratio} = 20 \times 1{\cdot}85 = 37.$$

$$\text{Actual } \frac{\text{span}}{\text{d}} \text{ ratio} = \frac{3000}{99} = 30{\cdot}3 < 37 \ldots\ldots\ldots \text{(O.K.)}$$

For span BC

$$\text{Basic } \frac{\text{span}}{\text{d}} \text{ ratio} = 7$$

$$\frac{100\, A_{st}}{bd} = \frac{100 \times 638}{1000 \times 99} = 0{\cdot}64.$$

Modification factor $= 1{\cdot}7$.

$$\frac{\text{span}}{\text{d}} \text{ ratio permissible} = 7 \times 1{\cdot}7 = 11{\cdot}9$$

$$\text{actual } \frac{\text{span}}{\text{d}} \text{ ratio} = \frac{1200}{99} = 12{\cdot}12.$$

This is critical and depth in cantilever portion shall be increased to

$$120 \times \frac{12{\cdot}12}{11{\cdot}9} = 122{\cdot}2 \text{ mm say } 125 \text{ mm.}$$

The calculations need not be rechecked.

However, increase in depth would result in increase of temperature steel.

For 125 mm thick slab,

$$\text{distribution steel} = \frac{0{\cdot}15}{100} \times 1000 \times 125 = 187{\cdot}5 \ \text{mm}^2.$$

Provide 6 ϕ about 140 c/c = 200 mm^2.

Note that the depth of cantilever portion is increased such that top of slabs remain the same. The formwork of cantilever portion shall be brought down. This is done to keep the floor above the casted slab in one level.

(g) Check for cracking:

(1) Main bars:

span *AB*: spacing permitted = 3 × 99 = 297 mm
spacing provided = 150 mm........(O.K.)

span *BC*: spacing permitted = 3 × 104 = 312 mm
spacing provided = 150 mm........(O.K.)

(2) Secondary bars:

span *AB*: spacing permitted = 5 ×99=495 or 450 mm
i.e. 450 mm
spacing provided = 150 mm.......(O.K.)

span *BC*: spacing permitted = 5 ×104=520 or 450 mm
i.e. 450 mm
spacing provided = 140 mm........(O.K.)

(h) Supplementary details:

(1) Stability against overturning may be checked using dead load on span *AB* and double the total load on span *BC*. If reaction at *A* is not negative, the slab is safe against overturning. If it is negative, the slab at point *A* requires counter-balance. In this type of case, the counter-balance is provided by the self weight of beam and the load from masonry walls on beam at *A*.

(2) To cast the concrete parapet, 6 mm ϕ bars are anchored in slab and erected. The parapet is casted afterwards. If reinforced brickwork (R.B.) parapet is to be provided, the spacing of 6 mm ϕ bars is provided to adjust with the size of brick. In any case it is advisable to provide parapet after the formwork is removed specially in case of cantilevers. Otherwise, after removing the formwork, as a

natural phenomenon, the cantilever slab will settle down and would result in cracks in masonry work at points such as x and y of fig. 6-9.

6-6. Concentrated load on slabs:

(*a*) *Simply supported and continuous slabs:* If a solid slab supported on two opposite edges carries concentrated loads, the maximum bending moment caused by the concentrated loads shall be assumed to be resisted by an effective width of slab (measured parallel to the supporting edges) as follows:

(1) For a single concentrated load, the effective width shall be calculated in accordance with the following equation provided that it shall not exceed the actual width of the slab:

$$b_{ef} = kx\ (l_{ef} - \frac{x}{l_{ef}}) + a$$

where

b_{ef} = effective width of slab,

k = constant having the values given in table 6-3 depending upon the ratio of width of slab (l') to the effective span l_{ef},

x = distance of the centroid of the concentrated load from nearer support,

l_{ef} = effective span, and

a = width of the contact area of the concentrated load measured parallel to the supported edge.

And provided further that in case of a load near the unsupported edge of a slab, the effective width shall not exceed the above value nor half the above value plus the distance of the load from the unsupported edge.

(2) For two or more concentrated loads placed in a line in the direction of span, the bending moment per metre width of slab shall be calculated separately for each load according to its appropriate effective width of slab calculated as in (1) above and added together for design calculations.

(3) For two or more loads not in a line in the direction of span, if the effective width of slab for one load does not overlap the effective width of slab for another load, both calculated as in (1) above, then the slab for each load can be designed separately. If the effective width of slab for one load overlaps the effective width of slab for an adjacent load, the overlapping portion of the slab shall be designed for combined effect of the two loads.

TABLE 6-3

VALUES OF k FOR SIMPLY SUPPORTED AND CONTINUOUS SLABS

$\frac{l'}{l_{ef}}$	k for simply supported slabs	k for continuous slabs
0·1	0·4	0·4
0·2	0·8	0·8
0·3	1·16	1·16
0·4	1·48	1·44
0·5	1·72	1·68
0·6	1·96	1·84
0·7	2·12	1·96
0·8	2·24	2·08
0·9	2·36	2·16
1·0 and above	2·48	2·24

(*b*) *Cantilever slabs:* For cantilever solid slabs, the effective width shall be calculated in accordance with the following equation:

$$b_{ef} = 1{\cdot}2\ a_1 + a$$

where

b_{ef} = effective width,

a_1 = distance of the concentrated load from the face of the cantilever support, and

a = width of contact area of the concentrated load measured parallel to the supporting edge.

Provided that the effective width of cantilever slab shall not exceed one-third the length of the cantilever slab measured parallel to the fixed edge.

And provided further that when the concentrated load is placed near the extreme ends of the length of cantilever slab in the direction parallel to the fixed edge, the effective width shall not exceed the above value, nor shall it exceed half the above value plus the distance of the concentrated load from the extreme end measured in the direction parallel to the fixed edge.

For slabs other than solid slabs, the effective width shall depend on the ratio of the transverse and longitudinal flexural rigidities of the slab. Where this ratio is one, that is, where the transverse and longitudinal flexural rigidities are approximately equal, the value of effective width as found for solid slabs may be used. But as the ratio decreases, proportionately smaller value shall be taken.

Any other recognized method of analysis for cases of slabs discussed above and for all other cases of slabs may be used with the approval of the engineer-in-charge.

6-7. Two-way slabs: The two-way spanning slab occurs when the slab is supported on all four edges. The two-way action can be well studied by considering a thin flexible membrane supported on all four edges. This is illustrated in fig. 6-12. If l_y is much greater than l_x, the membrane deflection is in direction of l_x. Now keeping l_x constant, l_y is reduced and the deflection of the membrane is observed. When $l_y \leqslant 2l_x$; from this point it can be observed

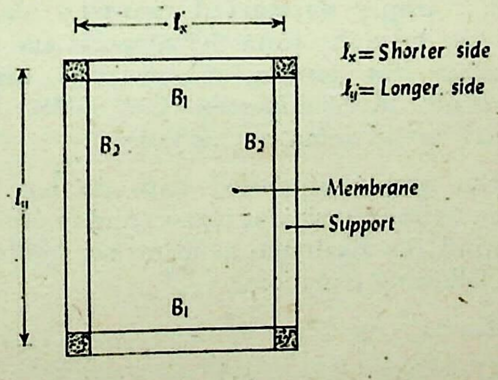


Fig. 6-12

that the membrane starts deflecting in the direction of l_y also. If l_y is further reduced, the deflection in direction of l_x decreases and in direction of l_y increases. When $l_y = l_x$ the membrane deflects as a saucer. The deflection in this case is smaller than what it was for the first case i.e. $l_y >> l_x$. When $l_y \geqslant 2l_x$ the case is a one-way spanning and when $l_y < 2l_x$; or $\frac{l_y}{l_x} < 2$; the case is a two-way spanning.

The above discussion shows that when a two-way slab is used, compared to one-way slab, the deflections and bending of slab are reduced; i.e. bending moment is reduced. In fact, bending moment is distributed in both the directions. This increases the load carrying capacity of the slab.

The two-way slabs can be simply supported or restrained slabs. In simply supported slabs, the corners can lift away from the support while for restrained slabs, the corners are held down by edge beams. The corners are held down by means of the stiffness of the beam and therefore at corners, torsion is induced. The slab should be capable of resisting torsion at corners. Also the edge beams shall be designed for torsion. When there is a continuity in slab, the corners are held down by means of continuity effect and hence, no provision for torsion is required for continuous slab on all four sides. It should be clear that this slab will not induce any torsion in supporting beams.

6-8. Simply supported two-way slabs: Here the corners can lift away from the supports and do not require the provision for torsion. These slabs can be designed in accordance with appendix C-2 of IS : 456-1978. The moments can be found out as follows:

When simply supported slabs do not have adequate provision to resist torsion at corners and to prevent the corners from lifting, the maximum moments per unit width are given by the following equations:

$$M_x = \alpha_x \cdot w \cdot l_x^2$$

$$M_y = \alpha_y \cdot w \cdot l_x^2$$

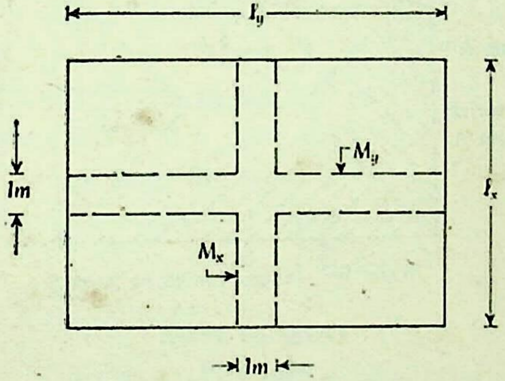
where referring fig. 6-13,

M_x, M_y = moments on strips of unit width spanning l_x and l_y respectively,

α_x, α_y = coefficients given in table 6-4,

l_x, l_y = lengths of the shorter span and longer span respectively, and

w = total design load per unit area.



Simply supported two-way slab

FIG. 6-13

The coefficients are given in table 23 of IS : 456 and are reproduced in table 6-4.

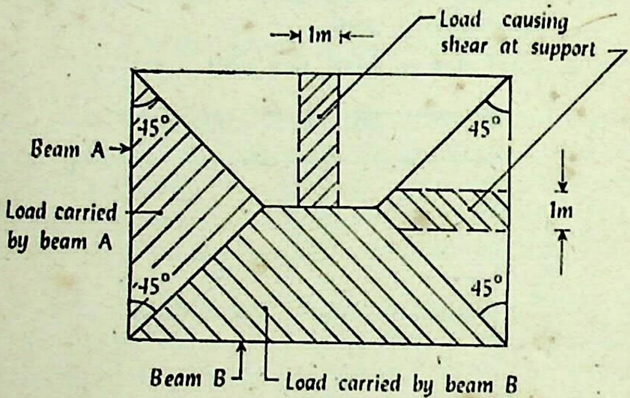
TABLE 6-4

BENDING MOMENT COEFFICIENTS FOR SLABS SPANNING IN TWO DIRECTIONS AT RIGHT ANGLES, SIMPLY SUPPORTED ON FOUR SIDES

$\frac{l}{l_y}$	1·0	1·1	1·2	1·3	1·4	1·5	1·75	2·0	2·5	3·0
αx	0·062	0·074	0·084	0·093	0·099	0·104	0·113	0·118	0·122	0·124
αy	0·062	0·061	0·059	0·055	0·051	0·046	0·037	0·029	0·020	0·014

IS : 456 also states that at least 50% of the tension reinforcement provided at mid-span should extend to within 0·1 l_x or 0·1 l_y of the support, as appropriate.

The shear stresses shall be checked as per one-way slab. The load causing maximum shear is shown in fig. 6-14. The loads on beams supporting solid slabs spanning in two directions at right angles and supporting uniformly distributed loads, may be assumed to be in accordance with fig. 6-14.



Loads on beam

Fig. 6-14

The deflection of two-way slabs shall be checked as per one-way slabs.

Note 1: For slabs spanning in two-directions, the shorter of the two spans should be used for calculating the span to effective depth ratios.

Note 2: For two-way slabs of small spans (upto 3·5 m) with mild steel reinforcement, the span to overall depth ratios given below may generally be assumed to satisfy vertical deflection limits for loading class upto 3000 N/m² (300 kg/m²):

Simply supported slabs	35
Continuous slabs	40

For high strength deformed bars of grade Fe 415, the values given above should be multiplied by 0·8.

The bar spacing controls remain same as per one-way slabs.

Example 6-4.

A drawing room of a residential building measures 4·3 m × 6·55 m. It is supported on 35 cm thick walls on all four sides. The slab is simply supported at edges with no provision to resist torsion at corners. Design the slab using grade M15 concrete and tor steel reinforcement of grade Fe 415.

Solution:

Consider 1 m wide strip. Assume 18 cm thick slab.

$l_x = 4{\cdot}3 + 0{\cdot}18 = 4{\cdot}48$ say $4{\cdot}5$ m.

$l_y = 6{\cdot}55 + 0{\cdot}18 = 6{\cdot}73$ say $6{\cdot}75$ m.

Dead load: self $0{\cdot}18 \times 25 = 4{\cdot}5$ kN/m²

floor finish $= 1{\cdot}0$ kN/m²

live load (residence) $= 2{\cdot}0$ kN/m²

Total $7{\cdot}5$ kN/m²

$$\frac{l_y}{l_x} = \frac{6{\cdot}75}{4{\cdot}5} = 1{\cdot}5$$

$$M_x = 0{\cdot}104 \times 7{\cdot}5 \times 4{\cdot}5^2 = 15{\cdot}8 \text{ kNm.}$$

$$M_y = 0{\cdot}046 \times 7{\cdot}5 \times 4{\cdot}5^2 = 6{\cdot}99 \text{ kNm.}$$

$$d_{required} = \sqrt{\frac{15{\cdot}8 \times 10^6}{1000 \times 0{\cdot}65}} = 156$$

$$d_{short} = 180 - 15 \text{ (cover)} - 5 = 160 > 156 \text{..(O.K.)}$$

$$d_{long} = 160 - 10 = 150$$

$$A_{st\ (short)} = \frac{15{\cdot}8 \times 10^6}{230 \times 0{\cdot}9 \times 160} = 477 \text{ mm}^2.$$

$$A_{st\ (long)} = \frac{6{\cdot}99 \times 10^6}{230 \times 0{\cdot}9 \times 150} = 225 \text{ mm}^2.$$

$$\text{Minimum steel} = \frac{0{\cdot}12}{100} \times 1000 \times 180 = 216 \text{ mm}^2.$$

Provide 10 mm Φ about 160 c/c = 491 mm² in short span and 8 mm Φ about 200 c/c = 250 mm² in long span.

The bars cannot be bent or curtailed because if 50% of long span bars are curtailed, the remaining bars will be less than minimum.

Check for development length:

Long span $V = 7{\cdot}5 \times 2{\cdot}25 = 16{\cdot}88$ kN.

$M_1 = 250 \times 230 \times 0{\cdot}9 \times 150 \times 10^{-6} = 7{\cdot}76$ kNm.

Assuming $L_o = 8\ \phi$,

$$1{\cdot}3 \times \frac{7{\cdot}76 \times 10^6}{16{\cdot}88 \times 10^3} + 8\ \phi \geqslant 69\ \phi$$

which gives $\phi \leqslant 9{\cdot}8$ mm........................(O.K.)

Short span $V = 7{\cdot}5 \times 2{\cdot}25 = 16{\cdot}88$ kN.

$M_1 = 491 \times 230 \times 0{\cdot}9 \times 160 \times 10^{-6} = 16{\cdot}26$ kNm.

Assuming $L_o = 8\ \phi$,

$$1{\cdot}3 \times \frac{16{\cdot}26 \times 10^6}{16{\cdot}88 \times 10^3} + 8\ \phi \geqslant 69\ \phi$$

which gives $\phi \leqslant 20{\cdot}53$ mm.....................(O.K.)

This shows usually bond is critical along long direction.

Check for shear:

This is critical along long span.

$$\text{Shear stress} = \frac{16{\cdot}88 \times 10^3}{1000 \times 150} = 0{\cdot}112 \text{ N/mm}^2.$$

$$\frac{100\ A_s}{bd} = \frac{100 \times 250}{1000 \times 150} = 0{\cdot}166.$$

$$k\tau_c = 0{\cdot}2 \times 1{\cdot}2 = 0{\cdot}24 \text{ N/mm}^2 \text{.........(O.K.)}$$

It shall be noted that IS : 456 gives value of τ_c minimum for 0·25% and SP : 16 gives value of τ_c minimum for 0·2% reinforcements. A formula to find out the permissible shear stress is given for limit state theory in SP : 16, however, nothing is mentioned for elastic theory. It is therefore considered that for a steel per cent less than 0·2, the value of τ_c for 0·2% steel will be adopted. It is hoped that new edition of IS : 456 will include this in table 17 and table 13 of code.

Check for deflection:

This check is critical along short span.

Basic $\dfrac{\text{span}}{\text{d}}$ ratio $= 20$.

$$\frac{100\,A_{st}}{bd} = \frac{491 \times 100}{1000 \times 160} = 0{\cdot}3.$$

Modification factor = 1·4.

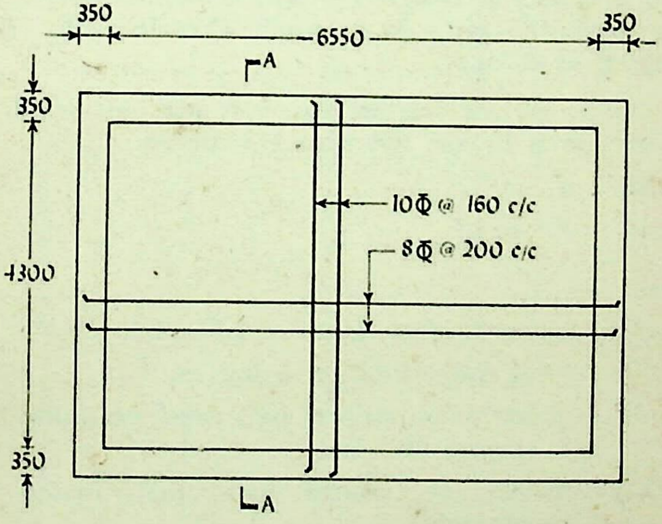
Permissible $\frac{\text{span}}{\text{d}}$ ratio = 20 × 1·4 = 28

actual $\frac{\text{span}}{\text{d}}$ ratio = $\frac{4480}{160}$ = 28................ (O.K.)

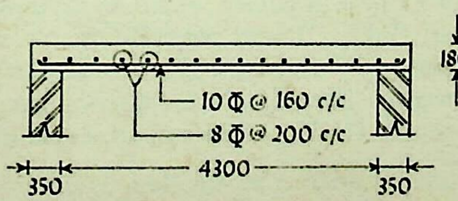
Check for cracking:

Maximum spacing permitted for short span steel
3 × 160 = 480 or 450 mm i.e. 450 mm.

Spacing provided = 160 mm..................(O.K.)



(a) Plan



(b) Section A-A

FIG. 6-15

Maximum spacing permitted for long span steel

$= 3 \times 150 = 450$ mm.

Spacing provided = 200 mm..................(O.K.)

Sketch: The designed reinforcement of slab is shown in fig. 6-15.

6-9. Restrained two-way slabs: In restrained two-way slabs, the corners are restrained and not allowed to lift away from the supports. If this is done, there induces a torsion at corners and the slab shall be suitably reinforced for torsion. According to IS : 456, the slabs spanning in two directions at right angles and carrying uniformly distributed load may be designed by any acceptable theory or by using coefficients given in Appendix C of IS : 456. This is explained as follows:

The maximum bending moments per unit width in a slab are given by the following equations:

$$M_x = \alpha_x \cdot w \cdot l_x^2$$

$$M_y = \alpha_y \cdot w \cdot l_x^2$$

where

α_x and α_y are coefficients given in table 6-5,

w = total design load per unit area,

M_x, M_y = moments on strip of unit width spanning l_x and l_y respectively, and

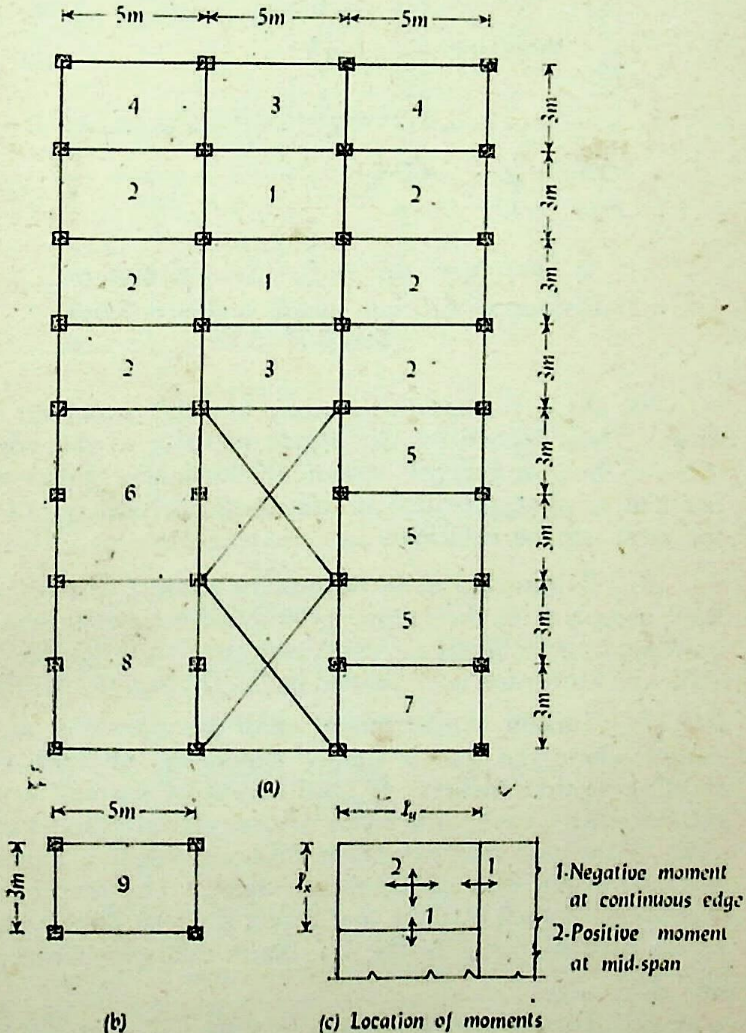
l_x, l_y = lengths of shorter span and longer span respectively.

Table 6-5 gives nine separate possible arrangement of two-way restrained slabs. These possibilities are illustrated in fig. 6-16(*a*) and (*b*). Fig 6-16(*c*) shows the location of different moments. The slabs are designed according to rules given in appendix C of IS : 456 and are explained below.

(1) Slabs are considered as divided in each direction into middle strips and edge strips as shown in fig. 6-17(*a*) and (*b*), the middle strip being three-quarters of the width and each edge strip one-eighth of the width.

(2) The maximum moments calculated in (1) apply only to the middle strips and no redistribution shall be made.

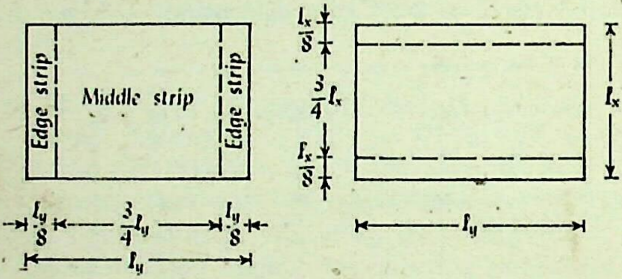
(3) Tension reinforcement provided at mid-span shall extend in the lower part of the slab to within $0{\cdot}25l$ of a continuous edge, or $0{\cdot}15l$ of a discontinuous edge.



Panel types for two-way slabs and location of moments

Fig. 6-16

(4) Over the continuous edges of a middle strip, the tension reinforcement shall extend in the upper part of the slab at a distance of $0{\cdot}15l$ from the support, and at least 50 per cent shall extend a distance of $0{\cdot}3\,l$.



(*a*) For span l_x (*b*) For span l_y

Division of slab into middle and edge strips

Fig. 6-17

(5) At a discontinuous edge, negative moments may arise. They depend on the degree of fixity at the edge of the slab but, in general, tension reinforcement equal to 50 per cent of that provided at mid-span extending $0{\cdot}1\,l$ into the span will be sufficient.

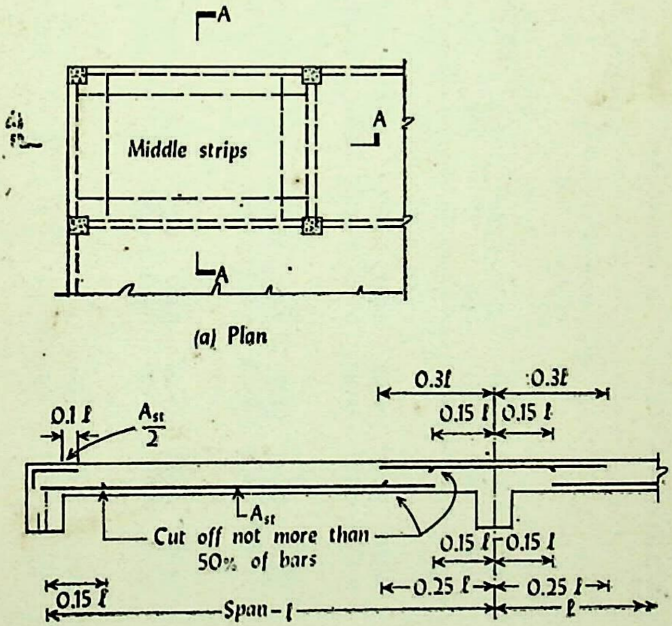
(6) Reinforcement in edge-strip, parallel to that edge, shall comply with the minimum reinforcement requirements, cracking requirements as explained previously for one way slabs and for torsion as explained in (7), (8) and (9) as follows.

(7) Torsion reinforcement shall be provided at any corner where the slab is simply supported on both edges meeting at that corner. It shall consist of top and bottom reinforcement, each with layers of bars placed parallel to the sides of the slab and extending from the edges a minimum distance of one-fifth of the shorter span. The area of reinforcement in each of these four layers shall be three quarters of the area required for the maximum mid-span moment in the slab.

(8) Torsion reinforcement equal to half that described in (7) shall be provided at a corner contained by edges over only one of which the slab is continuous.

(9) Torsion reinforcements need not be provided at any corner contained by edges over both of which the slab is continuous.

(10) Where $\frac{l_y}{l_x}$ is greater than 2, the slabs shall be designed as spanning one-way.



(a) Plan

(b) Section A-A

IS requirements for two-way slabs

FIG. 6-18

The above requirements are illustrated in fig. 6-18. *It is important to note that curtailment of bars can be done only if the continuing bars provide minimum reinforcements and satisfy the development length requirements.* Usually in most cases the bars cannot be curtailed as restricted by these requirements.

TABLE 6-5

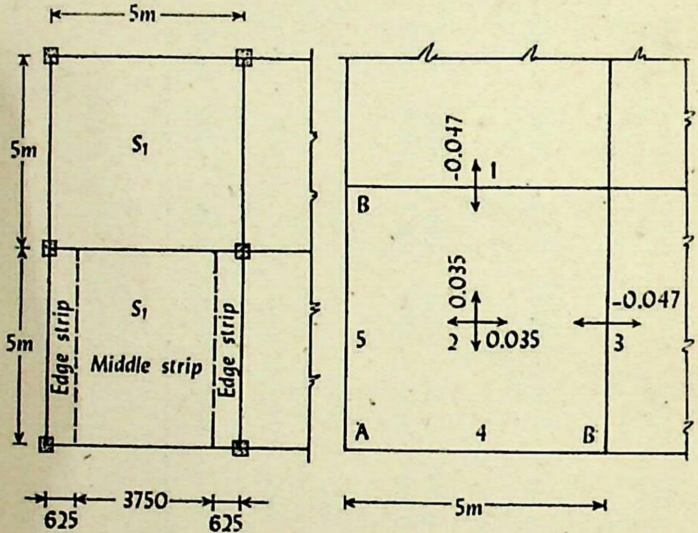
BENDING MOMENT COEFFICIENTS FOR RECTANGULAR PANELS SUPPORTED ON FOUR SIDES WITH PROVISION FOR TORSION AT CORNERS

Case No.	Type of panel and moments considered	Short span coefficients α_x (Values of l_y/l_x)								Long span coefficients α_y for all values of l_y/l_x
		1·0	1·1	1·2	1·3	1·4	1·5	1·75	2·0	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
1.	Interior panels:									
	Negative moment at continuous edge	0·032	0·037	0·043	0·047	0·051	0·053	0·060	0·065	0·032
	Positive moment at mid-span	0·024	0·028	0·032	0·036	0·039	0·041	0·045	0·049	0·024
2.	One short edge discontinuous:									
	Negative moment at continuous edge	0·037	0·043	0·048	0·051	0·055	0·057	0·064	0·068	0·037
	Positive moment at mid-span	0·028	0·032	0·036	0·039	0·041	0·044	0·048	0·052	0·028
3.	One long edge discontinuous:									
	Negative moment at continuous edge	0·037	0·044	0·052	0·057	0·063	0·067	0·077	0·085	0·037
	Positive moment at mid-span	0·028	0·033	0·039	0·044	0·047	0·051	0·059	0·065	0·028
4.	Two adjacent edges discontinuous:									
	Negative moment at continuous edge	0·047	0·053	0·060	0·065	0·071	0·075	0·084	0·091	0·047
	Positive moment at mid-span	0·035	0·040	0·045	0·049	0·053	0·056	0·063	0·069	0·035

5. Two short edge discontinuous:									
Negative moment at continuous edge	0·045	0·049	0·052	0·056	0·059	0·060	0·065	0·069	—
Positive moment at mid-span	0·035	0·037	0·040	0·043	0·044	0·045	0·049	0·052	0·035
6. Two long edges discontinuous:									
Negative moment at continuous edge	—	—	—	—	—	—	—	—	0·045
Positive moment at mid-span	0·035	0·043	0·051	0·057	0·063	0·068	0·080	0·088	0·035
7. Three edges discontinuous (One long edge continuous):									
Negative moment at continuous edge	0·057	0·064	0·071	0·076	0·080	0·084	0·091	0·097	—
Positive moment at mid-span	0·043	0·048	0·053	0·057	0·060	0·064	0·069	0·073	0·043
8. Three edges discontinuous (One short edge continuous):									
Negative moment at continuous edge	—	—	—	—	—	—	—	—	0·057
Positive moment at mid-span	0·043	0·051	0·059	0·065	0·071	0·076	0·087	0·096	0·043
9. Four edges discontinuous:									
Positive moment at mid-span	0·056	0·064	0·072	0·079	0·085	0·089	0·100	0·107	0·056

Example 6-5.

A part plan of a banking hall is shown in fig. 6-19(a). The slab is restrained with edge beams. Using M20 grade concrete and tor steel reinforcement of grade Fe 415, design slab S_1.



(a) Plan (b) Moment coefficients

FIG. 6-19

Solution:

Assume 15 cm thick slab.

Self load $0{\cdot}15 \times 25 = 3{\cdot}75$ kN/m²
Floor finish $= 1{\cdot}00$ kN/m²
Live load $= 3{\cdot}00$ kN/m²

Total $7{\cdot}75$ kN/m².

$$\frac{l_y}{l_x} = \frac{5}{5} = 1.$$

Middle strip:

$$M_1;\ M_3 = 0{\cdot}047 \times 7{\cdot}75 \times 5^2 = 9{\cdot}11 \text{ kNm}.$$

$$M_2 = 0{\cdot}035 \times 7{\cdot}75 \times 5^2 = 6{\cdot}78 \text{ kNm}.$$

For M20 mix and Fe 415 steel $Q = 0{\cdot}91$

$$d_{required} = \sqrt{\frac{9{\cdot}11 \times 10^6}{1000 \times 0{\cdot}91}} = 100 \text{ mm}.$$

$d_{provided}$ for positive reinforcement

$$= 150 - 15 - 10 - 5 = 120 \text{ mm (second layer)}.$$

For negative reinforcement $= 150 - 15 - 5 = 130$ mm ....(O.K.)

$$\text{Positive steel} = \frac{6\cdot78 \times 10^6}{230 \times 0\cdot9 \times 120} = 273 \text{ mm}^2.$$

Provide 8 mm Φ about 180 c/c = 278 mm².

$$\text{Negative steel} = \frac{9\cdot11 \times 10^6}{230 \times 0\cdot9 \times 130} = 339 \text{ mm}^2.$$

Provide 8 mm Φ about 140 c/c = 357 mm².

$$\text{Minimum steel} = \frac{0\cdot12}{100} \times 150 \times 1000 = 180 \text{ mm}^2.$$

At discontinuous edges 4 and 5, 50% of the positive steel is required at top $= \frac{1}{2} \times 273 = 136\cdot5$ mm².

This is less than minimum. Therefore, use minimum steel at locations 4 and 5.

Provide 8 mm Φ about 240 c/c = 208 mm². More steel is provided to match with the torsion reinforcement.

Edge strip:

In edge strip minimum reinforcements are provided equal to 8 mm Φ about 240 c/c.

Torsion steel:

At corner *A*,

Steel required $= \frac{3}{4} \times 273 = 205$ mm².

Use 8 mm Φ about 240 c/c = 208 mm². This will be provided by minimum steel of edge strip.

At corner *B*,

Steel required $= \frac{1}{2} \times 205 = 102\cdot5$ mm².

Use 8 mm Φ about 240 c/c = 208 mm², provided by the minimum steel.

Note that positive reinforcements are not curtailed because if they are curtailed, the remaining bars do not provide minimum steel.

Check for shear:

At point 1 or 3

$$\text{S.F.} = 7{\cdot}75 \times \frac{5}{2} + \frac{9{\cdot}11}{5} = 19{\cdot}38 + 1{\cdot}82 = 21{\cdot}2 \text{ kN.}$$

$$\frac{100\,A_s}{bd} = \frac{100 \times 357}{1000 \times 130} = 0{\cdot}275$$

$$\tau_c = 0{\cdot}228 \text{ N/mm}^2.$$

$$k\tau_c = 1{\cdot}3 \times 0{\cdot}228 = 0{\cdot}296 \text{ N/mm}^2$$

(M20 mix; 150 thick slab)

$$\text{Actual shear stress} = \frac{21{\cdot}2 \times 10^3}{1000 \times 130} = 0{\cdot}163 \text{ N/mm}^2.$$

At point 4 or 5

$$\text{S.F.} = 7{\cdot}75 \times \frac{5}{2} = 19{\cdot}38 \text{ kN}$$

$$\frac{100\,A_s}{bd} = \frac{100 \times 278}{1000 \times 120} = 0{\cdot}23$$

$$\tau_c = 0{\cdot}212 \text{ N/mm}^2$$

$$k\,\tau_c = 1{\cdot}3 \times 0{\cdot}212 = 0{\cdot}276 \text{ N/mm}^2.$$

$$\text{Actual shear stress} = \frac{19{\cdot}38 \times 10^3}{1000 \times 120} = 0{\cdot}162 \text{ N/mm}^2$$

........................(O.K.)

Check for development length:

This is critical at point 4 or 5.

At point 4 or 5

$$V = 19{\cdot}38 \text{ kN.}$$

No bar is curtailed or bent up

$$\therefore \quad M_1 = 278 \times 230 \times 0{\cdot}9 \times 120 \times 10^{-6} = 6{\cdot}9 \text{ kNm.}$$

Assume $L_o = 8\,\phi$

$1{\cdot}3\,\frac{M_1}{V} + L_o \geqslant L_d$ where L_d for M20 mix $= 51\,\phi$

$$\therefore \quad 1{\cdot}3 \times \frac{6{\cdot}9 \times 10^6}{19{\cdot}38 \times 10^3} + 8\,\phi \geqslant 51\,\phi$$

which gives $\phi \leqslant 10{\cdot}76$mm.................. (O.K.)

Check for deflection:

Basic $\frac{\text{span}}{\text{d}}$ ratio = 26.

Positive steel = 278 mm².

Actual $d = 150 - 15 - 8 - 4 = 123$ mm.

$$\frac{100\, A_{st}}{bd} = \frac{278 \times 100}{1000 \times 123} = 0{\cdot}22.$$

Modification factor = 1·55.

Permissible $\frac{\text{span}}{\text{d}}$ ratio $= 1{\cdot}55 \times 26 = 40{\cdot}30$.

Actual $\frac{\text{span}}{\text{d}}$ ratio $= \frac{5000}{123} = 40{\cdot}65$.

This is very near to the permissible value and no redesign is necessary. It shall be noted that for slabs, the deflection check is very important and may govern the thickness of slab.

Check for cracking:

Main reinforcement : Maximum spacing permitted

$= 3 \times 120 = 360$ mm.

Spacing provided = 180 mm..................(O.K.)

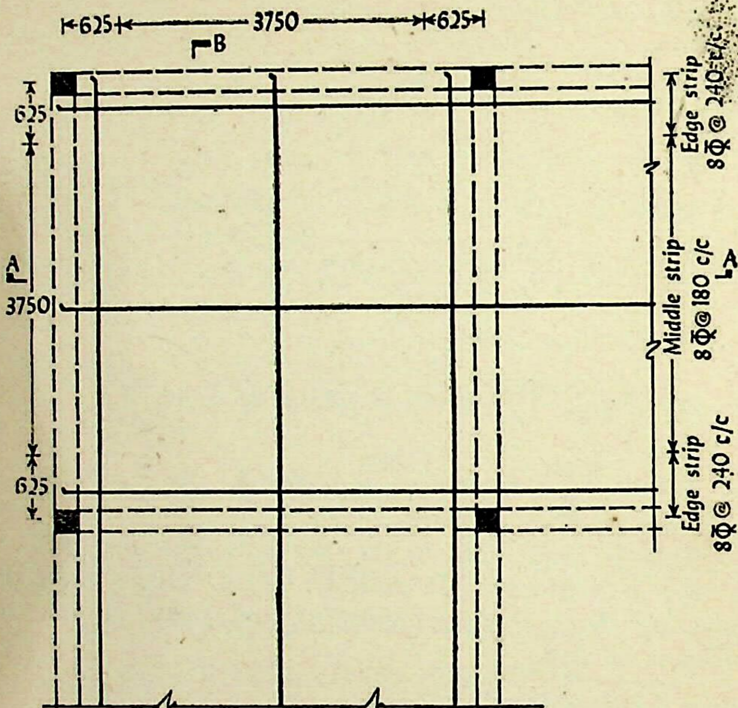
Secondary reinforcement:

The bottom reinforcement are both ways and therefore no necessity of secondary reinforcements. However, top reinforcement in edge strip requires the secondary steel for tying the bars.

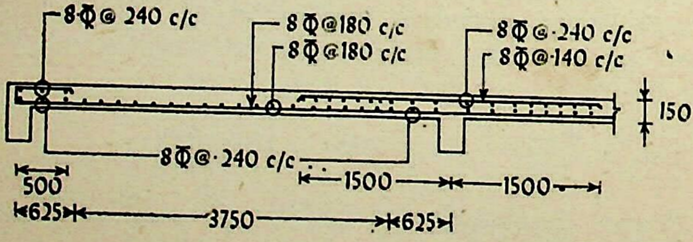
This is $\frac{0{\cdot}12}{100} \times 150 \times 1000 = 180$ mm².

Provide 8 mm ϕ about 240 c/c = 208 mm² for uniformity in spacing.

The arrangement of reinforcement is given in fig. 6-20. For clarity, top and bottom reinforcements are shown separately.



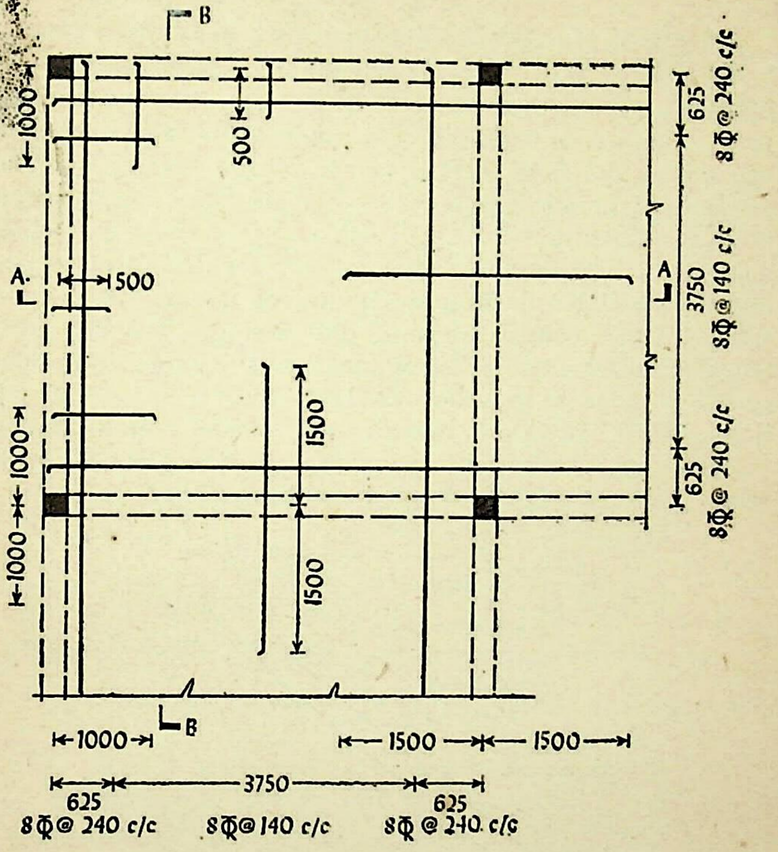
(a) Bottom plan



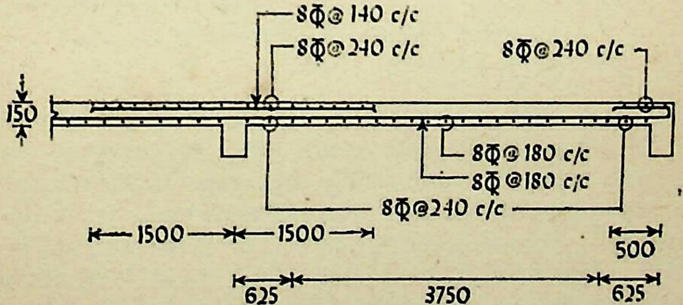
(c) Section A-A

Reinforcement details

FIG. 6-20



(b) Top plan

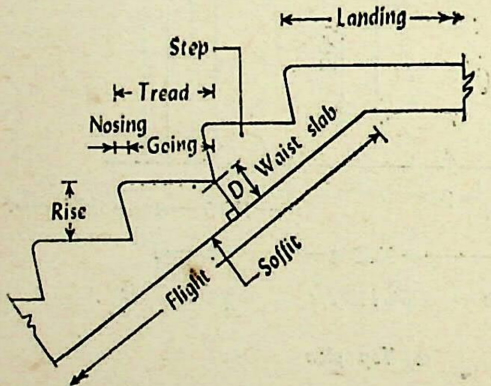


(d) Section B B

Reinforcement details

FIG. 6-20

6-10. Stair slabs: The staircases are used to give an access to different floors of a building. The components of stair are explained in fig. 6-21. The inclined slab of a stair is known as flight of stair while the straight portion other than the floor level is known as the landing. While going on flight, one travels vertically. The landing is provided midway either to turn the position and/or to relax while going up. The vertical height of the step is known as rise and the available horizontal distance on a step is known as tread. Tread consists of going and nosing. The net horizontal distance used in plan is known as going and additional nosing is done to get the required tread. This is shown in fig. 6-21. Nosing is not always provided. If the space for stair case is sufficiently available, then nosing is not necessary. However, this is decided by the Architect in charge.

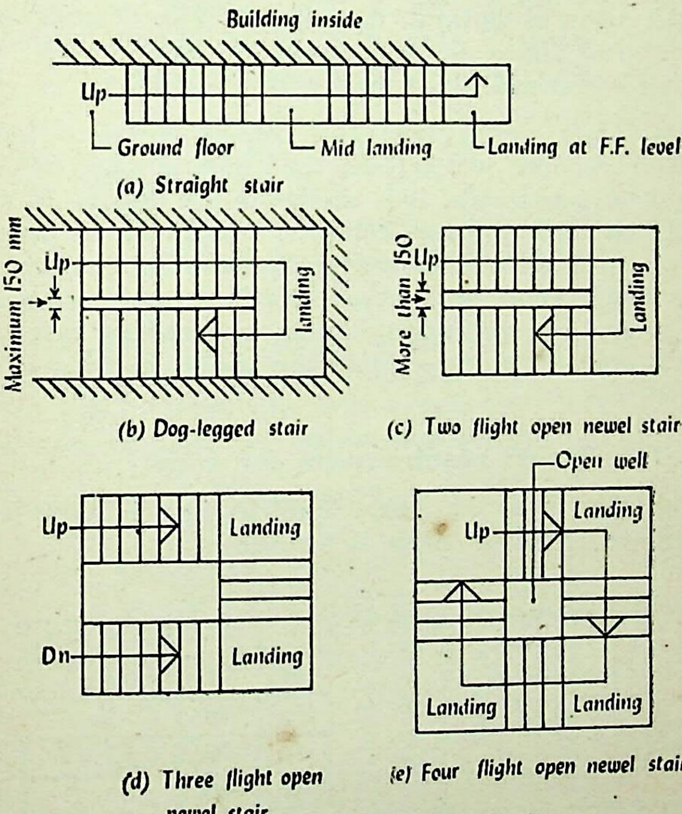


Nomenclature of a staircase
FIG. 6-21

The stairs are grouped into two types according to their use. They are (a) private stairs and (b) common stairs. *Private stair* is used for one family and *common stair* is used for more than one family or the stair of a commercial building, theatre halls, school buildings etc. For a private stair the care is taken that rise is not more than 200 mm and tread is not less than 230 mm. These are minimum requirements and usually a tread of 250 mm to 280 mm and a rise of 175 mm to 200 mm is provided depending on the space available. For common type stairs the rise is reduced and tr ead

is increased. This will depend on the use of the building and the space available. For any case the clear headway of 2·1 m is required on any step and for one flight the dimensions of tread and rise for all steps are kept same.

6-11. Classification of stairs: There are many types of staircases provided in buildings. Structurally speaking, the types of staircases are two:



(a) Straight stair

(b) Dog-legged stair

(c) Two flight open newel stair

(d) Three flight open newel stair

(e) Four flight open newel stair

Common types of staircases
Fig. 6-22

(a) Spanning longitudinally e.g. between floor beams of one floor to other floor or one floor to landing beam.

(b) Spanning in transverse direction i.e. each step is spanning between two parallel beams or cantilevered from one beam or wall.

According to arrangement of stair, some popular stairs are discussed below:

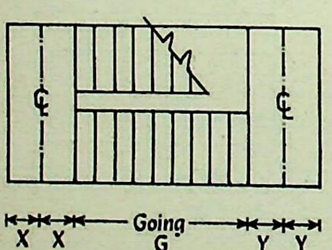
(1) Straight stair: This is a long narrow staircase that may or may not have landing. These stairs are popular in buildings, where the stairs are kept outside the building. This is shown in fig. 6-22(*a*).

(2) Dog-legged stair: This consists of two separate opposite flights as shown in fig. 6-22(*b*). The clear distance between two flights in plan may be zero to 150 mm. Landing is provided where the two flights meet.

(3) Open newel stair: This consists of two or more flights and an open well between the flights in plan. For two flight open newel stair, it is similar to the dog-legged stair except the clear distance between two flights in plan is more than 15 cm and is structurally spanning between the flights. In general, open newel stair has a stair well which may be used for ventilation purpose or sometimes it is used as a lift well. Open newel stairs are shown in fig. 6-22(*c*), (*d*) and (*e*).

6-12. Design requirements for stair:

(a) *Live loads on stair:* This is given in table 1 of IS : 875 and shall be taken as follows:



X	Y	Span in metres
<1m	<1m	G + X + Y
<1m	>1m	G + X + 1
>1m	<1m	G + Y + 1
>1m	>1m	G + 1 + 1

Effective span for stairs supported at each end by landing spanning parallel with the risers

Fig. 6-23

(1) Stairs, landing and corridors for floors in dwelling houses, tenements, hospital wards, bed rooms and private

sitting rooms in hostels and dormitories but not liable to over-crowding, the live load shall be 3 kN/m² of floor area.

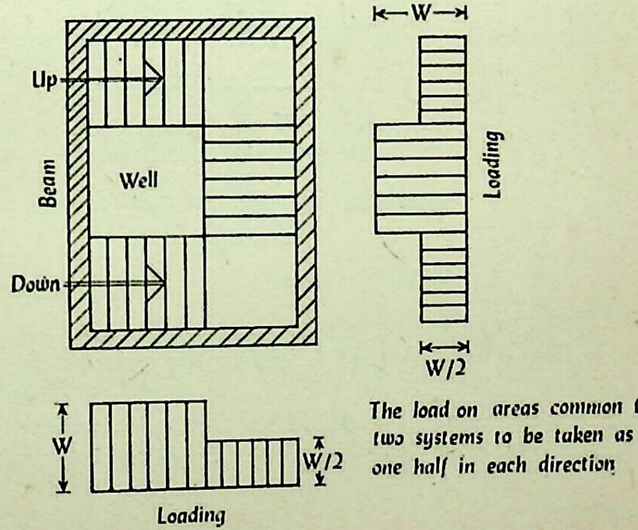
(2) Stairs, landing and corridors for above mentioned floors liable to over-crowding and for all other classes the live load shall be 5 kN/m² of floor area.

(3) The live load mentioned in (1) and (2) shall be subjected to minimum of 1·3 kN concentrated load at the unsupported end of each step for stairs constructed out of structurally independent cantilever steps.

(b) *Effective span of stairs:* The effective span of stair without stringer beams shall be taken as the following horizontal distances.

(1) Where supported at top and bottom risers by beams spanning parallel with the risers, the distance centre to centre of beams.

(2) Where spanning on to the edge of a landing slab, which spans parallel with the risers (fig. 6-23), a distance equal to the going of stairs plus at each end either half the width of landing or one metre, whichever is smaller.



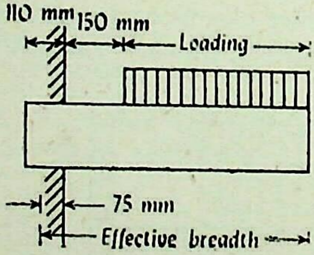
Loading on stairs with open wells

FIG. 6-24

(3) Where the landing slab spans in the same direction as the stairs, they shall be considered as acting together to form a single slab and the span determined as distance centre to centre of the supporting beams or walls, the going being measured horizontally.

(*c*) *Distribution of loading on stairs:* In the case of stairs with open wells, where spans crossing at right angles occur, the load on areas common to any two such spans may be taken as one-half in each direction as shown in fig. 6-24, where flights or landing are embedded into walls for a length of not less than 110 mm and are designed to span in the direction of the flight, a 150 mm strip may be deducted from the loaded area and the effective breadth of the section increased by 75 mm for purpose of design (fig. 6-25).

(*d*) *Depth of section:* The depth of section shall be taken as the minimum thickness perpendicular to the soffit of the stair case. This is shown in fig. 6-21.



Loading on stairs built into walls

Fig. 6-25

Example 6-6.

The arrangement of a dog-legged staircase in a residential building is shown in fig. 6-26. Rise of step is 16 cm and tread is 25 cm. Nosing is not provided. The materials are grade M15 concrete and mild steel reinforcement. Design the staircase.

Solution:

Assume 150 mm thick waist slab. Landing can span on walls.

Landing A or B

Self load $0{\cdot}15 \times 25 = 3{\cdot}75$ kN/m²
Floor finish $= 1{\cdot}00$ kN/m²
Live load (residence) $= 3{\cdot}00$ kN/m²
Total $7{\cdot}75$ kN/m²

Span $= 1950 + 150 = 2100$ i.e. $2{\cdot}1$ m.

$$M = \frac{2{\cdot}1^2}{8} \times 7{\cdot}75 = 4{\cdot}27 \text{ kNm.}$$

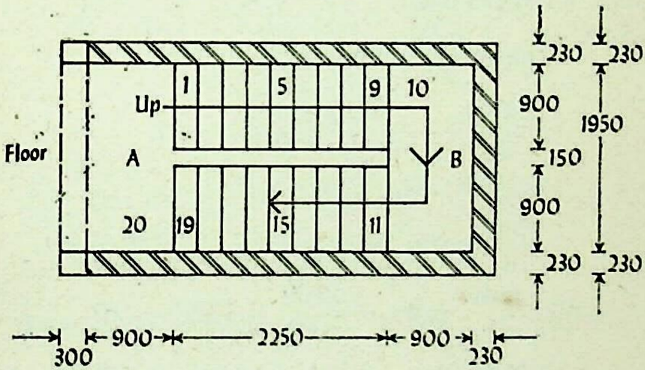


Fig. 6-26

Reinforcement will be in second layer. Assuming 12 ϕ bars, $d = 150 - 15$ (cover) $- 12 - 6 = 117$ mm.

$$d_{required} = \sqrt{\frac{4{\cdot}27 \times 10^6}{0{\cdot}87 \times 1000}} = 70 \text{ mm} \ldots\ldots\ldots \text{ (O.K.)}$$

$$A_{st} = \frac{4{\cdot}27 \times 10^6}{140 \times 0{\cdot}87 \times 117} = 300 \text{ mm}^2.$$

$$\text{Minimum steel} = \frac{0{\cdot}15}{100} \times 1000 \times 150 = 225 \text{ mm}^2.$$

Provide 10 mm ϕ about 250 c/c $= 314$ mm².

Maximum spacing $= 3 \times 117 = 351$ mm..... (O.K.)

Check for shear:

$$V = \frac{2{\cdot}1}{2} \times 7{\cdot}75 = 8{\cdot}14 \text{ kN.}$$

$$\text{Shear stress} = \frac{8{\cdot}14 \times 10^3}{1000 \times 117} = 0{\cdot}07 \text{ N/mm}^2 < 0{\cdot}2 \text{ N/mm}^2$$

(too small).

Check of development length:

Assuming $L_0 = 12\,\phi$ (mild steel)

$M_1 = 314 \times 140 \times 0{\cdot}87 \times 117 \times 10^{-6} = 4{\cdot}48$ kNm.

$V = 8{\cdot}14$ kN.

$$1{\cdot}3 \times \frac{4{\cdot}48 \times 10^6}{8{\cdot}14 \times 10^3} + 12\,\phi \geqslant 58\,\phi$$

which gives $\phi \leqslant 15{\cdot}55$ mm....................(O.K.)

Check for deflection:

$$\text{Basic } \frac{\text{span}}{\text{d}} \text{ ratio} = 20$$

$$\frac{100\ A_{st}}{bd} = \frac{100 \times 314}{1000 \times 117} = 0{\cdot}27$$

Modification factor = 2

$$\frac{\text{span}}{\text{d}} \text{ ratio permissible} = 2 \times 20 = 40$$

$$\text{actual } \frac{\text{span}}{\text{d}} \text{ ratio} = \frac{2100}{117} = 17{\cdot}95 < 40 \text{........(O.K.)}$$

Design of flight:

Span and loading on both the flights are same. Therefore same design will be adopted.

Loads:

Inclined length of waist slab for one step

$= \sqrt{25^2 + 16^2} = 29{\cdot}68$ cm.

Assuming 15 cm thick slab,

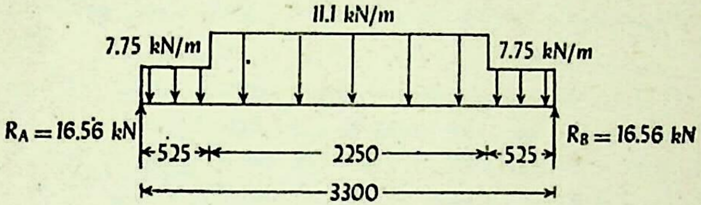
$$\text{self load in plan} = \frac{29{\cdot}68}{25} \times 0{\cdot}15 \times 25 = 4{\cdot}45 \text{ kN/m}^2.$$

Floor finish length for one step

$= 16 + 25 = 41$ cm.

Floor finish	$= \frac{41}{25} \times 1$	$= 1{\cdot}64$	kN/m²
Weight of step	$\frac{0 + 160}{2000} \times 25$	$= 2{\cdot}00$	kN/m²
Live load		$= 3{\cdot}00$	kN/m²
	Total	$11{\cdot}09$	kN/m²
	say	$11{\cdot}1$	kN/m².

Landing is spanning in transverse direction. The span of stair according to discussion made in art. 6-10 and loading for 1 m width of stair is shown in fig. 6-27.



Loading on flight

FIG. 6-27

$$R_A = R_B = 0{\cdot}525 \times 7{\cdot}75 + \frac{2{\cdot}25}{2} \times 11{\cdot}1$$

$$= 4{\cdot}07 + 12{\cdot}49 = 16{\cdot}56 \text{ kN.}$$

$$M = 16{\cdot}56 \times 1{\cdot}65 - \frac{1{\cdot}65^2}{2} \times 7{\cdot}75 - \frac{1{\cdot}125^2}{2} \times 3{\cdot}35$$

$$= 27{\cdot}32 - 10{\cdot}55 - 2{\cdot}12 = 14{\cdot}65 \text{ kNm.}$$

$$d_{required} = \sqrt{\frac{14{\cdot}65 \times 10^6}{1000 \times 0{\cdot}87}} = 129 \text{ mm.}$$

$$d_{provided} = 150 - 15 - 6 = 129 \text{ mm.............(O.K.)}$$

$$A_{st} = \frac{14{\cdot}65 \times 10^6}{140 \times 0{\cdot}87 \times 129} = 932 \text{ mm}^2.$$

Provide 12 mm ϕ about 120 c/c = 942 mm².

$$\text{Distribution steel} = \frac{0{\cdot}15}{100} \times 150 \times 1000 = 225 \text{ mm}^2.$$

$$\text{Spacing of 8 mm } \phi = \frac{50 \times 1000}{225} = 222 \text{ mm.}$$

Provide 8 mm ϕ about 220 c/c = 227 mm².

Check for shear:

$V = 16{\cdot}56$ kN.

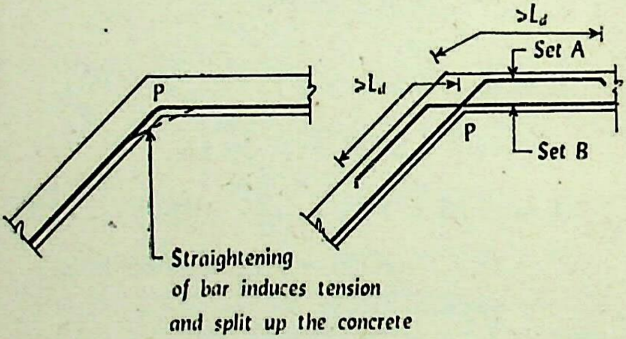
$$\text{Shear stress} = \frac{16{\cdot}56 \times 10^3}{1000 \times 129} = 0{\cdot}128 \text{ N/mm}^2.$$

$$\frac{100\,A_s}{bd} = \frac{100 \times 942}{1000 \times 129} = 0{\cdot}73$$

$$\tau_c = 0{\cdot}336 \text{ N/mm}^2 \qquad k = 1{\cdot}3.$$

Permissible shear stress $= 1{\cdot}3 \times 0{\cdot}336 = 0{\cdot}437$ N/mm^2.
..............(O.K.)

If the reinforcements are provided throughout at bottom, at landing, the bar will tend to split the concrete. To avoid this, two separate sets of bars are used as indicated in fig. 6-28(b). Note that this is equivalent to a lap at point P and the top bars on both sides should be extended upto L_d and bottom bars of course into the support.



(a) Wrong arrangement (b) Correct arrangement
Reinforcement details at landing
FIG. 6-28

At point P, the bending moment is reduced. If designer wishes and if check for shear, development length and curtailment rules can be satisfied; he can reduce the bars of set B. In such cases instead of reducing the number of bars, it is advisable to reduce the bar diameter keeping the spacing constant for the bars of set B e.g. for a given problem 10 mm ϕ about 120 c/c may be used in set B, if other checks are fulfilled. However, in this problem this reduction is not done and for set B also, 12 mm ϕ about 120 c/c are used.

Check for development length:

$$M_1 = 942 \times 140 \times 0{\cdot}87 \times 129 \times 10^{-6} = 14{\cdot}8 \text{ kNm.}$$

$$V = 16{\cdot}56 \text{ kN.}$$

Assume $L_o = 12\ \phi$

$$1{\cdot}3 \times \frac{14{\cdot}8 \times 10^6}{16{\cdot}56 \times 10^3} + 12\ \phi \geqslant 58\ \phi$$

which gives $\phi \leqslant 25{\cdot}26$mm.....................(O.K.)

From crossing of bars, the bars must extend upto $58 \times 12 = 696$ say 700 mm.

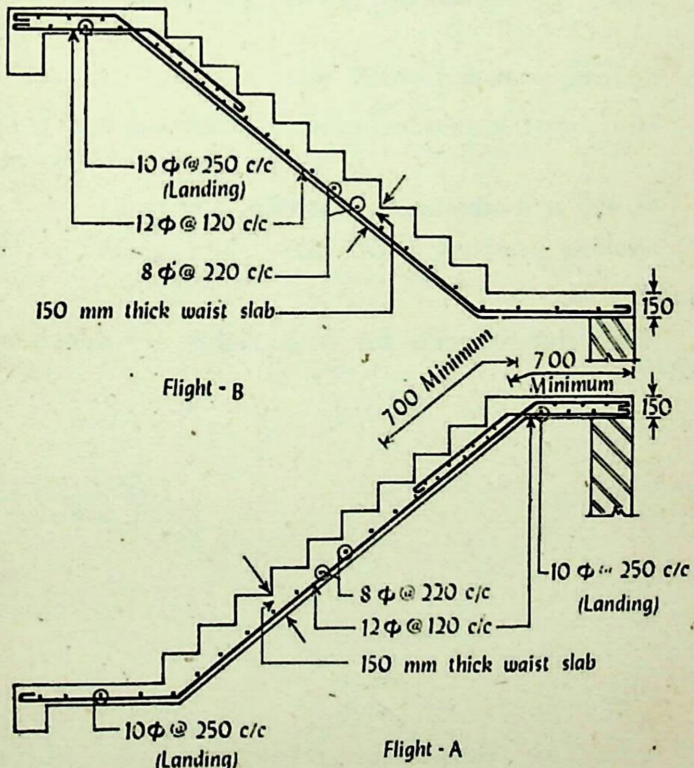


FIG. 6-29

Check for deflection:

Basic $\frac{\text{span}}{\text{d}}$ ratio $= 20$

$$\frac{100\ A_{st}}{bd} = \frac{100 \times 942}{1000 \times 129} = 0{\cdot}73.$$

Modification factor = 1·6.

Permissible $\frac{\text{span}}{\text{d}}$ ratio = 1·6 × 20 = 32.

Actual $\frac{\text{span}}{\text{d}}$ ratio = $\frac{3300}{129}$ = 25·58 < 32...... (O.K.)

Check for cracking:

Main bars: maximum spacing permitted = 3 × 129
= 387 mm.

Spacing provided = 120 mm..................(O.K.)

Dist. bars: maximum spacing permitted = 5 × 129
= 645 mm

or 450 mm whichever is less i.e. 450 mm.

Spacing provided = 220 mm..................(O.K.)

Sketch:

The reinforcements for both flights are shown in fig. 6-29.

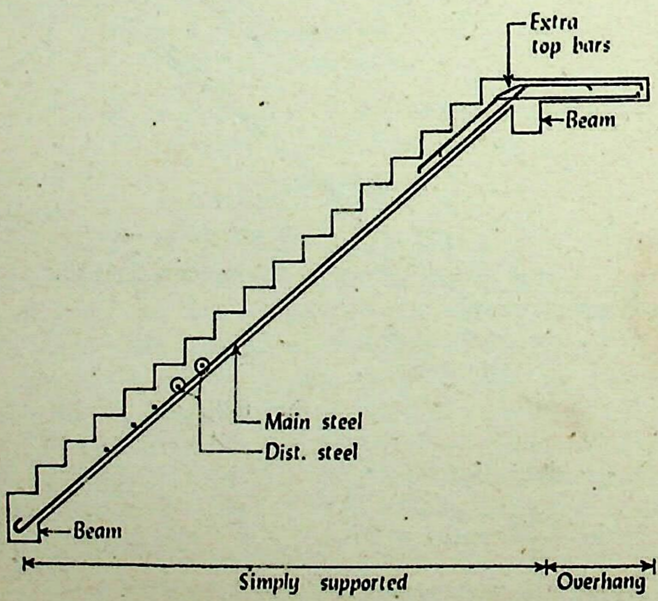


Fig. 6-30(*a*)

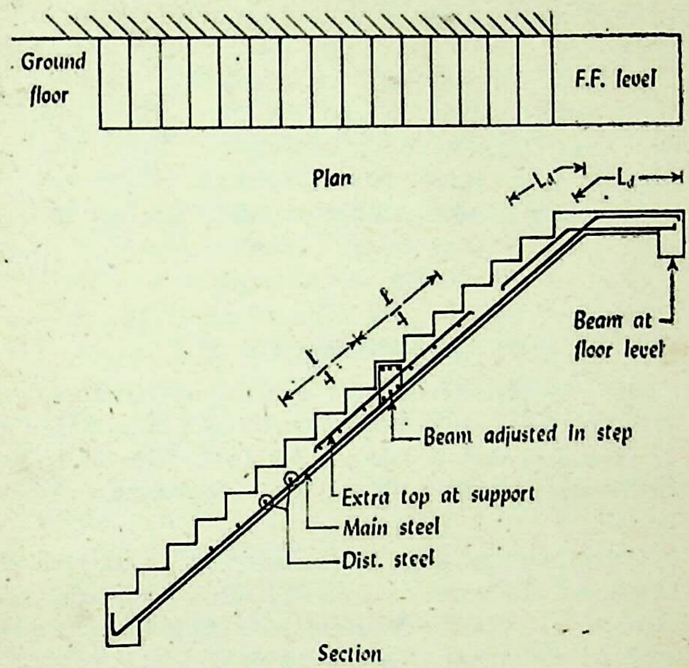
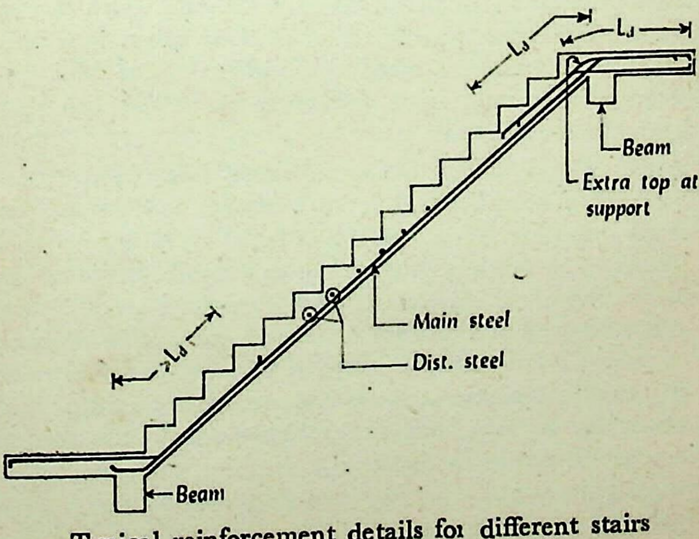


FIG. 6-30(*a*)



Typical reinforcement details for different stairs
FIG. 6-30(*b*)

6-13. Structural details for some stairs: Staircase is a special case of an inclined slab. While studying the slab, we have studied simply supported, cantilever or continuous slabs. Staircase also may be simply supported (Example 6-6), continuous or cantilever. These stairs may be designed by the same principles used for slabs. However, structural details of some stairs are shown in fig. 6-30.

EXAMPLES VI

(1) Design a simply supported one-way slab of span 2·8 m centre to centre of supporting beams of a corridor in office building. The live load shall be taken as 5 kN/m². The materials are M15 grade concrete and tor steel reinforcement of grade Fe 415.

(2) A five span continuous one-way slab is to be used for school building. The centre to centre distance of supporting beams is 3·0 m. Design the slab using M15 grade concrete and tor steel reinforcement of grade Fe 415.

(3) Design a terrace slab of a room 3·2 m × 4·3 m size and simply supported on all four sides on 230 mm thick masonry walls. The water proofing load shall be taken as 2 kN/m² and live load may be taken as 1·5 kN/m². Use M15 grade concrete and mild steel reinforcement. Floor finish may be taken as 1 kN/m².

(4) Design slab S_3 of fig. 6-9 cantilevered from bottom of the beam B_3. Size of beam B_3 is 300 mm wide × 600 mm overall depth and slab S_2 is at the top of beam B_3 (live load on this slab may be taken as 2 kN/m² as this slab is not to be used as balcony slab). Design the slab using M15 grade concrete and mild steel reinforcement. Sketch the details of reinforcement showing the anchorage of bars in beam. Also find the torsional moment to be resisted by beam B_3. If the beam B_3 is built into the columns, draw the torsional moment diagram for beam B_3.

(5) Design the staircase of fig. 6-26 if the live load is 5 kN/m². Use M15 grade concrete and tor steel reinforcement of grade Fe 415.

(6) An interior panel of a slab measures $4 \cdot 5$ m $\times$ $6 \cdot 0$ m centre to centre of supporting beams. Floor finish may be taken as 1 kN/m^2 and live load shall be considered as 4 kN/m^2. Design the slab using (a) M15 grade concrete and mild steel (b) M20 grade concrete and tor steel reinforcement of grade Fe 415.

(7) A simply supported one-way porch slab is spanning on two beams 3 m apart. Design the slab using M15 grade concrete and tor steel reinforcement of grade Fe 415.

(8) A slab *ABC* is supported at *A* and *B* and *BC* is overhang. $AB = 4$ m and $BC = 1 \cdot 2$ m. At *C*, there is a concrete parapet of 90 cm high. Design the slab for residential loads. The materials are M15 grade concrete and tor steel reinforcement of grade Fe 415.

(9) A four span continuous one-way slab is to be used for a marriage hall. The centre to centre distance of supporting beams is $3 \cdot 6$ m. Design the slab using M15 grade concrete and tor steel reinforcement of grade Fe 415.

(10) A continuous slab *ABCDEF* is supported at *A*, *B*, *C*, *D*, *E* and *F*. The span of exterior panels $AB = EF = 3$ m, while that of interior panels $BC = CD = DE = 3 \cdot 6$ m. The slab is to be used as an office floor with separate storage provision. Design the slab using M15 grade concrete and tor steel reinforcement of grade Fe 415. Analyse the slab using moment distribution.

(11) A two-way simply supported slab is resting on 23 cm thick masonry walls on all four sides. If the room dimensions are $4 \cdot 5$ m $\times$ $4 \cdot 5$ m, design the slab for residential loads. The materials are M15 grade concrete and mild steel reinforcement.

(12) Design a two-way restrained slab panel 3 of fig. 6-16 for office loads. The materials are M15 grade concrete and tor steel reinforcement of grade Fe 415.

(13) A stair-room is 3 m wide. A dog-legged stair is to be provided in two flights for a floor height of $4 \cdot 2$ m. The rise and tread shall be 15 cm and 30 cm respectively. The stair is to be used for an office building. Propose the architectural arrangement and design the flights. The materials are grade M15 concrete and tor steel reinforcement of grade Fe 415.

7

Columns

7-1. Introductory: The columns in a building are usually carrying axial compressive loads. In addition to the axial loads, usually columns also carry some moments. A column with only axial load is practically impossible. The moment in column may be due to gravity loads, wind loads or earthquake loads. Even the internal columns of a symmetrically framed building carry (may be small) the moments due to gravity loads when different types of live load arrangement is made. For wind and earthquake loads all columns carry some moments.

The shape of column may be square, rectangular, circular or any other shape depending on architectural requirements. Tee shape, ell shape or swastik columns are also used to match with the architectural requirements. Some popular shapes of column are shown in fig. 7-1.

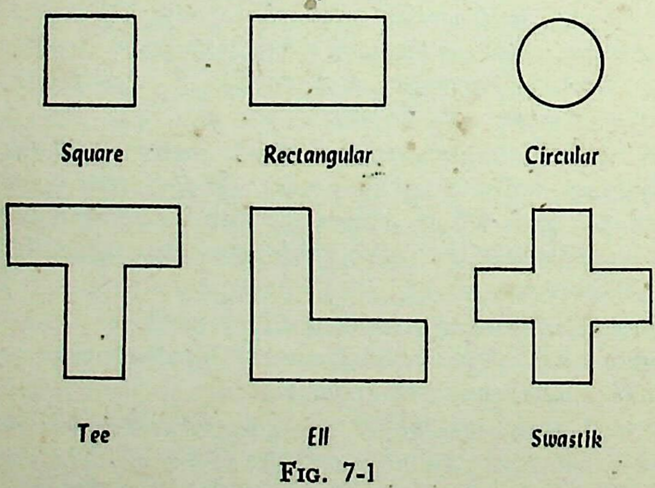


FIG. 7-1

7-2. Definitions and general requirements: The following are some definitions and general requirements as given by IS : 456.

(*1*) *Column:* A *column* or *strut* is a compression member, the effective length of which exceeds three times the least lateral dimension.

(*2*) *Short and slender column:* A compression member may be considered as short when both the slenderness ratios $\frac{l_{ex}}{D}$ and $\frac{l_{ey}}{b}$ are less than 12.

For more exact calculations a column is short if the slenderness ratios $\frac{l_{ex}}{i_{xx}}$ or $\frac{l_{ey}}{i_{yy}}$ are less than 40

where

l_{ex} = effective length in respect of the major axis

D = depth in respect of the major axis

l_{ey} = effective length in respect of the minor axis

b = width of the member

i_{xx} = radius of gyration in respect of the major axis

i_{yy} = radius of gyration in respect of minor axis.

It shall otherwise be considered as a slender compression member.

(*3*) *Unsupported length:* The unsupported length l, of a compression member shall be taken as the clear distance between end restraints. For more details IS : 456 may be consulted.

(*4*) *Effective length:* In the absence of more exact analysis, the effective length l_{ef} of columns may be obtained as described in table 24 of IS : 456. This is reproduced in table 7-1.

(*5*) *Slenderness limits for column:* The unsupported length between end restraints shall not exceed 60 times the least lateral dimension of a column.

If, in any given plane, one end of a column is unrestrained, its unsupported length l, shall not exceed $\frac{100\, b^2}{D}$

where

b = width of that cross-section

D = depth of the cross-section measured in the plane under consideration.

(6) *Minimum eccentricity:* All columns shall be designed for minimum eccentricity equal to the unsupported length of column/500 plus lateral dimension/30, subject to a minimum of 20 mm.

TABLE 7-1

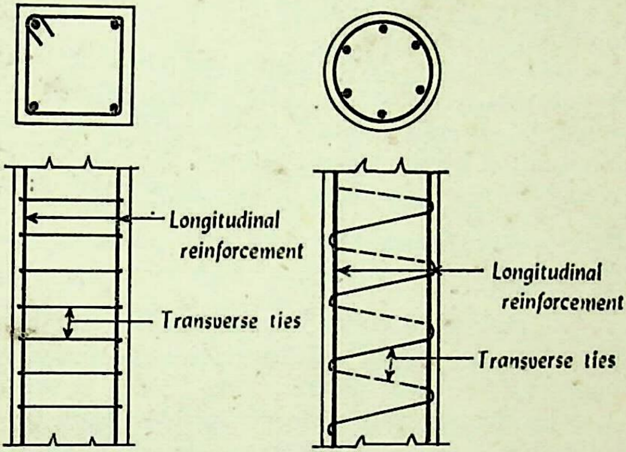
EFFECTIVE LENGTH OF COMPRESSION MEMBERS

No.	Degree of end restraint of compression member (1)	Theoretical value of effective length (2)	Recommended value of effective length (3)
(1)	Effectively held in position and restrained against rotation at both ends	0·5 l	0·65 l
(2)	Effectively held in position at both ends, restrained against rotation at one end	0·7 l	0·80 l
(3)	Effectively held in position at both ends, but not restrained against rotation	1·00 l	1·00 l
(4)	Effectively held in position and restrained against rotation at one end, and at the other restrained against rotation but not held in position	1·00 l	1·2 l
(5)	Effectively held in position and restrained against rotation at one end, and at the other partially restrained against rotation but not held in position	—	1·5 l
(6)	Effectively held in position at one end but not restrained against rotation, and at the other end restrained against rotation but not held in position	2·00 l	2·00 l
(7)	Effectively held in position and restrained against rotation at one end but not held in position nor restrained against rotation at the other end	2·00 l	2·00 l

Note: l is the unsupported length of compression member.

7-3. Reinforcement requirements: A column contains longitudinal and transverse reinforcements. Transverse reinforcements may be (a) transverse tie or (b) helical reinforcements. Usually transverse ties are used in

columns. Helical reinforcements are sometimes used in circular column. Fig. 7-2 shows typical reinforcements in columns.



Typical reinforcements in column

Fig. 7-2

The reinforcement requirements are set out in clause 25.5.3 of IS : 456. They are summarized as follows:

Longitudinal reinforcements:

(1) The bars shall not be less than 12 mm in diameter.

(2) There shall be minimum four bars in rectangular column and six bars in a circular or helically reinforced columns.

(3) Spacing of longitudinal bars along the periphery of column shall not exceed 300 mm. This is a requirement of cracking.

(4) Minimum cross-sectional area of longitudinal bars shall be 0·8 per cent of the gross cross-sectional area of the column. If a column has a larger cross-sectional area than that required to support the load, this minimum area of bars shall be based on the concrete area required to resist the direct stress and not upon the actual area.

(5) Maximum cross-sectional area of the longitudinal bars permitted is 6 per cent of gross cross-sectional area. However, at laps, the cross-sectional area shall not exceed 4 per cent. (In fact at laps, the total area of steel in other words is not to exceed 8 per cent.)

(6) In case of pedestals (which will be studied with foundations) in which the longitudinal reinforcement is not taken into account in strength calculations nominal longitudinal reinforcement not less than 0·15 per cent of the cross-sectional area shall be provided.

Transverse reinforcement:

(*a*) *General:*

A reinforced concrete compression member shall have transverse or helical reinforcement so disposed that every longitudinal bar nearest to the compression face has effective lateral support against buckling subject to provisions in (b). The effective lateral support is given by transverse reinforcement either in the form of circular rings capable of taking up circumferential tension or by polygonal links (lateral ties) with internal angles not exceeding 135°. The ends of the transverse reinforcement shall be properly anchored. (This has been explained in case of anchoring shear reinforcement in art. 7-3.)

(*b*) *Arrangement of transverse reinforcement:*

(1) If the longitudinal bars are not spaced more than 75 mm on either side, transverse reinforcement need only to go round corner and alternate bars for the purpose of providing effective lateral supports [fig. 7-3(*a*)].

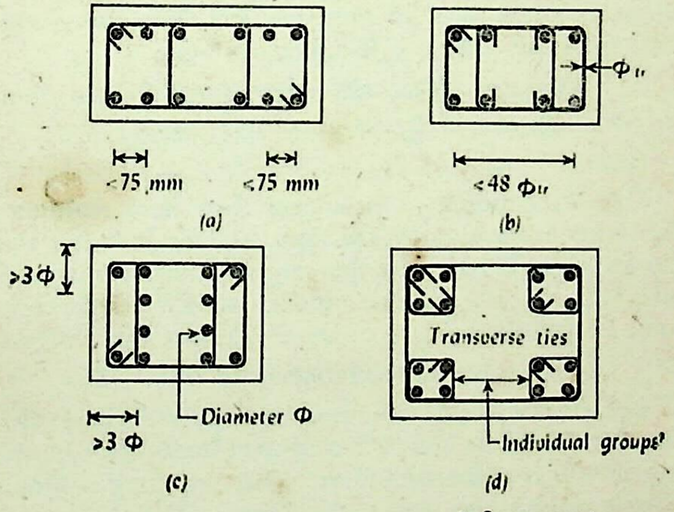
(2) If the longitudinal bars spaced at a distance of not exceeding 48 times the diameter of the tie are effectively tied in two directions, additional longitudinal bars in between these bars need to be tied in one direction by open ties [fig. 7-3(*b*)].

(3) Where the longitudinal reinforcing bars in a compression member are placed in more than one row, effective lateral support to the longitudinal bars in the inner rows may be assumed to have been provided if:

(i) transverse reinforcement is provided for outer most row in accordance with (2) and

(ii) no bar of the inner row is closer to the nearest compression face than three times the diameter of the largest bar in the inner row [(fig. 7-3(*c*)].

(4) Where the longitudinal bars in a compression member are grouped (not in contact) and each group adequately tied with transverse reinforcement in accordance with above requirements, the transverse reinforcement for the compression member as a whole may be provided on the assumption that each group is a single longitudinal bar for purpose of determining the pitch and diameter of the transverse reinforcement in accordance with above requirements. The diameter of such transverse reinforcement need not, however, exceed 20 mm [(fig. 7-3(*d*)].



Arrangment of transverse reinforcement
Fig. 7-3

(*c*) *Pitch and diameter of lateral tie:*

(1) *Pitch:* The pitch of transverse reinforcement shall be not more than the least of the following distances:

(i) The least lateral dimension of the compression member.

(ii) Sixteen times the smallest diameter of the longitudinal reinforcement bar to be tied.

(iii) Forty-eight times the diameter of the transverse reinforcement.

(2) *Diameter:* The diameter of the polygonal links or ties shall be not less than one-fourth of the diameter of the largest longitudinal bar and in no case less than 5 mm.

(*d*) *Helical reinforcement:*

(1) *Pitch:* Helical reinforcement shall be of regular formation with the turns of the helix spaced evenly and its ends shall be anchored properly by providing one and a half extra turns of the spiral bar. Where an increased load on the column on the strength of the helical reinforcement is allowed for, the pitch of helical turns shall be not more than 75 mm, nor more than one-sixth of the core diameter of the column, nor less than 25 mm, nor less than three times the diameter of the steel bar forming the helix.

(2) *Diameter:* The diameter of the helical reinforcement shall be as per lateral ties.

(*c*) *Cover:*

The longitudinal reinforcing bar in a column shall have concrete cover, not less than 40 mm, nor less than the diameter of such bar. In the case of columns the minimum dimensions of 200 mm or under, whose reinforcing bars do not exceed 12 mm, a cover of 25 mm may be used.

AXIALLY LOADED COLUMNS

The axially loaded column may be short or long (slender). In short column the concrete and steel reach their permissible value of stresses before failure while for long columns the failure is caused also due to buckling. The design methods for short and slender columns are given below:

7-4. Short columns: When a column is short, the safe axial load on a column reinforced with longitudinal bars and lateral ties is given by the following equation:

$$P = \sigma_{cc} A_c + \sigma_{sc} A_{sc} \quad \text{(7-1)}$$

where

σ_{cc} = permissible stress in concrete in direct compression

A_c = cross-sectional area of concrete excluding any finishing material and steel

σ_{sc} = permissible compressive stress for column bars

A_{sc} = cross-sectional area of the longitudinal steel.

Note: The minimum eccentricity mentioned in art. 7-2(6) may be deemed to be incorporated in the above equation.

The permissible load for columns with helical reinforcement shall be 1·05 times the permissible load for similar member with lateral ties or rings.

Example 7-1.

Determine a safe load on a short column 230 mm × 350 mm reinforced with 6 no. 16 mm dia. tor steel bars of grade Fe 415. The concrete is of grade M20.

Solution:

$A_{sc} = 6 \times 201 = 1206 \text{ mm}^2$

$A_c = 230 \times 350 - 1206 = 79294 \text{ mm}^2$

For grade M20 concrete $\sigma_{cc} = 5 \text{ N/mm}^2$

For Fe 415 steel $\sigma_{sc} = 190 \text{ N/mm}^2$

$$\begin{aligned}\text{Safe load} = P &= \sigma_{cc} A_c + \sigma_{sc} A_{sc} \\ &= 5 \times 79294 + 190 \times 1206 \\ &= 625{\cdot}6 \times 10^3 \text{ N} = 625{\cdot}6 \text{ kN.}\end{aligned}$$

00
62 tonne

Example 7-2.

Determine a safe load on a short circular column of 300 mm diameter reinforced with 6 no. of 16 mm dia. mild steel bars. The concrete is of grade M15 using (a) lateral ties and (b) helical reinforcement.

Solution:

$A_{sc} = 6 \times 201 = 1206 \text{ mm}^2$

$A_c = \frac{\pi}{4} \times 300^2 - 1206 = 69480 \text{ mm}^2.$

For M15 mix $\sigma_{cc} = 4 \text{ N/mm}^2$.

For mild steel $\sigma_{sc} = 130 \text{ N/mm}^2$.

$$\begin{aligned}P &= \sigma_{cc} A_c + \sigma_{sc} A_{sc} \text{ when lateral ties are used} \\ &= 4 \times 69480 + 130 \times 1206 \\ &= 434{\cdot}7 \times 10^3 \text{ N} = 434{\cdot}7 \text{ kN.}\end{aligned}$$

When helical reinforcements are used, the safe load can be increased by 5%.

Then safe load $= 1{\cdot}05 \times 434{\cdot}7 = 456{\cdot}44$ kN.

Example 7-3.

A short R.C.C. column is to carry an axial load of 625 kN. If the column is to be a square, design the column. The materials are grade M15 concrete and tor steel reinforcement of grade Fe 415.

Solution:

Assume 0·8% steel of gross area

$\therefore \quad A_{sc} = 0{\cdot}008\ A_g$

and $\quad A_c = 0{\cdot}992\ A_g \qquad$ where A_g = gross area of cross-section.

$P = \sigma_{cc}\, A_c + \sigma_{sc}\, A_{sc}$

For grade M15 concrete $\sigma_{cc} = 4\ \text{N/mm}^2$

For grade Fe 415 steel $\sigma_{sc} = 190\ \text{N/mm}^2$.

Then, $625 \times 10^3 = 4 \times 0{\cdot}992\ A_g + 190 \times 0{\cdot}008\ A_g$

which gives $\quad A_g = 113885\ \text{mm}^2$.

If column is square, side of column = 337 mm.

Adopt 325 × 325 size of column (if higher size say 350 × 350 is adopted, steel would be less than the minimum).

Then, $625 \times 10^3 = 4\ (325 \times 325 - A_{sc}) + 190\ A_{sc}$

which gives,

$$186\ A_{sc} = 220000$$

and $\quad A_{sc} = 1089\ \text{mm}^2$.

Provide 4 no. 20 mm $\phi = 1256\ \text{mm}^2$.

$$\text{Steel percentage} = \frac{1256}{325 \times 325} \times 100 = 1{\cdot}19 > 0{\cdot}8$$

.................(O.K)

Lateral ties:

$$\text{Minimum diameter of tie} = \frac{20}{4} = 5\ \text{mm}.$$

Use 6 mm ϕ mild steel tie bars.

Spacing of ties shall be minimum of:

(1) Least lateral dimension of column = 325 mm.

(2) 16 times the smallest diameter of the longitudinal reinforcement bar to be tied $= 16 \times 20 = 320$ mm.

(3) 48 times the diameter of transverse reinforcement $= 48 \times 6 = 288$ mm.

Use 6 mm ϕ about 280 mm c/c.

The arrangement of reinforcements is shown in fig. 7-4.

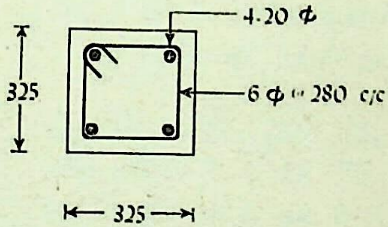


Fig. 7-4

Example 7-4.

A short R.C.C. column of size 450 mm × 450 mm has to carry an axial load of 600 kN. Design the column using M15 grade concrete and mild steel reinforcement.

Solution:

$$A_c = 450 \times 450 - A_{sc}$$
$$\sigma_{cc} = 4 \text{ N/mm}^2$$
$$\sigma_{sc} = 130 \text{ N/mm}^2$$
$$P = \sigma_{cc} A_c + \sigma_{sc} A_{sc}$$
$$600 \times 10^3 = 4\ (450 \times 450 - A_{sc}) + 130\ A_{sc}$$
$$= 810 \times 10^3 + 126\ A_{sc}$$

which gives $A_{sc} = -1667 \text{ mm}^2$.

Negative value indicates that there is no need of reinforcements. However, minimum reinforcement has to be provided. In this case minimum steel is based on required area for direct load.

Area of concrete required for direct load

$$\sigma_c A_c \Rightarrow \qquad A_c = \frac{600 \times 10^3}{\sigma_c = 4} = 150000 \text{ mm}^2.$$

$$\text{Minimum steel required} = \frac{0{\cdot}8}{100} \times 150000 = 1200 \text{ mm}^2.$$

Provide 4-16 ϕ + 4-12 ϕ bars = 1256 mm².

Note: If exact steel say 6 no. 16 mm ϕ is provided, on one side of column the centre to centre distance of bars will be 450 − 80 (cover) − 16 = 354 mm, which is greater than permissible value 300 mm.

Use 6 mm ϕ lateral ties.

Spacing should be lesser of:

(i) 450 mm

(ii) 16 × 12 = 192 mm

(iii) 48 × 6 = 288 mm.

Provide 6 mm ϕ ties about 280 c/c. This is shown in fig. 7-5.

Note that distance between bars in one face is more than 75 mm and distance between corner bars is (450 − 80 − 16 = 354 > 48 ϕ_{tr}). Therefore two sets of tie shall be used.

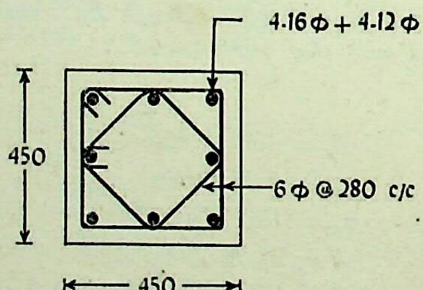


Fig. 7-5

7-5. Long columns: When a column is long, slendering effect becomes predominant and same section of column as used for short column can take a lesser load. IS : 456 specifies that the maximum permissible stress in a long reinforced concrete column shall not exceed that which

results from the multiplication of the appropriate maximum permissible stress by the coefficient C_r given by the formula:

$$C_r = 1{\cdot}25 - \frac{l_{ef}}{48b}$$

where C_r = reduction coefficient

l_{ef} = effective length of column

b = least lateral dimension of column, for a column with helical reinforcement, b is the diameter of core.

For more exact calculations, the maximum permissible stresses in a reinforced concrete column or part thereof having a ratio of effective column length to least lateral radius of gyration above 40 shall not exceed those which result from the multiplication of the appropriate maximum permissible stresses by coefficient C_r given by the following formula:

$$C_r = 1{\cdot}25 - \frac{l_{ef}}{160\ i_{min}}$$

where i_{min} is the least radius of gyration.

Using the value of C_r the safe load on a long column can be found out as follows:

Permissible stress in concrete $= C_r\ \sigma_{cc}$

Permissible stress in steel $= C_r\ \sigma_{sc}$

Area of concrete $= A_c$

Area of steel $= A_{sc}$

Safe load $= C_r\ \sigma_{cc} \cdot A_c + C_r\ \sigma_{sc} \cdot A_{sc}$

$= C_r\ (\sigma_{cc} A_c + \sigma_{sc} A_{sc})$ ....(7-2)

Thus, safe load on long column is the product of C_r and safe load on short column for the same section.

Example 7-5.

An R.C.C. column with effective height of 6 m and size 230 mm × 450 mm is subjected to an axial load of 600 kN. Design the column section. The materials are grade M20 concrete and tor steel reinforcement of grade Fe 415.

Solution:

$$l_{eff} = 6 \text{ m} \qquad b = 230 \text{ mm}$$

$$\frac{l_{eff}}{b} = \frac{6000}{230} = 26 > 12 \quad \therefore \quad \text{long column.}$$

$$C_r = 1{\cdot}25 - \frac{6000}{48 \times 230} = 0{\cdot}706.$$

Safe load on column

$$= C_r \; (\sigma_{cc} A_c + \sigma_{sc} A_{sc}).$$

Here $\sigma_{cc} = 5 \text{ N/mm}^2$

$A_c = (230 \times 450 - A_{sc})$

$\sigma_{sc} = 190 \text{ N/mm}^2$

$C_r = 0{\cdot}706.$

Equating,

$$600 \times 10^3 = 0{\cdot}706 \; [5 \; (230 \times 450 - A_{sc}) + 190 \; A_{sc}]$$
$$= 0{\cdot}706 \; (517{\cdot}5 \times 10^3 + 185 \; A_{sc})$$
$$185 \; A_{sc} = 849{\cdot}86 \times 10^3 - 517{\cdot}5 \times 10^3$$
$$= 332{\cdot}36 \times 10^3$$
$$A_{sc} = 1797 \text{ mm}^2.$$

Minimum reinforcement $= \dfrac{0{\cdot}8}{100} \times 230 \times 450 = 828 \text{ mm}^2.$

Provide 6 no. 20 mm Φ bars $= 6 \times 314 = 1884 \text{ mm}^2.$

Lateral ties:

Minimum diameter $= \dfrac{20}{4} = 5$ mm.

Use 6 mm ϕ M.S. ties. For the secondary reinforcement the use of lower grade of steel is permitted.

Spacing should be lesser of :

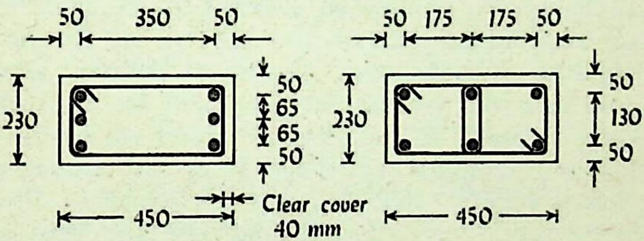
(i) 230 mm

(ii) $16 \times 20 = 320$ mm

(iii) $48 \times 6 = 288$ mm.

Use 6 mm ϕ lateral ties about 230 c/c.

Arrangement of lateral ties: Fig. 7-6(a) shows the arrangement of the designed reinforcement. In this case, the spacing of bars along width of the section is less than 300 mm. However, for the other direction, the spacing of bars exceed 300 mm. This does not satisfy the cracking requirements. The correct arrangement is shown in fig. 7-6(b). As the distance between bars exceed 75 mm, double tie is used. Also the distance between corner reinforcements along depth of section exceeds $48 \cdot \phi_{tr}$, two sets of closed tie are used.



(a) Wrong arrangement (b) Correct arrangement

FIG. 7-6

Example 7-6.

A circular column of 300 mm diameter is reinforced with 6 no. 20 mm diameter mild steel bars. If the effective length of column is 5 m, find the safe load on column. The concrete grade is M20.

Solution:

For circular column radius of gyration $= \dfrac{D}{4} = \dfrac{300}{4}$

$= 75$ mm.

$$\frac{l_{ef}}{i_{min}} = \frac{5000}{75} = 66{\cdot}66 > 40 \quad \therefore \text{ long column.}$$

$$C_r = 1{\cdot}25 - \frac{5000}{160 \times 75} = 0{\cdot}833.$$

For M20 mix $\sigma_{cc} = 5$ N/mm^2

For mild steel $\sigma_{sc} = 130$ N/mm^2

$A_{sc} = 6 \times 314 = 1884$ mm^2.

$$\text{Safe load} = 0{\cdot}833 \left[5 \left(\frac{\pi}{4} \times 300^2 - 1884\right) + 130 \times 1884\right]$$

$$= 490{\cdot}6 \times 10^3 \text{ N say } 490 \text{ kN.}$$

Supplementary details: Unless otherwise specified, for normal use, the concrete of M15 grade is used for all elements of a structure. For a column where size is not restricted, M15 mix with about 0·8% of reinforcement is economical. When there is a restriction of size, grade of concrete is increased or steel percentage is increased or the combination of both is used. For small structures, however, the concrete mix is not increased but steel is increased. This is little uneconomical, but used for two reasons:

(a) There are number of columns in one building and size of all columns cannot be different as formwork is uneconomical. Usually some groups of different sizes are made and then the columns are designed for the loads that they carry. Loads in columns or group of columns are different and designed separately. It is not necessary that all columns require higher grade of concrete. However, to keep the steel percentage around one per cent in all columns, if the mix is changed, it may be difficult, not impossible, to control the quality of concrete.

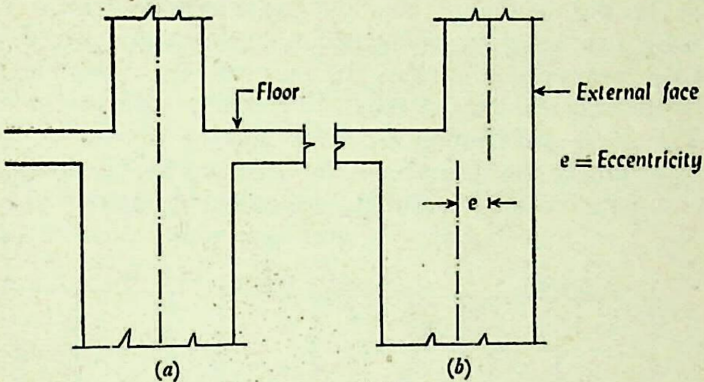
(b) On the other hand if mix is kept same but steel percentage is increased, sacrificing a small economy, it is not very difficult to control the reinforcements.

For a big job e.g. multi-storeyed building, the economy is very important and different economical combinations of size, mix and steel percentage may be adopted. For the size of the column, there can be *two* alternatives:

(1) Size of the colunn is kept same throughout all the floors. For lower storeys where loads and moments are more, richer mix and higher steel percentage is adopted. While for upper floors, the concrete mix and steel area are reduced. Usually for upper floors, the higher grade of concrete in a column than the lower floors for the same column is not used.

(2) Size of the column is reduced for upper floors and steel percentage is kept around 1 per cent. This can be done for the internal columns keeping the centroidal axis of the column same throughout. However, for exterior column, the external face of the column remains constant throughout and therefore, centroidal axis of the column changes. This induces moments in the lower column. Therefore, this method

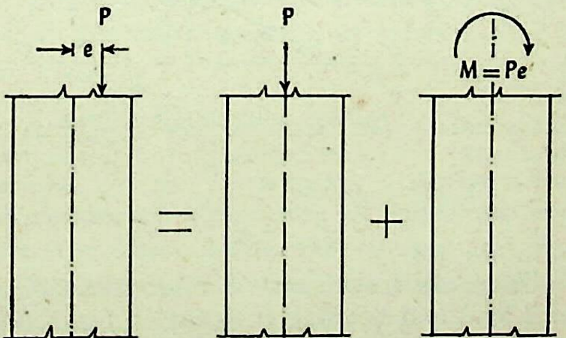
is used for internal columns only. This is shown in fig. 7-7.



Reduction in size of column
Fig. 7-7

ECCENTRICALLY LOADED COLUMNS

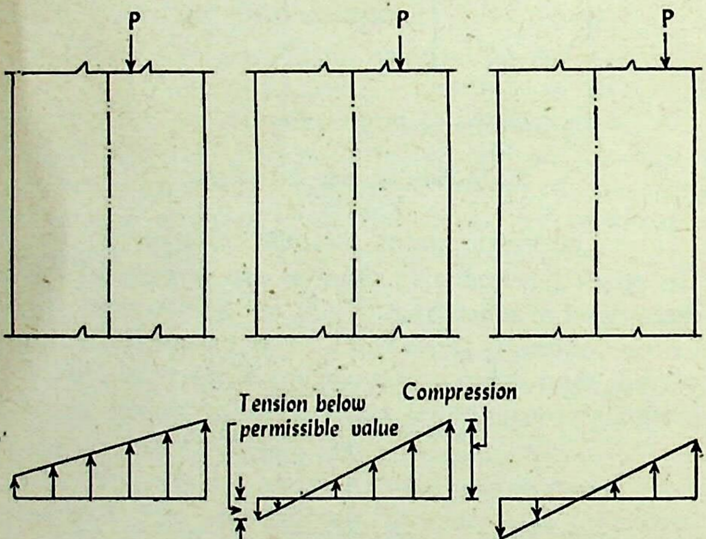
In general, usually all columns are eccentrically loaded i.e. direct load plus bending. There can be two cases: (a) The load P on column is at an eccentricity e from the centre line of column, then column is subjected to an axial load P plus the bending moment equal to Pe. This is shown in fig. 7-8.



Eccentrically loaded column
Fig. 7-8

(b) The load is axial but a column has a moment either due to gravity loads, wind loads, earthquake loads or any other loads. It can be said in this case that column is subjected to an axial load of P and bending moment M or the column is eccentrically loaded with a load P at an eccentricity $e = \frac{M}{P}$.

The bending in column can be either single axis bending or biaxial bending. For the eccentrically loaded column there can be three cases under consideration: (a) Compression over whole section (b) part of the concrete under permissible tension (c) part of concrete section under tension. The stress distribution for three types is shown in fig. 7-9. Accordingly the column can be designed by (a) design based on uncracked section and (b) design based on cracked section.



(*a*) Compression over the whole section (*b*) Part of section in limited tension (*c*) Part of section in tension

Stress distribution for eccentrically loaded columns

FIG. 7-9

7-6. Uncracked section: A column section subjected to the axial load and bending is shown in fig. 7-10(*a*). The transformed section is shown in fig. 7-10(*b*).

The sectional properties of transformed section are defined as:

$$A_T = A_C + 1{\cdot}5\ m\ A_{sc}$$
$$I_T = I_C + 1{\cdot}5\ m\ I_{sc}$$

where A_T = area of transformed section

A_C = net area of concrete section which is equal to gross area of concrete section minus A_{sc}

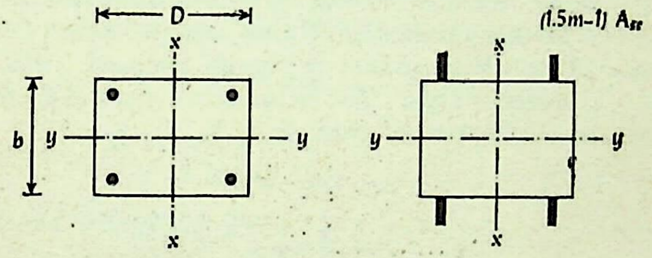
A_{sc} = cross-sectional area of longitudinal bar reinforcement

I_T = moment of inertia of transformed section.

The calculated direct compressive stress $\sigma_{cc,\ cal}$ in concrete is given by the equation

$$\sigma_{cc,\ cal} = \frac{P}{A_T} \quad \text{.......................} (7\text{-}3)$$

This stress is uniform all over the section.



(a) (b)

Eccentrically loaded column

FIG. 7-10

The calculated bending stress which is assumed zero at centroidal axis and maximum at extreme fibre shall be given by,

$$\sigma_{cbc,\ cal} = \frac{M_x}{Z_{xx}} \text{ if uniaxial bending} \quad \text{............} (7\text{-}4a)$$

and

$$= \frac{M_x}{Z_{xx}} + \frac{M_y}{Z_{yy}} \text{ if biaxial bending} \quad \text{......} (7\text{-}4b)$$

At extreme fibres where the stresses are maximum, following checks are to be made in accordance with IS : 456.

(a) $\dfrac{\sigma_{cc,\ cal}}{\sigma_{cc}} + \dfrac{\sigma_{cbc,\ cal}}{\sigma_{cbc}} \leqslant 1$

where

$\sigma_{cc,\ cal}$ = calculated direct compressive stress in concrete

σ_{cc} = permissible axial compressive stress in concrete

$\sigma_{cbc,\ cal}$ = calculated bending compressive stress in concrete

σ_{cbc} = permissible bending compressive stress in concrete.

(b) The maximum tensile stress σ_{ct} in concrete shall not exceed:

(1) $0 \cdot 25\ (\sigma_{cc,\ cal} + \sigma_{cbc,\ cal})$ for uniaxial bending
$0 \cdot 35\ (\sigma_{cc,\ cal} + \sigma_{cbc,\ cal})$ for biaxial bending.

(2) $0 \cdot 75 \times 7$ days modulus of rupture of concrete which may be taken from table 1-2.

Example 7-7.

The column section as shown in fig. 7-11 is subjected to an axial load of 600 kN and a moment of 12 kNm about y-y axis. Calculate maximum stresses in compression in concrete and steel. Also check whether the section is safe. The materials are grade M15 concrete and tor steel reinforcement of grade Fe 415.

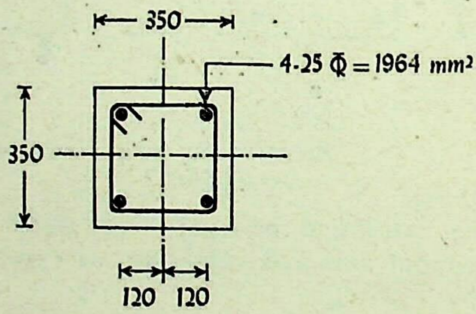


FIG. 7-11

Solution:

For M15 mix value of $m = 18 \cdot 66$. The section properties are worked out as follows:

$$\begin{aligned} A_T &= A + (1 \cdot 5\ m - 1)\ A_{sc} \\ &= 350 \times 350 + (1 \cdot 5 \times 18 \cdot 66 - 1) \times 1964 \\ &= 122500 + 53028 \\ &= 175528 \text{ mm}^2. \end{aligned}$$

$$\begin{aligned} I_{yy} &= \frac{1}{12} \times 350 \times 350^3 + (1 \cdot 5 \times 18 \cdot 66 - 1) \times 2 \times 982 \times 120^2 \\ &= 12 \cdot 5 \times 10^8 + 7 \cdot 64 \times 10^8 \\ &= 20 \cdot 14 \times 10^8 \text{ mm}^4. \end{aligned}$$

Concrete stresses:

$$\sigma_{cc,\,cal} = \frac{600 \times 10^3}{175528} = 3{\cdot}42 \text{ N/mm}^2$$

$$\sigma_{cbc,\,cal} = \frac{12 \times 10^6 \times 175}{20{\cdot}14 \times 10^8} = 1{\cdot}04 \text{ N/mm}^2.$$

Note that entire section is under compression.

Steel stresses:

Maximum compressive stress in steel = $(1{\cdot}5\ m - 1)$ $\times$ stress in concrete at level of steel reinforcement

$$= (1{\cdot}5\,m - 1)\left[3{\cdot}42 + \frac{12 \times 10^6 \times 120}{20{\cdot}14 \times 10^8}\right]$$

$$= 27\ (3{\cdot}42 + 0{\cdot}71)$$

$$= 111{\cdot}51 \text{ N/mm}^2.$$

Check: $\dfrac{\sigma_{cc,\,cal}}{\sigma_{cc}} + \dfrac{\sigma_{cbc,\,cal}}{\sigma_{cbc}} \leqslant 1.$

For M15 mix $\sigma_{cc} = 4$ N/mm²

$\sigma_{cbc} = 5$ N/mm².

Substituting

$$\frac{3{\cdot}42}{4} + \frac{1{\cdot}04}{5}$$

$$= 0{\cdot}855 + 0{\cdot}208 = 1{\cdot}063 > 1.$$

The section is not safe.

Example 7-8.

Check the column section of Ex. 7-7 for a load of 300 kN and a uniaxial moment of 32 kNm. The materials are grade M15 concrete and tor steel reinforcement of grade Fe 415.

Solution:

$$\sigma_{cc,\,cal} = \frac{300 \times 10^3}{175528} = 1{\cdot}71 \text{ N/mm}^2.$$

$$\sigma_{cbc,\,cal} = \frac{32 \times 10^6 \times 175}{20{\cdot}14 \times 10^8} = 2{\cdot}78 \text{ N/mm}^2.$$

Check: $\dfrac{1{\cdot}71}{4} + \dfrac{2{\cdot}78}{5}$

$$= 0{\cdot}428 + 0{\cdot}556 = 0{\cdot}984 < 1 \ldots\ldots\ldots\ldots \text{(O.K.)}$$

Now check for limiting tension is made as the bending stress is more than direct stress.

$\sigma_{ct} \not> 0{\cdot}25\ (\sigma_{cc,\ cal} + \sigma_{cbc,\ cal})$

and $\sigma_{ct} \not> 0{\cdot}75 \times 7$ days modulus of rupture.

Now $\sigma_{ct} = 2{\cdot}71 - 1{\cdot}71 = 1{\cdot}07\ \text{N/mm}^2$.

This should not exceed

$0{\cdot}25\ (1{\cdot}71 + 2{\cdot}71) = 1{\cdot}105$

and $0{\cdot}75\ (2{\cdot}1) = 1{\cdot}575$......................(O.K.)

The section is safe.

Stress in compression steel

$$= 1{\cdot}5\ (18{\cdot}66 - 1)\left[1{\cdot}71 + \frac{32 \times 10^6 \times 120}{20{\cdot}14 \times 10^8}\right]$$

$= 97{\cdot}74\ \text{N/mm}^2 < 190\ \text{N/mm}^2$................(O.K.)

Example 7-9.

The section of Example 7-7 is now subjected to a load of 320 kN and moments $M_{xx} = 10$ kNm and $M_{yy} = 20$ kNm. Check the section.

Solution:

$$A_T = 175528\ \text{mm}^4.$$

$$I_{xx} = I_{yy} = 20{\cdot}14 \times 10^8\ \text{mm}^4.$$

$$\sigma_{cc,\ cal} = \frac{320 \times 10^3}{175528} = 1{\cdot}82\ \text{N/mm}^2.$$

$$\sigma_{cbc,\ cal}\ (xx) = \frac{10 \times 10^6 \times 175}{20{\cdot}14 \times 10^8} = 0{\cdot}87\ \text{N/mm}^2.$$

$$\sigma_{cbc,\ cal}\ (yy) = \frac{20 \times 10^6 \times 175}{20{\cdot}14 \times 10^8} = 1{\cdot}74\ \text{N/mm}^2.$$

$\therefore$ $\sigma_{cbc} = 0{\cdot}87 + 1{\cdot}74 = 2{\cdot}61\ \text{N/mm}^2$.

As bending stress is more than direct stress, tension exists in concrete

$$\sigma_{ct} = 2{\cdot}61 - 1{\cdot}82 = 0{\cdot}79\ \text{N/mm}^2.$$

Check: (a) $\dfrac{1{\cdot}82}{4} + \dfrac{2{\cdot}61}{5}$

$= 0{\cdot}455 + 0{\cdot}522$

$= 0{\cdot}977 < 1$......................(O.K.)

(b) $\sigma_{ct} \not> 0.35\ (\sigma_{cc,\ cal} + \sigma_{cbc,\ cal})$

and $\sigma_{ct} \not> 0.75 \times 7$ days modulus of rupture

i.e. $\sigma_{ct} \not> 0.35\ (1.82 + 2.61)$

$0.79 \not> 1.55$ ....(O.K.)

and $\sigma_{ct} \not> 0.75\ (2.1)$

$0.79 \not> 1.571$ ....(O.K.)

The section is safe.

Stress in compressive steel

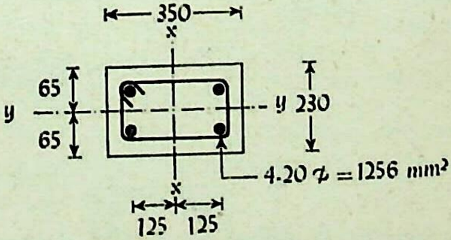
$$= (1.5 \times 18.66 - 1)\left[1.82 + \frac{10 \times 10^6 \times 125}{20.14 \times 10^8} + \frac{20 \times 10^6 \times 125}{20.14 \times 10^8}\right]$$

$$= 27\ (1.82 + 0.62 + 1.24)$$

$$= 99.36\ \text{N/mm}^2 < 190\ \text{N/mm}^2 \text{..........(O.K.)}$$

Example 7-10.

Design a column 230 mm × 350 mm size for a load of 400 kN and a moment $M_{xx} = 8$ kNm as shown in fig. 7-12. The materials are M15 grade concrete and mild steel.



Trial section

FIG. 7-12

Solution:

Try 4-20 mm ϕ bars $\quad A_{sc} = 1256\ \text{mm}^2$.

$$A_T = 230 \times 350 + (1.5 \times 18.66 - 1) \times 1256$$
$$= 80500 + 33912 = 114412\ \text{mm}^2.$$

$$I_{xx} = \frac{1}{12} \times 230 \times 350^3 + (1.5 \times 18.66 - 1) \times 1256 \times 125^2$$

$$I_{xx} = 8{\cdot}22 \times 10^8 + 5{\cdot}3 \times 10^8 = 13{\cdot}52 \times 10^8 \text{ mm}^4.$$

$$\sigma_{cc,\,cal} = \frac{400 \times 10^3}{114412} = 3{\cdot}5 \text{ N/mm}^2.$$

$$\sigma_{cbc,\,cal} = \frac{8 \times 10^6 \times 175}{13{\cdot}52 \times 10^8} = 1{\cdot}04 \text{ N/mm}^2.$$

Check : $$\frac{3{\cdot}5}{4} + \frac{1{\cdot}04}{5} = 0{\cdot}874 + 0{\cdot}208$$
$$= 1{\cdot}083 > 1.$$

Revise the section.

Now try 4-20 ϕ + 2-16 ϕ

$$A_{sc} = 1256 + 402 = 1658 \text{ mm}^2.$$
$$A_T = 230 \times 350 + 1{\cdot}5\ (18{\cdot}66 - 1) \times 1658$$
$$= 80500 + 67149 = 147649 \text{ mm}^2.$$

$$I_{xx} = \frac{1}{12} \times 230 \times 350^3 + (1{\cdot}5 \times 18{\cdot}66 - 1) \times 1658 \times 125^2$$
$$= 8{\cdot}22 \times 10^8 + 6{\cdot}99 \times 10^8 = 15{\cdot}21 \times 10^8 \text{ mm}^4.$$

$$\sigma_{cc,\,cal} = \frac{400 \times 10^3}{147649} = 2{\cdot}71 \text{ N/mm}^2.$$

$$\sigma_{cbc,\,cal} = \frac{8 \times 10^6 \times 175}{15{\cdot}21 \times 10^8} = 0{\cdot}92 \text{ N/mm}^2.$$

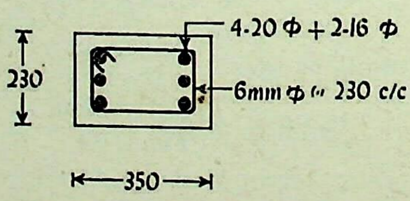


FIG. 7-13

Check: $$\frac{2{\cdot}71}{4} + \frac{0{\cdot}92}{5}$$
$$= 0{\cdot}678 + 0{\cdot}184$$
$$= 0{\cdot}862 < 1 \ldots\ldots\ldots\ldots\ldots\ldots \text{(O.K.)}$$

Maximum compressive stress in steel

$$= (1{\cdot}5 \times 18{\cdot}66 - 1)\left[2{\cdot}71 + \frac{8 \times 10^6 \times 125}{15{\cdot}21 \times 10^8}\right]$$

$$= 91 \text{ N/mm}^2 < 130 \text{ N/mm}^2 \ldots\ldots\ldots\ldots \text{(O.K.)}$$

The section with M15 mix concrete and 4-20 ϕ + 2-16 ϕ M.S. bars is adopted and is shown in fig. 7-13.

Ties:

Minimum diameter of tie $= \dfrac{20}{4} = 5$ mm.

Use 6 mm ϕ M.S. ties.

Spacing shall not exceed

(1) 230 mm

(2) $16 \times 16 = 256$ mm

(3) $48 \times 6 = 288$ mm.

Use 6 mm ϕ ties about 230 mm c/c.

7-7. Cracked section: When a section cannot be checked by the theory of uncracked section, it shall be checked by the theory of cracked section. Here the stresses are found out using the theory of cracked sections where the tensile strength of concrete is ignored i.e. concrete is assumed to be cracked. The actual stresses in steel or concrete shall not exceed the permissible stresses.

The maximum stress in concrete and steel may be found from tables and charts based on the cracked section theory or directly by determining the no-stress line which should satisfy the following requirements:

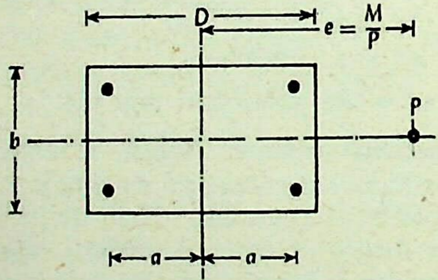
(a) The direct load should be equal to the algebraic sum of the forces on concrete and steel.

(b) The moment of the external loads about any reference line should be equal to the algebraic sum of the moment of the forces in concrete (ignoring the tensile force in concrete) and steel about the same line.

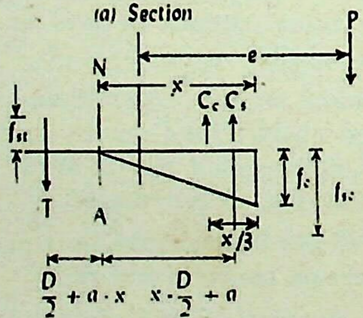
(c) The moment of the external loads about any other reference lines should be equal to the algebraic sum of the moment of the forces in concrete (ignoring the tensile force in concrete) and steel about the same line.

7-8. Uniaxial bending — cracked section: Consider a column section subjected to an axial load P and uniaxial moment M. Eccentricity e is given by $\frac{M}{P}$. This is shown in fig. 7-14. Let the depth of neutral axis be x.

Define: f_c = maximum compressive stress in concrete
f_{sc} = compressive stress in steel
f_{st} = tensile stress in steel
C_c = total compressive force in concrete
C_s = Total compressive force in steel
T = total tensile force in steel.



(a) Section



(b) Stresses and internal forces

FIG. 7-14

Referring fig. 7-14,

total compressive force in concrete

$$C_c = \tfrac{1}{2} f_c\, bx.$$

Compressive stress in concrete at the level of compressive steel

$$= f_c \cdot \frac{(x - \frac{D}{2} + a)}{x}.$$

Then compressive stress in steel

$$f_{sc} = (1·5\,m - 1)\, f_c\, \frac{(x - \frac{D}{2} + a)}{x}.$$

Total compressive force in steel

$$C_s = f_{sc} \cdot \frac{A_{sc}}{2}$$

$$= (1·5\,m - 1)\, f_c\, \frac{(x - \frac{D}{2} + a)}{x} \cdot \frac{A_{sc}}{2}.$$

Tensile stress in steel

$$f_{st} = m\, f_c\, \frac{(\frac{D}{2} + a - x)}{x}.$$

Total tensile force in steel

$$T = f_{st} \cdot \frac{A_{sc}}{2}$$

$$= m f_c\, \frac{(\frac{D}{2} + a - x)}{x} \cdot \frac{A_{sc}}{2}.$$

(a) Equating sum of vertical forces to zero

$$\Sigma V = 0$$

$$C_c + C_s - T - P = 0$$

i.e. $$P = C_c + C_s - T \ldots\ldots\ldots\ldots \quad (7\text{-}5a)$$

(b) Equating sum of moments about centre line of the column to zero

$$P\,e - C_c\,(\frac{D}{2} - \frac{x}{3}) - C_s \cdot a - T \cdot a = 0.$$

Substituting P from equation (7-5a)

$$(C_c + C_s - T)\, e = C_c\,(\frac{D}{2} - \frac{x}{3}) + (C_s + T)\, a \ldots \quad (7\text{-}5b)$$

Example 7-11.

A column 350 mm × 350 mm is subjected to a direct load of 280 kN and a moment of 28 kNm. It is reinforced with 4 no. 25 φ bars. Determine the stresses in concrete and steel. The materials are M15 grade concrete and tor steel reinforcement of grade Fe 415.

Solution:

The column section, stresses and internal forces are shown in fig. 7-15.

$$M = 28 \text{ kNm}$$

$$P = 280 \text{ kN}$$

$$\therefore \quad e = \frac{28 \times 10^6}{280 \times 10^3} = 100 \text{ mm.}$$

For M15 mix and tor steel $m = 18{\cdot}66$.

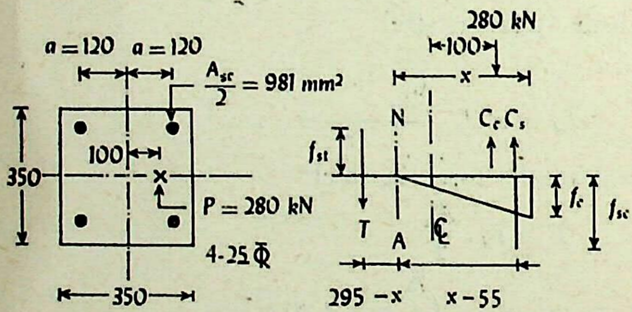


Fig. 7-15

Referring fig. 7-15

$$C_c = \frac{1}{2} f_c \cdot b \cdot x$$

$$= \frac{1}{2} f_c \cdot 350\, x = 175\, f_c x.$$

$$f_{sc} = (1{\cdot}5\, m - 1)\, f_c \frac{x - 55}{x} = 27\, f_c \frac{x - 55}{x}$$

$$C_s = f_{sc} \cdot \frac{A_{sc}}{2} = 27\, f_c \times \frac{x - 55}{x} \times 981$$

$$= 26487\, f_c \frac{x - 55}{x}$$

$$f_{st} = mf_c \cdot \frac{295 - x}{x} = 18{\cdot}66\ f_c\ \frac{295 - x}{x}$$

$$T = f_{st} \cdot \frac{A_{sc}}{2} = 18{\cdot}66\ f_c \times \frac{295 - x}{x} \times 981$$

$$= 18306\ f_c\ \frac{295 - x}{x}.$$

Equating vertical forces

$$P = C_c + C_s - T \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Sum of moments about centre line of column gives

$$P.e - C_c\ \left(\frac{D}{2} - \frac{x}{3}\right) - C_s a - T \,.\, a = 0.$$

Substituting P from (1) gives

$$(C_c + C_s - T)\ e = C_c \left(\frac{D}{2} - \frac{x}{3}\right) + (C_s + T)\ a \ldots\ldots\ldots (2)$$

Substituting above values

$$100 \left[175\ f_c x + 26487\ f_c\ \frac{x - 55}{x} - 18306\ f_c\ \frac{295 - x}{x}\right]$$

$$= 175\ f_c x\ \left(175 - \frac{x}{3}\right) + 120 \left[26487\ f_c\ \frac{x - 55}{x} + 18306\ f_c \times \frac{295 - x}{x}\right]$$

Eliminating f_c from both sides and multiplying both sides by x, gives

$$17500x^2 + 2648700x - 145678500 - 540027000 + 1830600x$$
$$= 30625x^2 - 58{\cdot}33x^3 + 3178440x - 174814200 + 648032400$$
$$- 2196720x$$

which gives

$$58{\cdot}33x^3 - 13125x^2 + 3497580x - 1158923700 = 0.$$

Dividing both sides by 58·33 gives

$$x^3 - 225\ x^2 + 59962\ x - 19868398 = 0.$$

Solving by trial and error

$$x = 273 \text{ mm.}$$

Substitute this value in equation (1)

$$P = C_c + C_s - T$$

$$280000 = 175\, f_c x + 26487\, f_c \frac{x-55}{x} - 18306\, f_c \frac{295-x}{x}$$

$$= 47775\, f_c + 21151\, f_c - 1475\, f_c$$

$$= 67451\, f_c$$

which gives

$$f_c = 4{\cdot}15 \text{ N/mm}^2 < 5 \text{ N/mm}^2 \ldots\ldots\ldots\ldots \text{(O.K.)}$$

Then $f_{sc} = 27\, f_c \dfrac{x-55}{x} = 89{\cdot}48 \text{ N/mm}^2 < 190 \text{ N/mm}^2$

$\ldots\ldots\ldots\ldots\ldots$(O.K.)

$$f_{st} = 18{\cdot}66\, f_c \frac{295-x}{x}$$

$$= 6{\cdot}24 \text{ N/mm}^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(O.K.)}$$

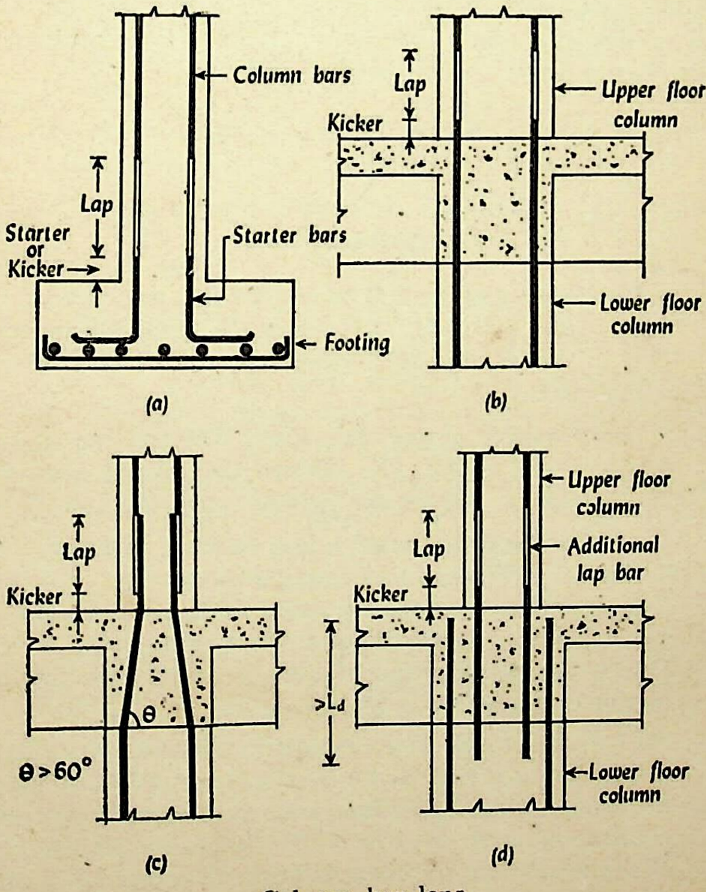
The section is safe in cracked theory.

7-9. Biaxial bending — cracked section: It can be seen that while designing column with cracked theory, the method is very cubersome. Example on biaxial bending involves more calculations than above. Moreover, according to IS : 456 clause 46.3, "Members subjected to combined direct load and flexure and designed by the methods based on elastic theory should be further checked for their strength under ultimate load conditions to ensure the desired margin of safety; this check is specially necessary when the bending moment is due to horizontal loads".

Considering the above requirements, it is advisable to use directly limit state method for direct load and bending in columns. The example on biaxial bending is considered outside the scope of this book and is not considered.

7-10. Column bar splices: Wherever required, the column bars shall be spliced or lapped for a length equal to the development length of bar. When bars of two different diameters are to be spliced, the lap length shall be calculated on the basis of diameter of smaller bar. Usually the lapping

of bars is adjusted such that lap is just above the slab. If the lap bars are provided at beam level, it will create difficulties in placing and compacting the concrete. Because of the reduction in loads in upper floor, usually the bars are reduced in number or the diameter is changed. This requires lapping of bars. If the bars are lapped in between the floors, again the laps are needed in upper floor and this is uneconomical due to increase in bar length. Sometimes it is economical, because of the length of lap required, to reduce the bars at alternate floors in multistoreyed building. Four general cases of lap bars are detailed in fig. 7-16.



Column bar laps
FIG. 7-16

Fig. 7-16(*a*) shows the lap of column bars at foundation level. To provide an exact alignment to the upper column, *starter* or *kicker* is casted above the footing or above the slab. Thickness of kicker is usually 8 to 10 cm and casted in a rich mix than lower column or footing. The bars shall be lapped above the kicker.

Fig. 7-16(*b*) shows the bar laps where the size of column is equal in lower and upper floors. Fig. 7-16(*c*) shows the bar laps where the size of upper column is less than that of lower column. Note that angle θ should not be less than 60°. If the size of column reduced, is such that $\theta > 60°$ is not possible, arrangement (d) is to be used where additional lap bars of length L_d in lower column and L_d + thickness of kicker in the upper column is provided.

EXAMPLES VII

(1) A short column of size 230 mm × 300 mm is reinforced with 6 no. 16 mm dia. bars. Determine the safe load on column. The materials are M15 grade concrete and mild steel reinforcement.

(2) A short column of size 300 mm × 300 mm has to carry an axial load of 650 kN. Design the column using M15 grade concrete and tor steel reinforcement of grade Fe 415.

(3) A ground floor column of a four storeyed building of size 230 mm × 450 mm has to carry an axial load of 850 kN. Two floors are to be constructed in first six months and remaining two floors are to be constructed after 2 years. Design the column using M15 grade concrete and tor steel reinforcement of grade Fe 415.

(4) An R.C.C. column of size 300 mm × 450 mm with effective height of 6 m is subjected to an axial load of 750 kN. Design the section using M15 grade concrete and tor steel reinforcement of grade Fe 415. Also design the section if the effective height is increased to 9 m. The column is hinged at both ends.

(5) Design column in Example (2) if a circular column section is to be adopted of size 300 mm diameter using (a) lateral ties and (b) helical reinforcement.

(6) Design a circular R.C.C. column to carry an axial load of 600 kN. The effective length of column is 6 m and there is no restriction in size of column. Use M20 mix and mild steel reinforcement.

(7) A column section of size 300 mm × 300 mm is subjected to an axial load of 500 kN and a moment of 12 kNm about one of the axis. Calculate maximum stresses in compression in concrete and steel if it is reinforced with 4-25 ϕ bars. Also check whether the section is safe. The materials are M15 grade concrete and tor steel reinforcement of grade Fe 415.

(8) Check the column section of Example (7) if it is subjected to a load of 260 kN and a moment of 10 kNm.

(9) Check the column section of Example (7) if it is subjected to a load of 260 kN and moments $M_{xx} = 6$ kNm and $M_{yy} = 5$ kNm.

(10) Design a column section 230 mm × 400 mm size for a load of 450 kN and M_x about major axis = 10 kNm. The materials are M15 grade concrete and tor steel of grade Fe 415. Also design the same section using mild steel. Compare the economy and comment.

(11) A column 350 mm × 350 mm is subjected to a direct load of 300 kN and a moment of 30 kNm. It is reinforced with 4 no. 25 mm dia. bars. Check whether the section is safe using theory of cracked section if necessary. The materials are M15 grade concrete and mild steel reinforcement.

(12) A tee shaped column of flange 300 mm × 600 mm and web size 300 mm × 300 mm is subjected to an axial load of 2000 kN. If the column is short, design the reinforcement. The materials are M15 grade concrete and tor steel reinforcement of grade Fe 415.

(13) A rectangular column of size 230 mm × 600 mm is subjected to a direct load of 500 kN and a moment of 40 kNm about major axis. Design the section. The materials are M15 grade concrete and tor steel reinforcement of grade Fe 415.

8

Foundations

8-1. Introductory: Foundations also called footings are provided under the column or a wall of the structure to transmit the loads from column to the soil. In addition to the axial loads, the foundations may have to resist moment due to gravity loads, wind loads or earthquake loads. The size of the footing depends on the safe bearing capacity of soil which differs from soil to soil. The safe bearing capacity (S.B.C.) of the soil can be determined either by practical experience or by soil testing. Dimensions of footings are selected such that the centre of gravity of column loads and that of footing coincide (if they do not coincide, a moment due to eccentricity of loads is induced). Various types of footings are used depending on type of soil and availability of land. Some types of footings are listed below:

(1) Continuous wall footing
(2) Isolated footing
(3) Combined footing
(4) Strap footing
(5) Raft foundation
(6) Pile foundation.

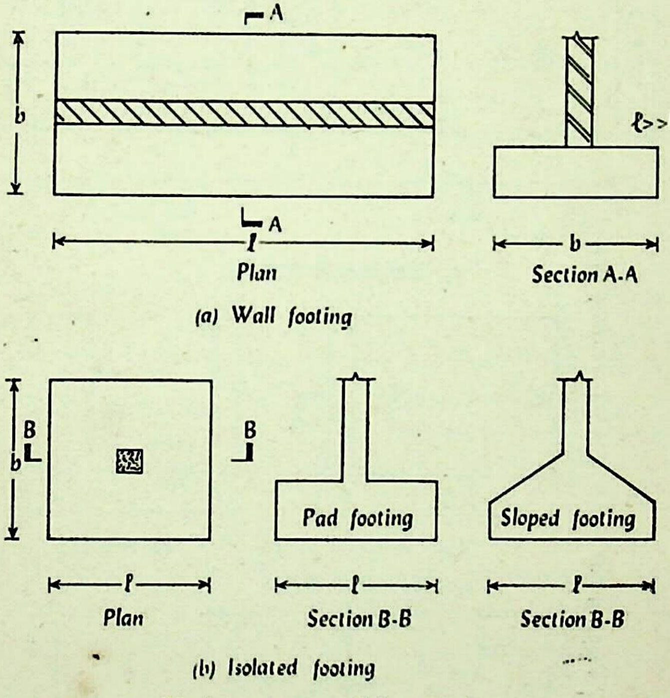
Each footing will now be briefly discussed.

Continuous wall footing: Continuous footing is provided under a long wall. In this case the width of footing will be very less than the length of footing [fig. 8-1(*a*)].

Isolated footing: Isolated footing is an individual one under the column. Where sufficient area and good soil is available, this is economical [fig. 8-1(*b*)].

Combined footing: Where the distance between two columns is small and if the isolated footings for these columns coincide, a combined footing is used. In this case the footings are combined such that centre of gravity of column loads and

that of footing coincide. This may result in a rectangular or trapezoidal shape of footing [fig. 8-1(*c*)]. This footing also can be used for the column on the property line where it is combined with the footing of internal column [fig. 8-1(*d*)].



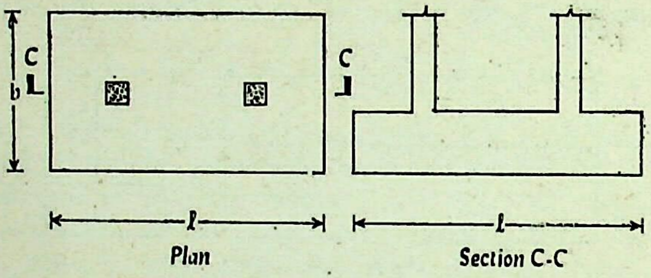
Various types of foundations
Fig. 8-1

Note that in this case the footing of the column on property line has to be combined with the footing of internal column because the area for an isolated footing such that c.g. of column loads coinciding with that of footing is not available.

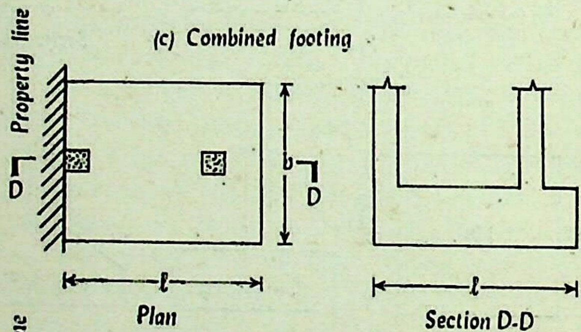
Strap footing: If the combined footing is to be adopted but the distance between the columns is large, the strap footing is used [fig. 8-1(*e*)].

Raft foundation: When the safe bearing capacity of soil is low and columns carry heavy loads, then footings of a group of columns or all the columns in a structure are combined to form a raft foundation. Differential settlement

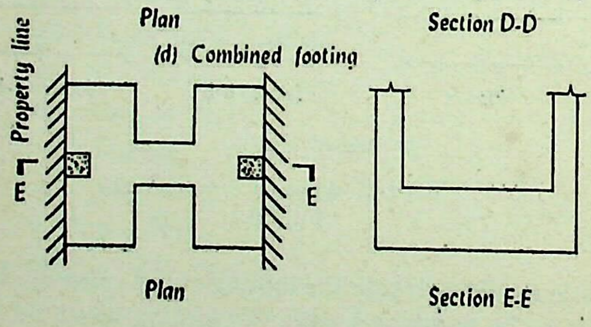
in the structure can be minimised by this kind of footing. This is frequently used for multistoreyed building on poor soil or soil having a higher water-table [fig. 8-1 (f)].



(c) Combined footing



(d) Combined footing

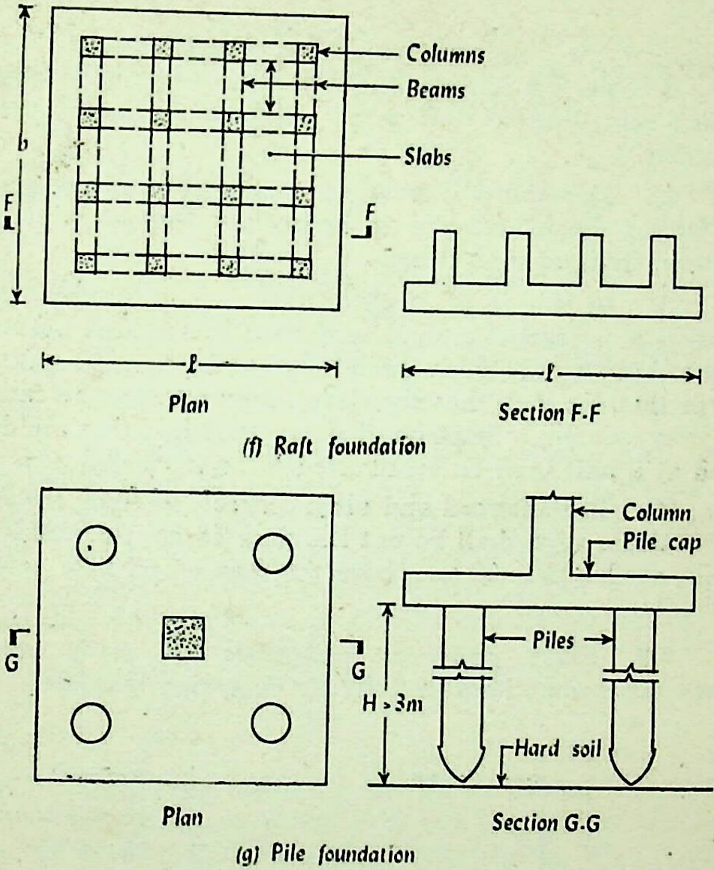


(e) Strap footing

Various types of foundations

Fig. 8-1

Pile foundation: If the good soil is available at a higher depth (more than 3 m) below the ground level, the pile foundations are economical. Piles transfer the loads from columns to the hard soil by bearing and to the surrounding soil by friction [fig. 8-1 (g)].



Various types of foundations
FIG. 8-1

8-2. General design considerations: While designing the R.C.C. footings, the following points shall be considered:

(1) The settlement of footings shall be as nearly uniform as possible and upward soil pressure under footing shall not exceed the safe bearing capacity of soil.

To minimise the differential settlement, the footings are proportioned to get equal soil pressure under each column. This is done by providing footing area very near to the required area considering S.B.C. of soil. If under one column, the exact required area of footing is provided and for another

column in the same structure, a larger area than required is provided, the soil pressure under both the columns is different which may lead to differential settlement. This is usually avoided.

(2) The centre of gravity of loads and centre of gravity of footing should coincide (if they do not coincide, a moment will be induced in footing).

(3) In sloped or stepped footings the effective cross-section in compression shall be limited by the area above the neutral plane, and the angle of slope or depth and location of steps shall be such that the design requirements are satisfied at every section. Sloped and stepped footings that are designed as a unit shall be constructed to assure action as a unit.

(4) In reinforced and plain concrete footings, the thickness at the edge shall be not less than 15 cm for footings on soils, nor less than 30 cm above the tops of piles for footings on piles.

8-3. Plain concrete pedestal: A plain concrete pedestal is sometimes used for the following reasons:

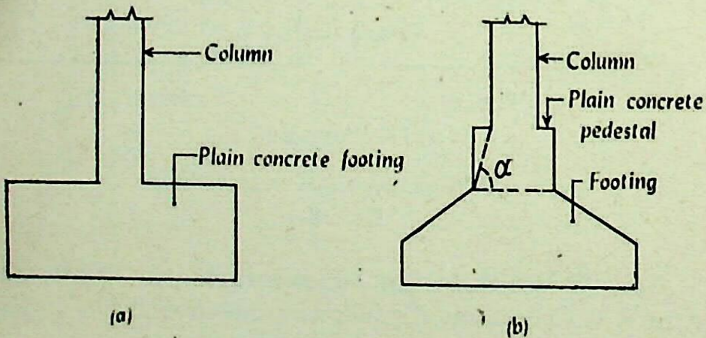


FIG. 8-2

(1) When the load on column is small, concrete pedestal is used as a "plane concrete footing" [fig. 8-2(*a*)].

(2) To reduce the effective cantilevers of footing and thus to obtain an economical design.

According to IS : 456, "In the case of plain concrete pedestals, the angle α between the plane passing through the

bottom edge of the pedestal and the corresponding junction edge of the column with pedestal and the horizontal plane fig. 8-2(*b*) shall be governed by the expression:

$$\tan\alpha \nless 0.9 \sqrt{\frac{100\, q_o}{f_{ck}} + 1}$$

where

q_o = calculated maximum bearing pressure at the base of the pedestal in N/mm², and

f_{ck} = characteristic strength of concrete at 28 days in N/mm²."

8-4. Transfer of load at the base of column: The compressive stress in concrete at the base of a column or pedestal shall be considered as being transferred by bearing to the top of the surrounding pedestal or footing. The bearing pressure on the loaded area shall not exceed the permissible bearing stress in direct compression multiplied by a value equal to $\sqrt{\frac{A_1}{A_2}}$ but not greater than 2

where

A_1 = supporting area for bearing of footing, which in sloped or stepped footing may be taken as the area of the lower base of the largest frustum of a pyramid or cone contained wholly within the footing and having for its upper base, the area actually loaded and having side slope of one vertical to two horizontal; and

A_2 = Loaded area at the column base.

For working stress method of design the permissible bearing stress on full area of concrete shall be taken as $0.25 f_{ck}$.

Let

Sides of column	b_c, l_c
Sides of pedestal	b_p, l_p
Sides of footing	b_f, l_f
Depth of footing	D

(1) When a column rests directly on footing [fig. 8-3(*a*)], $A_1 = b_f l_f$ or $(b_c + 4D)$ $(l_c + 4D)$ whichever is less, and $A_2 = b_c l_c$.

(2) When a pedestal is used [fig. 8-3(*b*)], $A_1 = b_p l_p$ and $A_2 = b_c l_c$.

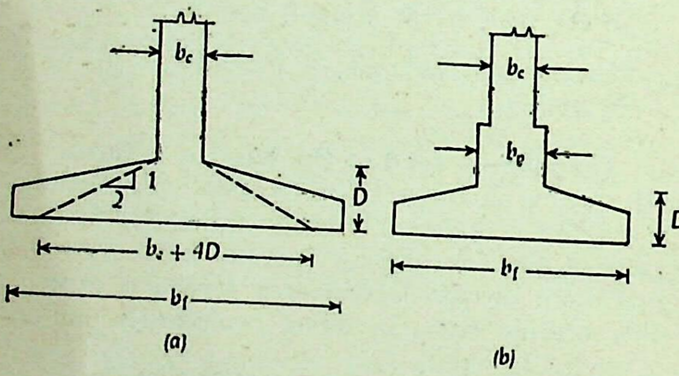


FIG. 8-3

The bearing stress shall not exceed the permissible stress. However, if it exceeds the permissible stress, according to IS : 456,

"Where the permissible bearing stress on the concrete in the supporting or supported member would be exceeded, reinforcement shall be provided for developing the excess force, either by extending the longitudinal bars in the supporting member, or by dowels.

Where the transfer of load is accomplished by reinforcement, the development length of the reinforcement shall be sufficient to transfer the compression or tension to the supporting member.

Extended longitudinal reinforcement or dowels of at least 0·5 per cent of the cross-sectional area of the supported column or pedestal and a minimum of four bars shall be provided. Where dowels are used, their diameter shall not exceed the diameter of column bars by more than 3 mm.

Column bars of diameters larger than 36 mm, in compression only can be dowelled at the footings with bars of smaller size of the necessary area. The dowel shall extend into the column, a distance equal to the development length of the column bar and into the footing, a distance equal to the development length of the dowel".

Even if the bearing stress at the base of the column is less than the permissible value, the column reinforcement is usually extended in the footing. The footing is casted earlier than the column. It is inconvenient to hold the column bars till the column is casted. Therefore the dowels are used to transfer the load from the column to footing.

ISOLATED FOOTINGS

Isolated footing may be a pad footing or a sloped footing. It may be axially loaded or eccentrically loaded. Sometimes a pedestal is used to achieve economical design. The procedure of design for all kinds of footings is similar and explained in detail for the isolated footing. In this book only isolated footings on soils are considered.

8-5. Axially loaded pad footing: An axially loaded pad footing is shown in fig. 8-4. The footings are designed as inverted cantilevers from column loaded with upward soil pressure. The design procedure is explained below:

(1) *Proportioning the size:* Assume the weight of the footing as 10% of the axial load on column. If column load is W, the load on soil is $1 \cdot 1 W$.

$$\text{Area of footing} = \frac{\text{load on soil}}{\text{safe bearing capacity of soil}}$$

From this area, fix the size of the footing. The footing may be square or rectangular. If the column is square, usually the square footing is adopted. If the column is rectangular, a square footing or a rectangular footing can be adopted. When a rectangular footing is adopted, select the size of the footing such that the effective cantilever on all four sides is equal. This will give same bending moment in x and y directions leading to an economical solution. Check the transfer of load at the base of column (assuming depth of footing) in accordance with art. 8-4.

(2) *Bending moment:* This is determined in accordance with clause 33.2.3 of IS : 456 as follows:

The bending moment at any section shall be determined by passing through the section a vertical plane which extends completely across the footing and computing the moment of forces acting over the entire area of footing on one side of the said plane.

The greatest bending moment to be used in the design of an isolated concrete footing which supports a column, pedestal or wall, shall be the moment computed as above at sections located as follows:

(a) At the face of the column, pedestal or wall, for footings supporting a concrete column, pedestal or wall.

(b) Half-way between the centre line and the edge of the wall, for footings under masonry walls.

(c) Half-way between the face of the column or pedestal and the edge of the gussetted base, for footings under gussetted bases.

The critical sections for moment are shown in [fig. 8-4(*a*)].

(3) *Shear:* Two checks for shear force are required.

(a) One-way shear: The sum of the vertical forces due to soil pressure on footing outside the critical section is called one-way shear. According to IS : 456, "The footing acting essentially as a wide beam with a potential diagonal crack extending in a plane across the entire width; the critical section for this condition shall be assumed as a vertical section located from the face of the column, pedestal or wall at a distance equal to the effective depth of footing in case of footings on soils, and a distance equal to half the effective depth of footing for footings on piles". For isolated footing on soils, this is shown in fig. 8-4(*b*). The permissible shear stress for one-way shear depends on the percentage of reinforcement provided as discussed in chapter 3.

(b) Two-way shear: The sum of vertical loads outside the appropriate perimeter as defined by IS : 456 is known as two-way shear. The critical section for shear in this case

is at a distance $\frac{d}{2}$ from the periphery of the column or pedestal where d is the effective depth of footing. In case of sloped footing d may be taken as the effective depth at the face of column or pedestal. This is shown in fig. 8-4(c).

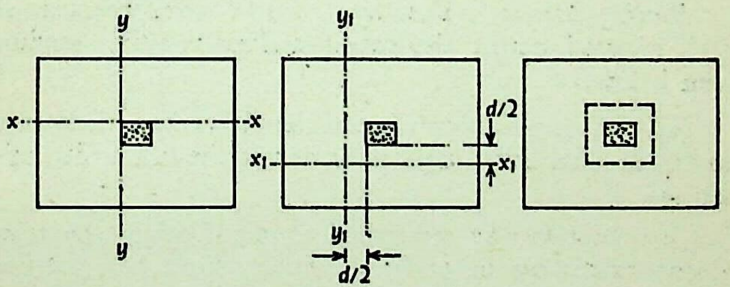
The permissible stress in this case shall be taken equal to $k_s\tau_c$

where

$k_s = (0{\cdot}5 + \beta_c)$ but not greater than 1, β_c being the ratio of short side to long side of the column or pedestal, and

$\tau_c = 0{\cdot}16\sqrt{f_{ck}}$ in working stress method and $0{\cdot}25\ \sqrt{f_{ck}}$ in limit state method of design.

The depth of footing is chosen such that the shear reinforcement is not required. Thus shear check may govern the depth of footing.



(a) Critical sections for moment (b) Critical sections for one-way shear (c) Critical sections for two-way shear

Fig. 8-4

(4) *Development length:* The critical section for checking the development length in footing shall be assumed at the same planes as those described for bending moment [fig. 8-4(a)] and also at all other vertical planes where abrupt changes of section occur. If the reinforcement is curtailed, the anchorage requirements shall be checked in accordance with the curtailment rules as explained for beams in art. 5-7.

(5) *Deflection:* This is not important in footing and may not be checked.

(6) *Cracking:* Footing is described by the code as "*essentially acting as a wide beam*". Hence, cracking of footing shall be checked as per beam design. For the footings, redistribution of moment is not made. Hence, the clear distance between bars shall not exceed

300 mm for the bars of grade Fe 250
180 mm for the bars of grade Fe 415
150 mm for the bars of grade Fe 500.

The above requirements shall be satisfied. However, if the depth of footing exceeds 750 mm, side reinforcement as provided in beams is not necessary.

(7) *Cover:* Clear cover to main reinforcement of the footing bars may be provided as 50 mm.

(8) *Reinforcement requirements:* The following are the general reinforcement requirements for footings:

Bending moment: The total tensile reinforcement shall be distributed across the corresponding resisting section as given below:

(a) In one-way reinforced footing, the reinforcement should be distributed uniformly across the full width of the footing.

(b) In two-way reinforced square footing, the reinforcement extending in each direction shall be distributed uniformly across the full width of footing.

(c) In two-way reinforced rectangular footing, the reinforcement in the long direction shall be distributed uniformly across the full width of footing. For reinforcement in the short direction, a central band equal to the width of footing shall be marked along the length of footing and portion of the reinforcement determined in accordance with the equation given below should be uniformly distributed across the central band:

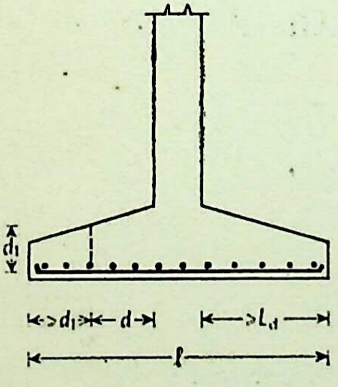
$$\frac{\text{reinforcement in central band width}}{\text{total reinforcement in short direction}} = \frac{2}{\beta + 1}$$

where β is the ratio of the long side to the short side of the footing. The remainder of the reinforcement shall be

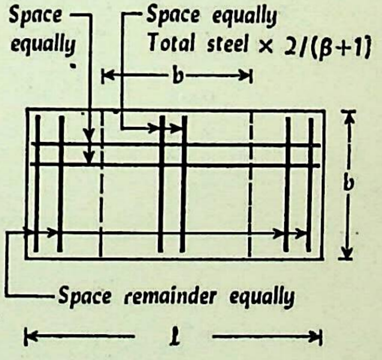
uniformly distributed in the outer portions of the footing. This is illustrated in fig. 8-5(b). As the footing is designed essentially as wide beam, the minimum reinforcement in the footing shall be as discussed for beams.

Shear force: One-way shear check is made at distance d from the face of the column. From this point, the bar must extend upto a minimum distance of d_1 where d_1 is the effective depth of footing at a critical section for checking one-way shear. This is required as the permissible shear stress is based on percentage of reinforcement at a section continuing at least for a distance d_1. This is shown in fig. 8-5(a).

Development length: From the point of maximum bending moment, the bar must extend in both directions for a length equal to its development length. Critical development length is as shown in fig. 8-5(a). This check will control the diameter of the bar.



(a) Section



(b) Plan

FIG. 8-5

(9) *Weight of footing:* After completing the design, self weight of footing is found and shall be comparable with that of assumed one.

Example 8-1.

An R.C.C. column of size 350 mm × 350 mm carries a load of 800 kN. The safe bearing capacity of soil is 200 kN/m². Design an isolated pad footing. The materials are grade M15 concrete and mild steel reinforcement.

Solution :

(a) Size of footing:

Load on column $= 800$ kN

Assume dead load of

footing (10% of column load) $= 80$ kN

Total load on soil 880 kN

Area of footing required $= \dfrac{880}{200} = 4{\cdot}4$ m^2.

Adopt $2{\cdot}1$ m $\times$ $2{\cdot}1$ m footing $= 4{\cdot}41$ m^2.

(b) Transfer of load at the base of column:

$A_1 = 2{\cdot}1 \text{ m} \times 2{\cdot}1 \text{ m} = 4{\cdot}41 \text{ m}^2$

or $(0{\cdot}35 + 4 \times 0{\cdot}5) \times (0{\cdot}35 + 4 \times 0{\cdot}5) = 5{\cdot}52 \text{ m}^2$ whichever is less i.e. $4{\cdot}41$ m^2 (assuming depth of footing to be 500 mm).

$A_2 = 0{\cdot}35 \text{ m} \times 0{\cdot}35 \text{ m} = 0{\cdot}1225 \text{ m}^2$

$$\sqrt{\frac{A_1}{A_2}} = \sqrt{\frac{4{\cdot}41}{0{\cdot}1225}} = 6 > 2.$$

Use $\sqrt{\dfrac{A_1}{A_2}} = 2.$

Permissible bearing pressure

$$= 0{\cdot}25\, f_{ck} \sqrt{\frac{A_1}{A_2}}$$

$$= 0{\cdot}25 \times 15 \times 2 = 7{\cdot}5 \text{ N/mm}^2.$$

Actual bearing pressure

$$= \frac{\text{column load}}{\text{area at top of footing}}$$

$$= \frac{800 \times 10^3}{2{\cdot}1 \times 2{\cdot}1 \times 10^6}$$

$$= 0{\cdot}181 \text{ N/mm}^2 < 7{\cdot}5 \text{ N/mm}^2 \ldots\ldots\ldots \text{(O.K.)}$$

(c) Moment steel:

Net upward pressure $= \dfrac{800}{2{\cdot}1 \times 2{\cdot}1} = 181{\cdot}4$ kN/m^2.

Note that weight of footing does not induce any shear or moment as footing dead load is in opposite direction to the soil pressure. Thus in calculating the net upward pressure, only column load is considered.

Net cantilever [fig. 8-6(*a*)]

$$= \frac{2100 - 350}{2} = 875 \text{ mm}$$

$$M_{xx} = M_{yy} = \frac{0{\cdot}875^2}{2} \times 181{\cdot}4 \times 2{\cdot}1 = 145{\cdot}83 \text{ kNm.}$$

$$\text{Depth required} = \sqrt{\frac{145{\cdot}83 \times 10^6}{2100 \times 0{\cdot}87}} = 282{\cdot}5 \text{ mm.}$$

It will be realised after solving some problems on pad footing without pedestal that the depth is usually governed by check of two-way shear.

Try an overall depth of 500 mm and assuming 16 mm diameter bars

$$d = 500 - 50 \text{ (cover)} - 16 - 8 = 426 \text{ mm (second layer)}$$

$$A_{st} = \frac{145{\cdot}83 \times 10^6}{140 \times 0{\cdot}87 \times 426} = 2811 \text{ mm}^2.$$

Minimum steel

$$= \frac{0{\cdot}34}{100} \times 2100 \times 426 = 3042 \text{ mm}^2.$$

Provide 15 no. 16 mm diameter bars $= 3015$ mm^2.

This is very near to the required minimum area and may be adopted.

Development length $= 58\ \phi$

$$= 58 \times 16 = 928 \text{ mm.}$$

Available anchorage referring fig. 8-6(*d*)

$= 875 - 25$ (cover) $- 3\ \phi$ (bend allowance) $+ 16\ \phi$ (U bend anchorage)

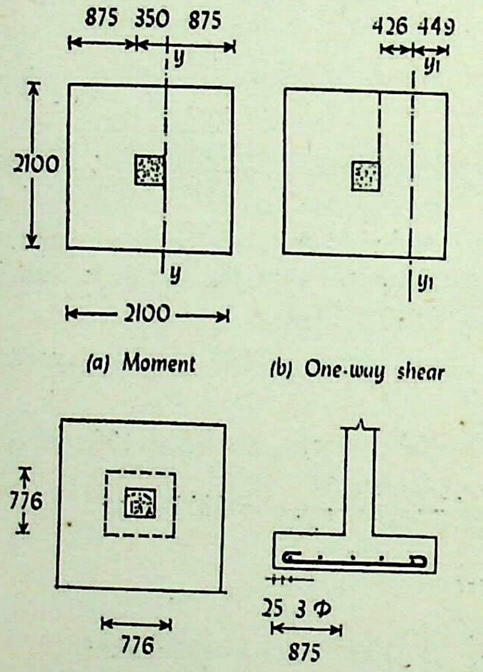
$= 850 + 13 \times 16 = 1058$ mm > 928 mm....(O.K.)

(d) One-way shear:

Shear at 426 mm from face of the column [fig. 8-6(*b*)],

$$= 0.449 \times 2.1 \times 181.4 = 171.04 \text{ kN}$$

$$\frac{100\, A_s}{bd} = \frac{100 \times 3015}{2100 \times 426} = 0.34.$$



(a) Moment (b) One-way shear

(c) Two-way shear (d) Development length

Fig. 8-6

Permissible shear stress by interpolation

$$= 0.22 + \frac{0.29 - 0.22}{0.25}\,(0.34 - 0.25)$$

$$= 0.245 \text{ N/mm}^2.$$

Actual shear stress

$$= \frac{171.04 \times 10^3}{2100 \times 426} = 0.191 \text{ N/mm}^2 \ldots\ldots\ldots \text{(O.K.)}$$

The bars from the point where the shear is checked, are extended (anchored) upto a distance

$$= 1058 - 426 = 632 \text{ mm} > 426 \text{ mm} \ldots \text{(O.K.)}$$

(e) Two-way shear:

$$V = (2{\cdot}1^2 - 0{\cdot}776^2) \times 181{\cdot}4 = 690{\cdot}7 \text{ kN}$$

$$d = 426 \text{ mm}$$

$$b = 4 \times 776 = 3104 \text{ mm [fig. 8-6}(c)]$$

$$\tau_v = \frac{690{\cdot}7 \times 10^3}{3104 \times 426} = 0{\cdot}52 \text{ N/mm}^2.$$

Permissible shear stress

$$= k_s\ \tau_c \qquad \text{where}$$

$$k_s = (0{\cdot}5 + \beta_c) \text{ and } \beta_c = \frac{\text{short side of column}}{\text{long side of column}}$$

$$\beta_c = \frac{1}{1} = 1$$

$$k_s = (0{\cdot}5 + 1) = 1{\cdot}5$$

$$k_s \not> 1$$

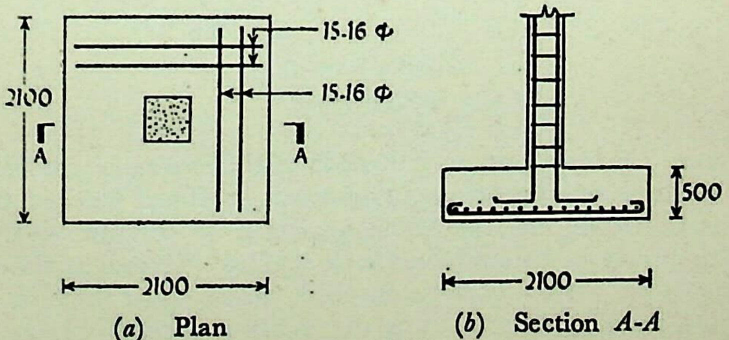
$$\therefore \qquad k_s = 1$$

$$\tau_c = 0{\cdot}16\sqrt{f_{ck}} = 0{\cdot}16\sqrt{15} = 0{\cdot}62.$$

Permissible shear stress

$$= 1 \times 0{\cdot}62 = 0{\cdot}62 \text{ N/mm}^2$$

$$\tau_v < k_s\,\tau_c \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(O.K.)}$$



(a) Plan (b) Section A-A

FIG. 8-7

(f) Cracking:

Clear distance between bars

$$= \frac{2100-50-16}{14} - 16 = 129 \cdot 3 \text{ mm} < 300 \text{ mm (O.K.)}$$

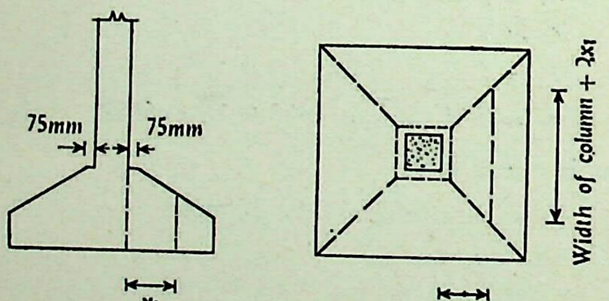
(g) Weight of base:

weight $= 2 \cdot 1^2 \times 0 \cdot 5 \times 25 = 55 \cdot 1$ kN < 80 kN....(O.K.)

(h) Sketch:

Designed footing is shown in fig. 8-7.

8-6. Axially loaded sloped footing: The design method for axially loaded sloped footing is similar to the pad footing. However, for sloped footing where the B.M. is maximum, width of the resisting section is minimum. An axially loaded sloped footing is shown in fig. 8-8. The moment shall be calculated at the face of the column. When footing is casted, a straight width of 75 mm on all four sides of the column is made to facilitate the seating of formwork for



(*a*) Section (*b*) Plan

Sloped footing

Fig. 8-8

column. In pad footing, full width of the footing is available for resisting bending moment whereas, in sloped footing, the resisting width may be taken as width of column + extra width casted for column formwork seating (150 mm as shown in fig. 8-8). When shear is checked, the depth at the critical section shall be found out as the depth in sloped footing is varying. The width of footing resisting shear at a distance x_1

from the face of the column may be taken as width of column $+ 2x_1$ (or the actual width of footing whichever is less) as shown in fig. 8-8(*b*).

In sloped footing a pedestal is sometimes used to have economy in footing design. The pedestal has following advantages.

(1) It reduces effective cantilever of footing and thus reduces the bending moment and shear.

(2) It gives larger width to resist the bending moment.

(3) It gives larger perimeter while checking two-way shear.

Pedestal is casted after casting the footing. Usually the concrete mix used in footing is M15 grade. However, the column may have higher grade of concrete. The concrete mix of pedestal shall be that used in column. To facilitate casting of the pedestal, 75 mm straight length at top of footing is casted. While using the pedestal, width resisting bending moment may be taken as width of pedestal + 150 mm.

Example 8-2.

Design a sloped footing for the column of Ex. 8-1.

Solution:

(a) Size of the footing:

2·1 m × 2·1 m is adopted from Example 8-1.

(b) Transfer of load at the base of column:

$A_2 = 0{\cdot}35 \times 0{\cdot}35 = 0{\cdot}1225 \text{ m}^2$

A_1 = Smaller of (i) $2{\cdot}1 \text{ m} \times 2{\cdot}1 \text{ m} = 4{\cdot}41 \text{ m}^2$

(ii) (width of column + 4 × depth of footing) × (length of column + 4 × depth of footing)

$= (0{\cdot}35 + 4 \times 0{\cdot}6)\ (0{\cdot}35 + 4 \times 0{\cdot}6)$ assuming $d = 0{\cdot}6$ m

$= 7{\cdot}5625 \text{ m}^2.$

$\therefore\ A_1 = 4{\cdot}41 \text{ m}^2$

$$\sqrt{\frac{A_1}{A_2}} = \sqrt{\frac{4{\cdot}41}{0{\cdot}1225}} = 6 > 2 \qquad \therefore \text{ adopt } \sqrt{\frac{A_1}{A_2}} = 2.$$

Permissible bearing pressure

$$= 0{\cdot}25\ f_{ck} \times \sqrt{\frac{A_1}{A_2}}$$

$$= 0{\cdot}25 \times 15 \times 2 = 7{\cdot}5\ \text{N/mm}^2.$$

Actual bearing pressure

$$= \frac{\text{column load}}{\text{area at top of footing}}$$

$$= \frac{800 \times 10^3}{500 \times 500}$$

$$= 3{\cdot}2\ \text{N/mm}^2 < 7{\cdot}5\ \text{N/mm}^2 \ldots\ldots\ldots\ldots\ldots \text{(O.K.)}$$

(c) Moment steel:

$M_{xx} = M_{yy} = 145{\cdot}83$ kNm from Ex. 8-1.

The resisting section has a width

$= 350$ mm $+ 150$ mm $= 500$ mm as discussed in art. 8-5.

$$\text{Depth required} = \sqrt{\frac{145{\cdot}83 \times 10^6}{500 \times 0{\cdot}87}}$$

$$= 579 \text{ mm.}$$

Try an overall depth $= 700$ mm

$d = 700 - 50 - 16 - 8 = 626$ mm (second layer)

$$A_{st} = \frac{145{\cdot}83 \times 10^6}{140 \times 0{\cdot}87 \times 626} = 1913 \text{ mm}^2.$$

Minimum area of steel required

$$= \frac{0{\cdot}34}{100} \times 500 \times 626 = 1064 \text{ mm}^2.$$

Provide 17 nos. 12 mm diameter bars

$= 1921$ mm$^2 > 1913$ mm^2 .... (O.K.)

Development length

$= 58 \times 12 = 696$ mm.

Available anchorage $= 875 - 25$ (cover) $- 3\,\phi$ (bend allowance) $+ 16\,\phi$ (U bend anchorage)

$= 875 - 25 + 13 \times 12$

$= 1006$ mm > 696 mm .... (O.K.)

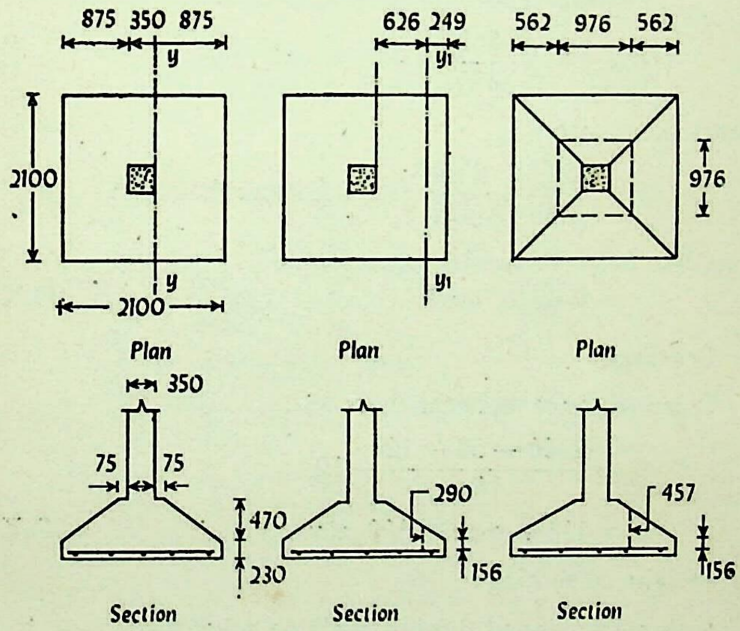
(d) One-way shear:

Assume the depth at the edge of footing = 230 mm. Shear force at 626 mm from face of the column [fig. 8-9(*b*)]

$$= 0{\cdot}249 \times 2{\cdot}1 \times 181{\cdot}4 = 94{\cdot}85 \text{ kN.}$$

$$b = 350 + 2 \times 626 = 1602 \text{ mm}$$

$$d = 156 + \frac{249}{875} \times 470 = 290 \text{ mm.}$$



(*a*) Moment (*b*) One-way shear (*c*) Two-way shear

Fig. 8-9

$$\frac{100\ A_s}{bd} = \frac{100 \times 1921}{1602 \times 290} = 0{\cdot}41$$

$$\tau_c = 0{\cdot}22 + \frac{0{\cdot}29 - 0{\cdot}22}{0{\cdot}25} \times 0{\cdot}16$$

$$= 0{\cdot}265 \text{ N/mm}^2.$$

Actual shear stress

$$\tau_v = \frac{94{\cdot}85 \times 10^3}{1602 \times 290} = 0{\cdot}20 \text{ N/mm}^2 < \tau_c \ldots\ldots \text{(O.K.)}$$

The bar extends $249 - 3\phi + 16\ \phi = 249 + 13 \times 12 = 405$ mm from the point of critical shear which is greater than 290 mm..............................(O.K.)

(e) Two-way shear:

Referring fig. 8-9(*c*)

shear force $= (2{\cdot}1^2 - 0{\cdot}976^2) \times 181{\cdot}4 = 627{\cdot}18$ kN

$$b = 4 \times 976 = 3904 \text{ mm}$$

$$d = 156 + \frac{562}{875} \times 470 = 457 \text{ mm.}$$

Actual shear stress

$$\tau_v = \frac{627{\cdot}18 \times 10^3}{3904 \times 457} = 0{\cdot}35 \text{ N/mm}^2.$$

From Ex. 8-1, permissible shear stress

$= 0{\cdot}62$ N/mm².........................(O.K.)

(f) Cracking:

Clear distance between bars

$$= \frac{2100 - 50 - 12}{16} - 12$$

$= 115{\cdot}4$ mm $<$ 300 mm................(O.K.)

(g) Weight of footing:

Volume of sloped footing shall be taken as:

Volume of lower prism + volume of upper frustum which shall be taken as

$$\frac{h}{6}(A_1 + A_2 + 4A_m)$$

where

A_1 = area of top of frustum

A_2 = area of bottom of frustum

A_m = area at mid-height of frustum

h = height of frustum.

In this case

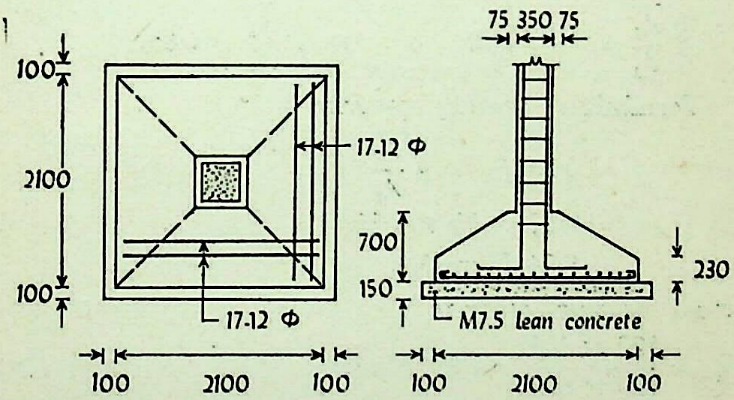
$$V = 2{\cdot}1 \times 2{\cdot}1 \times 0{\cdot}23 + \frac{0{\cdot}47}{6}\,(0{\cdot}35^2 + 2{\cdot}1^2$$
$$+ 4 \times 1{\cdot}225^2)$$
$$= 1{\cdot}01 + 0{\cdot}83 = 1{\cdot}84 \text{ m}^3.$$

Weight of footing $= 25 \times 1{\cdot}84 = 46$ kN $<$ 80 kN....(O.K.)

Note: Redesign Ex. 8-1 and Ex. 8-2 critically by reducing the depth and observe that sloped footing gives an economical solution.

(h) Sketch:

The designed footing is shown in fig. 8-10.



(*a*) Plan (*b*) Section

FIG. 8-10

(i) Supplementary details:

Under the R.C.C. footing usually a lean concrete is laid first and then the footing is casted. This is done to prevent direct contact of footing with soil and to act as levelling coarse. The lean concrete is projected beyond the footing and this projection varies from 0 to 150 mm. This is particularly useful for the sloped footing to have a space for formwork for the depth at end of the slope. The thickness of such lean concrete should be not less than the projection beyond the footing. For the footing designed in this problem lean concrete of grade M7·5 is used and shown in fig. 8-10.

Example 8-3.

In Ex. 8-2, a concrete pedestal is now used to transfer the load from column to foundation. Design the footing.

Solution:

(a) Size of the base:

2·1 m × 2·1 m is adopted from Example 8-2.

(b) Transfer of load at the base of column:

A pedestal of 650 mm × 650 mm is used.

$$A_1 = 0{\cdot}65 \times 0{\cdot}65 = 0{\cdot}4225 \text{ m}^2$$

$$A_2 = 0{\cdot}35 \times 0{\cdot}35 = 0{\cdot}1225 \text{ m}^2$$

$$\sqrt{\frac{A_1}{A_2}} = 1{\cdot}85 < 2 \qquad \therefore \text{ use } \sqrt{\frac{A_1}{A_2}} = 1{\cdot}85.$$

Permissible bearing pressure

$$= 0{\cdot}25\, f_{ck} \times \sqrt{\frac{A_1}{A_2}}$$

$$= 0{\cdot}25 \times 15 \times 1{\cdot}85$$

$$= 6{\cdot}94 \text{ N/mm}^2.$$

Actual bearing pressure

$$q_o = \frac{800 \times 10^3}{650 \times 650} = 1{\cdot}89 \text{ N/mm}^2 \ldots\ldots\ldots\ldots\ldots\ldots \text{(O.K.)}$$

Now for a pedestal referring fig. 8-2(*b*),

$$\tan \alpha \nless 0{\cdot}9 \sqrt{\frac{100 \times 1{\cdot}89}{15} + 1}$$

which gives

$$\tan \alpha \nless 3{\cdot}32 \qquad \text{i.e. } \alpha \geqslant 73{\cdot}2°.$$

Projection of pedestal = 150 mm.

Depth of pedestal = 0·15 × 3·32 = 0·5 m.

Use 600 mm deep pedestal.

(c) Moment steel:

$$\text{Cantilever} = \frac{2100 - 650}{2} = 725 \text{ mm.}$$

$$M_{xx} = M_{yy} = \frac{0{\cdot}725^2}{2} \times 2{\cdot}1 \times 181{\cdot}4 = 100{\cdot}12 \text{ kNm.}$$

The resisting section has a width

$$= 650 + 150 = 800 \text{ mm.}$$

$$\text{Depth required} = \sqrt{\frac{100{\cdot}12 \times 10^6}{0{\cdot}87 \times 800}}$$

$$= 379 \text{ mm.}$$

Try an overall depth = 500 mm

$d = 500 - 50 - 16 - 8 = 426$ mm (second layer)

$$A_{st} = \frac{100{\cdot}12 \times 10^6}{140 \times 0{\cdot}87 \times 426} = 1930 \text{ mm}^2.$$

Minimum area of steel required

$$= \frac{0{\cdot}34}{100} \times 800 \times 426 = 1159 \text{ mm}^2.$$

Provide 18 no. 12 mm dia. bars = 2034 mm²

development length = $58 \times 12 = 696$ mm²

available anchorage $= 725 - 25 - 3\,\phi + 16\,\phi$
$= 700 + 13 \times 12$
$= 856 \text{ mm} > 696$ mm......(O.K.)

(d) One-way shear:

Referring fig. 8-11(*b*)

shear at 426 mm from face of the pedestal

$$= 0{\cdot}299 \times 2{\cdot}1 \times 181{\cdot}4 = 113{\cdot}9 \text{ kN}$$

$$b = 650 + 2 \times 426 = 1502 \text{ mm}$$

$$d = 156 + \frac{299}{725} \times 270 = 267 \text{ mm}$$

$$\frac{100\,A_s}{bd} = \frac{100 \times 2034}{1502 \times 267} = 0{\cdot}5$$

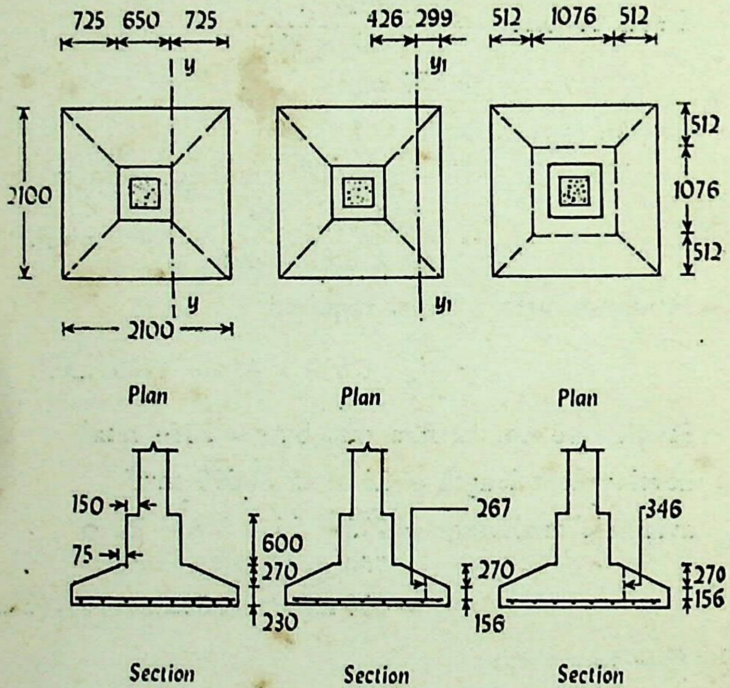
$$\tau_c = 0{\cdot}29 \text{ N/mm}^2.$$

Actual shear stress

$$\tau_v = \frac{113{\cdot}9 \times 10^3}{1502 \times 267} = 0{\cdot}28 \text{ N/mm}^2$$

$\tau_v < \tau_c$.............................. (O.K.)

The bar extends, $299 - 3\phi + 16\phi = 299 + 13 \times 12 =$ 455 mm from the point of critical shear which is greater than 267 mm.......................................(O.K.)



(*a*) Moment (*b*) One-way shear (*c*) Two-way shear

FIG. 8-11

(c) Two-way shear:

Referring fig. 8-11(*c*),

shear force $= (2{\cdot}1^2 - 1{\cdot}076^2) \times 181{\cdot}4 = 590$ kN.

$b = 4 \times 1076 = 4304$ mm

$$d = 156 + \frac{512}{725} \times 270 = 346 \text{ mm.}$$

Actual shear stress

$$\tau_v = \frac{590 \times 10^3}{4304 \times 346} = 0{\cdot}4 \text{ N/mm}^2.$$

Permissible shear stress $= 0{\cdot}62$ N/mm².........(O.K.)

(f) Cracking:

Clear distance between bars

$$= \frac{2100 - 50 - 12}{17} - 12 = 107 \cdot 8 \text{ mm} < 300 \text{ mm} \ldots \text{(O.K.)}$$

(g) Weight:

$$\text{Weight of upper prism} = \frac{0 \cdot 27}{6}(0 \cdot 8^2 + 2 \cdot 1^2 + 4 \times 1 \cdot 45^2) \times 25$$
$$= 15 \cdot 14 \text{ kN.}$$

$$\text{Weight of lower prism} = 2 \cdot 1 \times 2 \cdot 1 \times 0 \cdot 23 \times 25$$
$$= 25 \cdot 36 \text{ kN.}$$

$$\text{Total weight} = 15 \cdot 14 + 25 \cdot 36$$
$$= 40 \cdot 50 \text{ kN} < 80 \text{ kN} \ldots \text{(O.K.)}$$

Example 8-4.

Design a rectangular isolated sloped footing for a column of size 250 mm × 750 mm carrying an axial load of 1700 kN. The safe bearing capacity of soil is 200 kN/m². The materials are grade M15 concrete and tor steel reinforcement of grade Fe 415.

Solution:

(a) Size of footing:

Column load	= 1700 kN
Assume footing load	= 170 kN
Load on soil	1870 kN

$$\text{Area of footing required} = \frac{1870}{200} = 9 \cdot 35 \text{ m}^2.$$

As the footing is rectangular, size of footing may be selected such that the effective cantilevers on both sides are equal.

If b is the width of footing

$$b\ (b + 0 \cdot 5) = 9 \cdot 35$$

i.e. $$b^2 + 0 \cdot 5b = 9 \cdot 35$$

which on solving gives

$$x = 2 \cdot 82 \text{ m.}$$

Provide 2·85 m × 3·35 m footing

$$A = 9 \cdot 55 \text{ m}^2 \ldots \ldots \ldots \text{(O.K.)}$$

(b) Transfer of load at the base of column:

This is not usually critical and also the column bars are carried in footing, therefore this need not be checked.

(c) Moment steel:

Net upward pressure $= \dfrac{1700}{2{\cdot}85 \times 3{\cdot}35} = 178$ kN/m².

Net cantilever on xx or $yy = 1{\cdot}3$ m.

$$M_{xx} = \frac{1{\cdot}3^2}{2} \times 2{\cdot}85 \times 178 = 428{\cdot}67 \text{ kNm.}$$

The resisting section has a width
$= 250 + 150 = 400$ mm.

$$\text{Depth required} = \sqrt{\frac{428{\cdot}67 \times 10^6}{0{\cdot}65 \times 400}} = 1284 \text{ mm.}$$

$$M_{yy} = \frac{1{\cdot}3^2}{2} \times 3{\cdot}35 \times 178 = 503{\cdot}9 \text{ kNm.}$$

The resisting section has a width
$= 750 + 150 = 900$ mm.

$$\text{Depth required} = \sqrt{\frac{503{\cdot}9 \times 10^6}{0{\cdot}65 \times 900}} = 928 \text{ mm.}$$

Try an overall depth $= 1350$ mm

$d_{xx} = 1350 - 50 - 8 = 1292$ mm

$d_{yy} = 1292 - 16 = 1276$ mm assuming 16 mm dia. bars.

Adopt a depth of 300 mm at the edge of footing.

For M_{xx}

$$A_{st} = \frac{428{\cdot}67 \times 10^6}{230 \times 0{\cdot}9 \times 1292} = 1603 \text{ mm}^2.$$

$$\text{Minimum steel} = \frac{0{\cdot}205}{100} \times 400 \times 1292 = 1060 \text{ mm}^2.$$

Provide 15 no. 12 mm Φ bars = 1695 mm².

Clear distance between bars

$$= \frac{2850 - 50 - 12}{14} - 12 = 187 \text{ mm} > 180 \text{ mm.}$$

Therefore provide 16 no. 12 mm Φ bars.

Clear distance between bars

$$= \frac{2850 - 50 - 12}{15} - 12 = 173{\cdot}9 \text{ mm} < 180 \text{ mm}$$

................(O.K.)

For M_{yy}

$$A_{st} = \frac{503{\cdot}9 \times 10^6}{230 \times 0{\cdot}9 \times 1276} = 1908 \text{ mm}^2.$$

$$\text{Minimum steel} = \frac{0{\cdot}205}{100} \times 900 \times 1276 = 2354 \text{ mm}^2.$$

The reinforcement parallel to longer direction (for M_{xx}) are spaced equally. For the reinforcement parallel to shorter direction

$$\beta = \frac{3{\cdot}35}{2{\cdot}85} = 1{\cdot}175$$

$$\frac{2}{\beta + 1} = \frac{2}{2{\cdot}175} = 0{\cdot}92.$$

Reinforcement in central band of 2·85 m width

$$= 0{\cdot}92 \times 2354 = 2166 \text{ mm}^2.$$

Remaining steel 2354 − 2166 = 198 mm² shall be distributed in end band of 3·35 − 2·85 = 0·5 m width.

If exact calculations are followed, a designer can provide the steel according to above calculations. This has a meaning only if longer side is very long than the shorter side giving reasonable end band reinforcement. In this problem the spacing of bars for central band is found out and the same spacing is adopted in end bands also.

For a central band, steel required per metre

$$= \frac{2166}{2{\cdot}85} = 760 \text{ mm}^2$$

i.e. 12 mm Φ about 148 mm c/c.

For total length of footing, no. of 12 mm diameter bars required

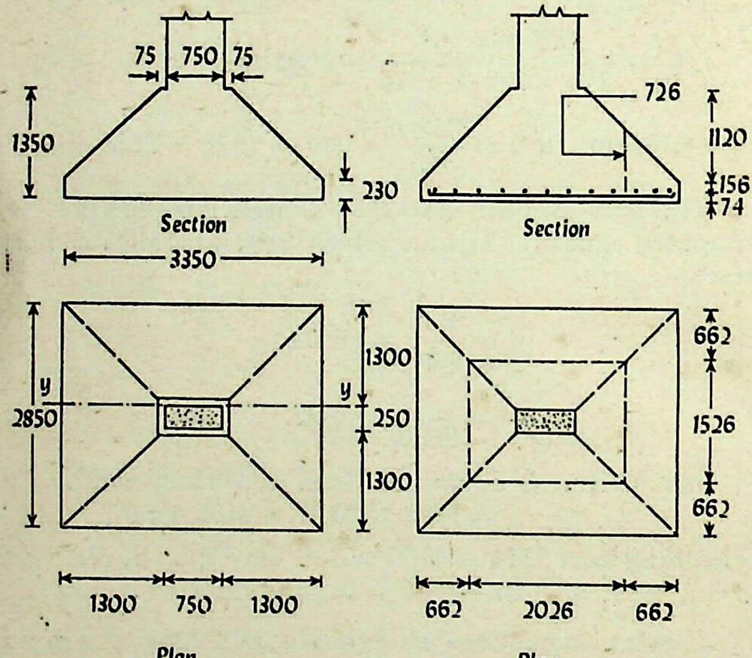
$$= \frac{3350}{148} = 22{\cdot}6.$$

Provide 23 no. 12 mm Φ equally spaced bars.

Clear distance between bars

$$= \frac{3350 - 50 - 12}{22} - 12$$

$$= 137{\cdot}45 \text{ mm} < 180 \text{ mm} \ldots\ldots\ldots (\text{O.K.})$$



(*a*) Moment (*b*) Two-way shear

Fig. 8-12

(d) Development length:

The tor steel bars are used without end anchorage.

Development length

$= 69 \times 12 = 828$ mm.

Anchorage available

$= 1300 - 25$ (cover) $= 1275$ mm..........(O.K.)

(e) One-way shear:

One-way shear is checked at a distance of effective depth from the face of the column. In this case, the effective

cantilever and effective depth are almost same and there is no necessity of checking one-way shear.

(f) Two-way shear:

Two-way shear is checked at distance $\frac{d}{2}$ from the face of column. The properties of section at $\frac{d}{2}$ are shown in fig. 8-12(*b*).

$$\text{S.F.} = (3{\cdot}35 \times 2{\cdot}85 - 1{\cdot}526 \times 2{\cdot}026) \times 178$$
$$= 1149{\cdot}2 \text{ kN.}$$
$$b = 2\ (1526 + 2026) = 7104 \text{ mm.}$$
$$d = 156 + 1120 \times \frac{662}{1300} = 726 \text{ mm.}$$

Actual shear stress

$$\tau_v = \frac{1149{\cdot}2 \times 10^3}{7104 \times 726} = 0{\cdot}223 \text{ N/mm}^2.$$

Permissible shear stress

$$= k_s\ \tau_c \qquad \text{where}$$

$$k_s = (0{\cdot}5 + \beta_c) \qquad \text{and } \beta_c = \frac{\text{short side of column}}{\text{long side of column}}$$

and also $k_s \ngtr 1$

$$\tau_c = 0{\cdot}16\ \sqrt{f_{ck}}$$
$$\beta_c = \frac{250}{750} = 0{\cdot}33$$
$$k_s = 0{\cdot}5 + 0{\cdot}33 = 0{\cdot}83 < 1$$
$$\tau_c = 0{\cdot}16\ \sqrt{15} = 0{\cdot}62 \text{ N/mm}^2.$$

Permissible shear stress

$$= 0{\cdot}83 \times 0{\cdot}62 = 0{\cdot}51 \text{ N/mm}^2$$

$\tau_v < k_s\ \tau_c$ ....(O.K.)

(g) Cracking:

This was incorporated in calculations of moment steel.

(h) Weight of footing:

Weight of upper prism

$$= \frac{1 \cdot 12}{6} (0 \cdot 4 \times 0 \cdot 9 + 2 \cdot 85 \times 3 \cdot 35 + 4 \times 1 \cdot 625 \times 2 \cdot 125) \times 25$$

$$= 110 \cdot 7 \text{ kN.}$$

Weight of lower prism

$$= 2 \cdot 85 \times 3 \cdot 35 \times 0 \cdot 23 \times 25 = 54 \cdot 9 \text{ kN.}$$

Total weight $= 110 \cdot 7 + 54 \cdot 9 = 165 \cdot 6$ kN.

Assumed weight $= 170$ kN.................... (O.K.)

8-7. Eccentrically loaded isolated footing: A column may transfer to the footing, an axial load P and bending moment M. Alternatively, the footing is subjected to an eccentric load P with an eccentricity $e = \frac{M}{P}$. In such cases, there can be two alternatives for design of footing:

(1) The footing is made eccentric from the column at an eccentricity of $e = \frac{M}{P}$. In such a case, the soil pressure distribution under the footing will be uniform. This is illustrated in fig. 8-13(*a*).

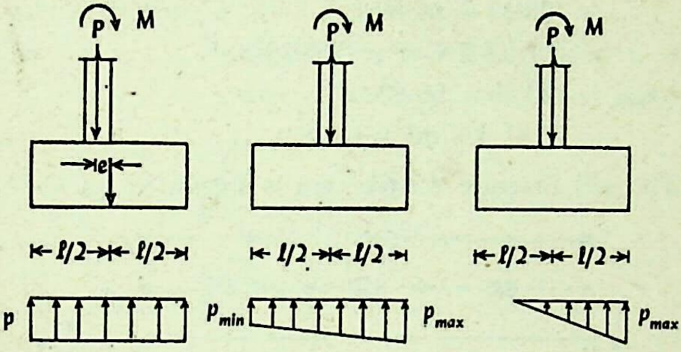
(2) The footing is designed for an axial load and moment. In such a case, the soil pressure distribution under the footing will be varying. This is illustrated in fig. 8-13(*b*) and (*c*). The maximum and minimum pressures under the footing are given by

$$p_{max} = \frac{P}{A} + \frac{M}{Z}$$

$$p_{min} = \frac{P}{A} - \frac{M}{Z}$$

p_{max} should not exceed the soil bearing capacity. If $e < \frac{l}{6}$, the whole footing is subjected to pressure as shown in fig. 8-13(*b*). However, if $e > \frac{l}{6}$, a part of the footing is under tension causing overturning of the footing. This is indicated in fig. 8-13(*c*). Here the footing loses its contact

with soil. Such a condition is not allowed and the footing shall be redesigned.



(*a*) Moment is eliminated (*b*) $e < \frac{l}{6}$ (*c*) $e < \frac{l}{6}$

FIG. 8-13

Example 8-5.

A 230 mm × 530 mm column carries a load of 600 kN and a moment of 100 kNm about major axis. The safe bearing capacity of soil is 200 kN/m². Design an isolated rectangular sloped footing without pedestal. The materials are grade M15 concrete and tor steel reinforcement of grade Fe 415.

Solution:

(a) Size of footing:

Column load	= 600 kN
Self wt. of footing	= 60 kN
Total load on soil	660 kN.

$$\text{Area of footing required} = \frac{660}{200} = 3{\cdot}3 \text{ m}^2.$$

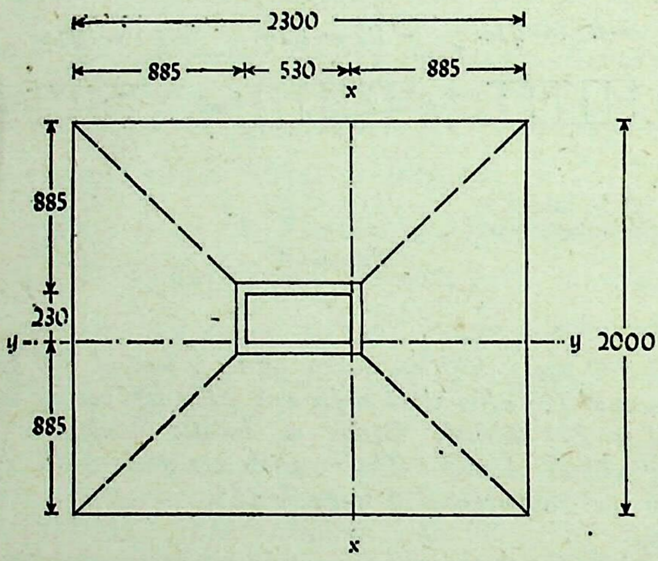
Try $2{\cdot}0$ m × $2{\cdot}3$ m size of footing to have equal cantilevers on both axis.

$A = 4{\cdot}6$ m² (larger area is assumed to accommodate for soil pressure due to bending moment).

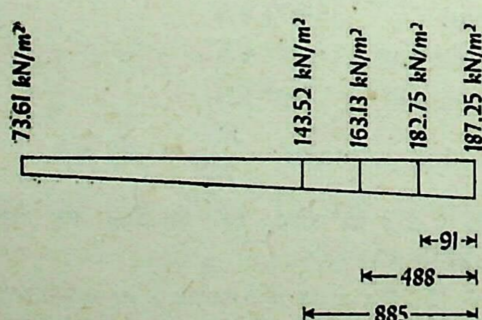
$$Z = \frac{1}{6}\,bd^2 = \frac{1}{6} \times 2{\cdot}0 \times 2{\cdot}3^2 = 1{\cdot}76 \text{ m}^3.$$

$$p_{max} = \frac{600}{2 \times 2{\cdot}3} + \frac{100}{1{\cdot}76}$$
$$= 130{\cdot}43 + 56{\cdot}82$$
$$= 187{\cdot}25 \text{ kN/m}^2 < 200 \text{ kN/m}^2 \ldots\ldots\ldots\ldots \text{(O.K.)}$$
$$p_{min} = 130{\cdot}43 - 56{\cdot}82$$
$$= 73{\cdot}61 \text{ kN/m}^2.$$

The soil pressure distribution is shown in fig. 8-14.



(a) Critical sections for moment



(b) Soil pressure distribution

FIG. 8-14

(b) Transfer of load at the base of column:

$A_2 = 0{\cdot}23 \times 0{\cdot}53 = 0{\cdot}1229 \text{ m}^2$

$A_1 =$ smaller of (i) $2 \times 2{\cdot}3 = 4{\cdot}6 \text{ m}^2$

(ii) $(0{\cdot}23 + 4 \times 0{\cdot}8)\,(0{\cdot}53 + 4 \times 0{\cdot}8)$

$= 12{\cdot}8 \text{ m}^2$ assuming $d = 0{\cdot}8$ m

$$\therefore \quad A_1 = 4{\cdot}6 \text{ m}^2$$

$$\sqrt{\frac{A_1}{A_2}} = \sqrt{\frac{4{\cdot}6}{0{\cdot}1229}} = 6{\cdot}14 > 2.$$

Adopt $\sqrt{\dfrac{A_1}{A_2}} = 2.$

Permissible bearing pressure

$$= 0{\cdot}25\, f_{ck} \sqrt{\frac{A_1}{A_2}}$$

$$= 0{\cdot}25 \times 15 \times 2.$$

$$= 7{\cdot}5 \text{ N/mm}^2.$$

Size of footing at top

$$= (230 + 150) \times (530 + 150)$$

$$= 380 \text{ mm} \times 680 \text{ mm}.$$

Bearing pressure at the top of footing

$$= \frac{600 \times 10^3}{380 \times 680} + \frac{6 \times 100 \times 10^6}{380 \times 680^2}$$

$$= 2{\cdot}32 + 3{\cdot}41$$

$$= 5{\cdot}73 \text{ N/mm}^2 < 7{\cdot}5 \text{ N/mm}^2$$

......................(O.K.)

(c) Moment steel:

$$M_{xx} = \frac{0{\cdot}885^2}{2} \times 2 \times 143{\cdot}52 + 43{\cdot}73 \times \frac{0{\cdot}885}{2} \times \frac{2}{3} \times 0{\cdot}885$$

$$= 112{\cdot}4 + 11{\cdot}42$$

$$= 123{\cdot}82 \text{ kNm}.$$

Width of resisting section

$$= 230 + 150$$

$$= 380 \text{ mm}.$$

Depth required for flexure

$$= \sqrt{\frac{123 \cdot 82 \times 10^6}{0 \cdot 65 \times 380}}$$

$= 780$ mm.

For the moment about yy axis

$$\text{average soil pressure} = \frac{73 \cdot 61 + 187 \cdot 25}{2}$$

$= 130 \cdot 43 \text{ kN/m}^2.$

$$M_{yy} = \frac{0 \cdot 885^2}{2} \times 2 \cdot 3 \times 130 \cdot 43$$

$= 117 \cdot 48$ kNm.

Width of resisting section

$= 530 + 150$

$= 680$ mm.

Depth required for flexure

$$= \sqrt{\frac{117 \cdot 48 \times 10^6}{0 \cdot 65 \times 680}}$$

$= 515 \cdot 6$ mm

Try an overall depth $= 850$ mm

$d_{xx} = 850 - 50 - 6 \quad = 794$ mm

$d_{yy} = 794 - 12 \quad = 782$ mm

For M_{xx}

$$A_{st} = \frac{123 \cdot 82 \times 10^6}{230 \times 0 \cdot 9 \times 794} = 753 \text{ mm}^2.$$

Minimum steel

$$= \frac{0 \cdot 204}{100} \times 380 \times 794 = 616 \text{ mm}^2.$$

Provide 12 no. 10 mm Φ bars $= 942 \text{ mm}^2$.

(Larger area is provided to satisfy the cracking requirement.)

Clear distance between bars

$$= \frac{2000 - 50 - 10}{11} - 10$$

$= 166 \cdot 4 \text{ mm} < 180$ mm. . . . . (O.K.)

For M_{yy}

$$A_{st} = \frac{117 \cdot 48 \times 10^6}{230 \times 0 \cdot 9 \times 782}$$
$$= 726 \text{ mm}^2.$$

Minimum steel

$$= \frac{0 \cdot 204}{100} \times 580 \times 782$$
$$= 926 \text{ mm}^2$$
$$\beta = \frac{2 \cdot 3}{2 \cdot 0} = 1 \cdot 15$$
$$\frac{2}{\beta + 1} = \frac{2}{1 \cdot 15 + 1} = 0 \cdot 93.$$

Reinforcement in central band of 2 m width

$$= 0 \cdot 93 \times 926$$
$$= 861 \cdot 2 \text{ mm}^2.$$

Steel required per metre

$$= \frac{861 \cdot 2}{2 \cdot 0} = 430 \cdot 6 \text{ mm}^2$$

i.e. 10 mm Φ about 182 mm c/c.

For a total length of footing, no. of 10 mm diameter bars required

$$= \frac{2300}{182} = 13 \text{ (say)}.$$

Provide 13 no. 10 mm Φ equally spaced bars.

Clear distance between bars

$$= \frac{2300 - 50 - 10}{12} - 10$$
$$= 176 \cdot 7 \text{ mm} < 180 \text{ mm} \ldots\ldots\ldots\ldots\ldots \text{(O.K.)}$$

(d) Development length:

For 10 mm diameter bars, anchorage required

$$= 69 \times 10 = 690 \text{ mm}.$$

Anchorage available

$$= 885 - 25 = 860 \text{ mm} \ldots\ldots\ldots\ldots\ldots\ldots \text{(O.K.)}$$

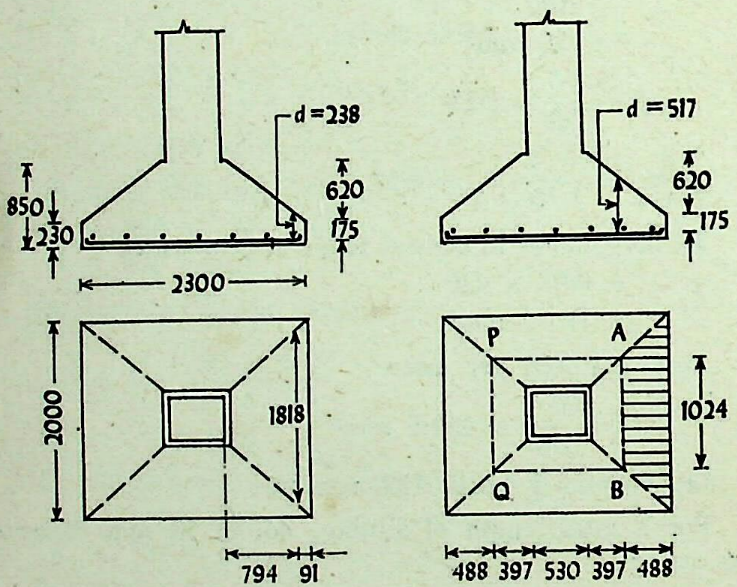
(e) One-way shear:

XX: Shear force at 794 mm from face of support

$$= 0{\cdot}091 \times 2 \times \frac{182{\cdot}75 + 187{\cdot}25}{2} = 33{\cdot}67 \text{ kN.}$$

$$b = 230 + 2 \times 794 \quad = 1818 \text{ mm}$$

$$d = 175 + 620 \times \frac{91}{885} = 238 \text{ mm.}$$



(*a*) One-way shear (*b*) Two-way shear

FIG. 8-15

$$\frac{100\, A_s}{bd} = \frac{100 \times 942}{1818 \times 238} = 0{\cdot}218$$

$$\tau_c = 0{\cdot}2 + \frac{0{\cdot}22 - 0{\cdot}2}{0{\cdot}05} \times (0{\cdot}218 - 0{\cdot}2)$$

$$= 0{\cdot}2072 \text{ N/mm}^2.$$

Actual shear stress

$$= \frac{33{\cdot}67 \times 10^3}{1818 \times 238}$$

$= 0{\cdot}078$ N/mm^2........................ (O.K.)

YY: One-way shear need not be checked.

(f) Two-way shear:

The soil pressure distribution below the footing is varying, hence, the two-way shear stress on perimeter lines AB, BQ, QP and PA as shown in fig. 8-15(b) will be different. The maximum two-way shear will be along the line AB. Two way shear, therefore, may be checked along line AB,

$$\beta_c = \frac{\text{short side of column}}{\text{long side of column}}$$

$$= \frac{2}{2\cdot3} = 0\cdot87$$

$$k_s = (0\cdot5 + B_c) = 0\cdot5 + 0\cdot87$$

$$= 1\cdot37 \text{ but not greater than } 1$$

$$\therefore \quad k_s = 1$$

$$\tau_c = 0\cdot16\ \sqrt{f_{ck}} = 0\cdot16\ \sqrt{15}$$

$$= 0\cdot62\ \text{N/mm}^2.$$

Permissible shear stress

$$= 1\cdot0 \times 0\cdot62 = 0\cdot62\ \text{N/mm}^2.$$

Two-way shear along line AB

$$= \frac{(1\cdot024 + 2\cdot0)}{2} \times 0\cdot488 \times \frac{163\cdot13 + 187\cdot25}{2}$$

$$= 129\cdot26\ \text{kN}.$$

$$b = 1024\ \text{mm}$$

$$d = 517\ \text{mm}$$

$$\tau_v = \frac{129\cdot26 \times 10^6}{1024 \times 517}$$

$$= 0\cdot244\ \text{N/mm}^2 < 0\cdot62\ \text{N/mm}^2 \ldots\ldots\ldots \text{(O.K.)}$$

(g) Cracking:

This was incorporated in calculations of moment steel.

(h) Weight of footing:

Weight of upper prism

$$= \frac{0\cdot62}{6}\,[0\cdot38 \times 0\cdot68 + 2\cdot0 \times 2\cdot3 + 4 \times 1\cdot19 \times 1\cdot49] \times 25$$

$$= 30\cdot87\ \text{kN}.$$

Weight of lower prism

$= 2 \times 2{\cdot}3 \times 0{\cdot}23 \times 25$

$= 26{\cdot}45$ kN.

Total weight

$= 30{\cdot}87 + 26{\cdot}45$

$= 57{\cdot}32$ kN $<$ 60 kN....................(O.K.)

EXAMPLES VIII

(1) An R.C.C. column of size 300 mm × 300 mm carries a load of 660 kN. The safe bearing capacity of soil is 180 kN/m². Design an isolated pad footing. The materials are

(a) M15 grade concrete and mild steel

(b) M15 grade concrete and tor steel of grade Fe 415.

(2) Design an isolated sloped footing for data given in Example (1)

(a) Without pedestal

(b) Using pedestal.

(3) An R.C.C. column of size 230 mm × 530 mm carries a load of 1200 kN. The safe bearing capacity of soil is 200 kN/m². Design an isolated sloped footing:

(a) Without pedestal

(b) Using pedestal.

The materials are M15 grade concrete and tor steel reinforcement of grade Fe 415.

(4) A 250 mm × 750 mm column carries a load of 1200 kN and a moment of 250 kNm about major axis. The safe bearing capacity of soil is 180 kN/m². Design an isolated rectangular sloped footing:

(a) Without pedestal

(b) Using pedestal.

The materials are M15 grade concrete and tor steel reinforcement of grade Fe 415.

9

Limit State Method

9-1. Limit state method: In the method of design based on limit state concept, the structure shall be designed to withstand safely all loads liable to act on it throughout its life. It shall also satisfy the serviceability requirements, such as limitations on deflection and cracking. The acceptable limit for the safety and serviceability requirements before failure occurs is called a *Limit State.* The aim of design is to achieve acceptable probabilities that the structure will not become unfit for the use for which it is intended, that is, it will not reach a limit state.

All relevant limit states shall be considered in design to ensure an adequate degree of safety and serviceability. In general, the structure shall be designed on the basis of the most critical limit state and shall be checked for other limit states.

For ensuring the above objective, the design should be based on characteristic values for material strengths and applied loads, which take into account the variations in the material strengths and in the loads to be supported. The characteristic values should be based on statistical data if available; where such data are not available they should be based on the experience. The 'design values' are derived from the characteristic values through the use of partial safety factors, one for material strength and the other for loads. In the absence of special considerations these factors should have the values given in art. 9-4 according to the material, the type of loading and the limit state being considered.

9-2. Limit state of collapse: The limit state of collapse of the structure or part of the structure could be assessed from rupture of one or more critical sections and from buckling due to elastic or plastic instability (including the

effects of sway where appropriate) or overturning. The resistance to bending, shear, torsion and axial loads at every section shall not be less than the appropriate value at that section produced by the probable most unfavourable combination of loads on the structure using the appropriate partial safety factors.

9-3. Limit state of serviceability: To satisfy the limit state of serviceability the deflection and cracking in a structure shall not be excessive. This limit state corresponds to deflection and cracking.

(a) *Deflection:* The deflection of a structure or part thereof shall not adversely affect the appearance or efficiency of the structure or finishes or partitions. This has been discussed in detail in chapter 4.

(b) *Cracking:* Cracking of concrete should not adversely affect the appearance or durability of the structure; the acceptable limits of cracking would vary with the type of structure and environment. The actual width of cracks will vary between wide limits and prediction of absolute maximum width is not possible. In general, the surface width of cracks should not exceed 0·3 mm.

The acceptable criteria of design for cracking shall be as given in chapter 4 for various concrete structure elements.

9-4. Characteristic and design values and partial safety factors: These are explained as below:

(a) *Characteristic strength of materials:* The value of the strength of the material below which not more than 5 per cent of the test results are expected to fall is known as the characteristic strength of the material and is denoted by f.

The characteristic strength for concrete (f_{ck}) shall be in accordance with table 1-1, modified by the age factor as given in art. 1-3.

The characteristic strength for steel (f_y) shall be assumed as minimum yield stress or 0·2 per cent proof stress.

(b) *Characteristic loads:* The value of load which has a 95 per cent probability of not being exceeded during the life of the structure is known as characteristic load and is denoted by F.

The characteristic loads shall be worked out using statistical methods. Since data are not available to express the loads in statistical terms, the loads that have given safe designs in past shall be used and are set out in relevant Indian Standards.

(1) Characteristic *dead loads* are the weight of the structure itself. These shall be found out using the unit weights of materials as given in IS : 1911-1967.

(2) Characteristic *live loads* and *wind loads* in the absence of statistical data, shall be taken from IS : 875-1964.

(3) Characteristic *seismic loads* in the absence of statistical data shall be taken from IS : 1893-1975.

(c) *Partial safety factors:* These are the factors when applied to loads and materials give the design values. The partial safety factors take into account the possible over-loads, the limit state considered and inaccurate assessment of the effects of loading.

For limit state of collapse and limit state of serviceability the partial safety factor γ_f for loads are as given in table 9-1.

TABLE 9-1

VALUES OF PARTIAL SAFETY FACTORS γ_f FOR LOADS

Load combination	Limit state of collapse			Limit state of serviceability		
	DL	*LL*	*WL*	*DL*	*LL*	*WL*
(1)	(2)	(3)	(4)	(5)	(6)	(7)
DL + *LL*	1·5	1·5	—	1·0	1·0	—
DL + *WL*	1·5 or 0·9*	—	1·5	1·0	—	1·0
DL + *LL* + *WL*	1·2	1·2	1·2	1·0	0·8	0·8

Note 1: While considering earthquake effects, substitute *EL* for *WL*.

Note 2: For the limit states of serviceability, the values of γ_f given in this table are applicable for short term effects. While assessing the long term effects due to creep, the dead load and that part of live load likely to be permanent may only be considered.

* This value is to be considered when stability against overturning or stress reversal is critical.

When assessing the strength of a structure or structural member for the limit state of collapse, the values of partial safety factor γ_m should be taken as 1·5 for concrete and 1·15 for steel.

The above partial safety factors shall normally be used. However, where the consequences of a structure attaining a limit state are of a serious nature such as huge loss of life and disruption of the economy, higher values of γ_f and γ_m than above may be applied.

(d) *Design values:* The design values are obtained when partial safety factors are applied to characteristic loads and strengths. These are obtained as below:

(1) *Materials:* The design strength of the materials, f_d is given by

$$f_d = \frac{f}{\gamma_m} \qquad \text{(9-1)}$$

where

f = characteristic strength of the material

γ_m = partial safety factor appropriate to the material and limit state being considered.

(2) *Loads:* The design load F_d is given by

$$F_d = F\gamma_f \qquad \text{(9-2)}$$

where

F = characteristic load

γ_f = partial safety factor appropriate to the nature of loading and the limit state being considered.

9-5. Limit state of collapse: Flexure:

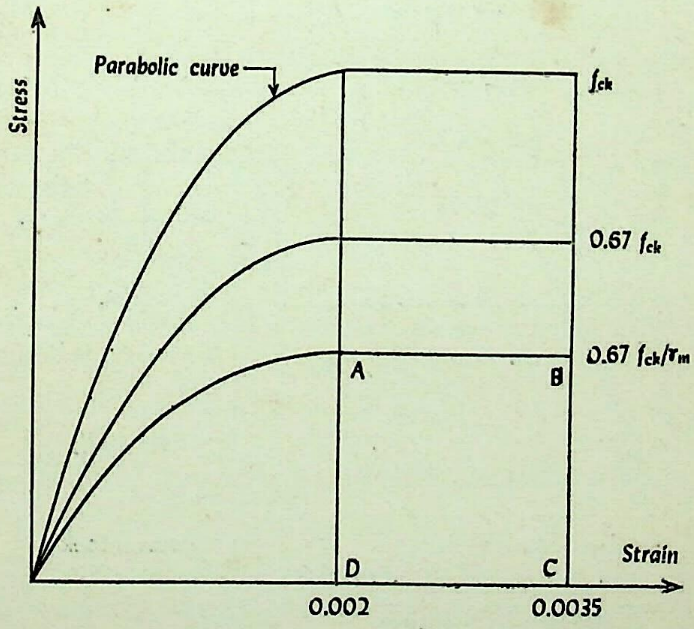
Assumptions: These are set out in clause 37.1 of IS : 456 and explained as below:

(1) Plane sections normal to the axis remain plane after bending.

This assumption means that strain at any point on the cross-section is directly proportional to its distance from the neutral axis.

(2) The maximum strain in concrete at the outermost compression fibre is taken as 0·0035 in bending.

(3) The relationship between the compressive stress distribution in concrete and the strain in concrete may be assumed to be rectangle, trapezoid, parabola or any other shape which results in prediction of strength in substantial agreement with the results of tests. An acceptable stress-strain curve is given in fig. 9-1. For design purpose, the compressive strength of concrete in the structure shall be assumed to be 0·67 times the characteristic strength. The partial safety factor $\gamma_m = 1·5$ shall be applied in addition to this.



Stress-strain curve for concrete
FIG. 9-1

The design stress block and strain diagram are separately shown in fig. 9-2. The design stress block parameters are as follows:

Area of stress block $= 0·36\ f_{ck}\ x_u$

Depth of centre of compressive force from the extreme fibre in compression $= 0·42\ x_u$

where

f_{ck} = characteristic compressive strength of concrete
x_u = depth of neutral axis.

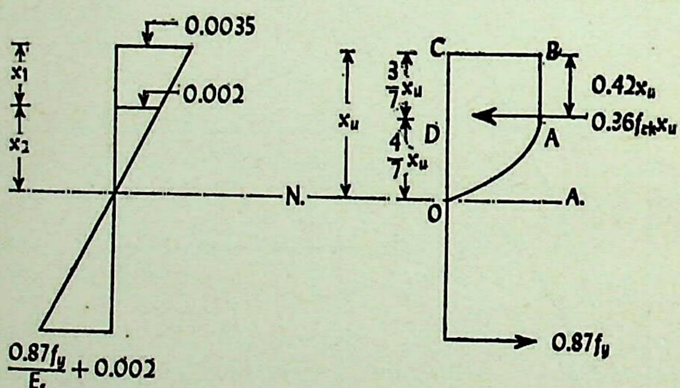
The stress block parameters are derived as follows:

As the strain in concrete is proportional to the distance from the neutral axis, the depth of parabolic portion, referring fig. 9-2

$$x_2 = \frac{0{\cdot}002\, x_u}{0{\cdot}0035} = \frac{4}{7}\, x_u, \text{ and}$$

the depth of rectangular portion

$$x_1 = x_u - \frac{4}{7}\, x_u$$
$$= \frac{3}{7}\, x_u.$$



(a) Strain diagram (b) Stress block

FIG. 9-2

Area of parabolic portion

$$= \frac{2}{3} \times 0{\cdot}446\, f_{ck} \times \frac{4}{7}\, x_u$$
$$= 0{\cdot}17\, f_{ck}\, x_u.$$

Area of rectangular portion

$$= 0{\cdot}446\, f_{ck} \times \frac{3}{7}\, x_u$$
$$= 0{\cdot}19\, f_{ck}\, x_u.$$

Total area of stress block

$$= 0{\cdot}17\, f_{ck}\, x_u + 0{\cdot}19\, f_{ck}\, x_u$$
$$= 0{\cdot}36\, f_{ck}\, x_u \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (9\text{-}3a)$$

Let $\bar{y}$ be the distance of c.g. of stress block from the extreme compression fibre, then

$$\bar{y} = \frac{(0{\cdot}17\, f_{ck}\, x_u)\ (x_1 + \frac{3}{8} x_2) + 0{\cdot}19\, f_{ck}\, x_u\ (\frac{x_1}{2})}{0{\cdot}36\, f_{ck}\, x_u}.$$

Substituting $x_1 = \frac{3}{7} x_u$ and $x_2 = \frac{4}{7} x_u$ and simplifying,

$$\bar{y} = 0{\cdot}416\, x_u \simeq 0{\cdot}42\, x_u \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (9\text{-}3b)$$

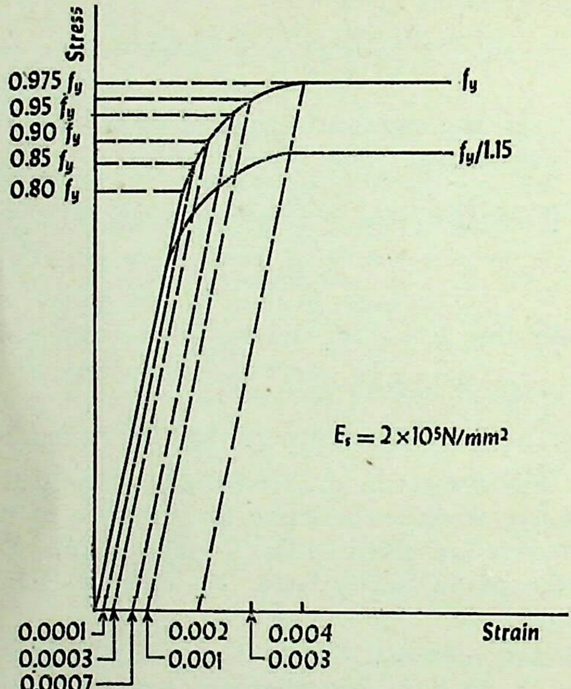
(4) The tensile strength of concrete is ignored.

(5) The stresses in the reinforcement are derived from representative stress-strain curve for the type of steel used. Typical curves are given in fig. 9-3(*a*) and (*b*). For design purpose the partial safety factor γ_m, equal to 1·15 shall be applied.

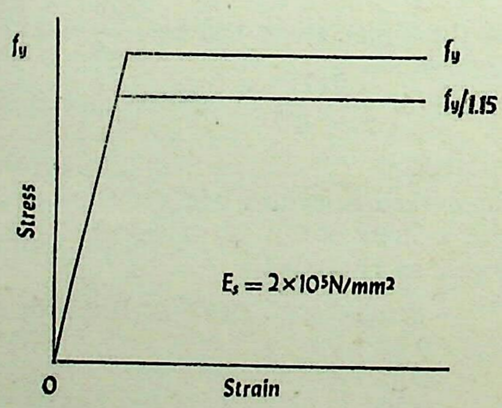
The close observation of these curves shows that for mild steel, the stress is proportional to strain upto yield point and then upto failure, strain increases at a constant stress f_y. (Note that this is not the actual stress-strain curve for mild steel, but is idealised for design purpose.) For mild steel of grade Fe 250 the design stress will be $\frac{250}{1{\cdot}15} = 217$ N/mm² for the strain value of $\frac{217}{2 \times 10^5} = 0{\cdot}00109$ and above (i.e. upto failure).

For cold-worked deformed bar, the stress is proportional to strain upto a stress of $0{\cdot}8\, f_y$. Thereafter, the stress-strain curve is defined as given below:

Stress	Inelastic strain
$0{\cdot}80\ f_y$	Nil
$0{\cdot}85\ f_y$	0·0001
$0{\cdot}90\ f_y$	0·0003
$0{\cdot}95\ f_y$	0·0007
$0{\cdot}975\ f_y$	0·0010
$1{\cdot}00\ f_y$	0·0020



(*a*) Cold-worked deformed bar



(*b*) Mild steel bar

Representative stress-strain curves for reinforcement

FIG. 9-3

To find out the design stress for a given strain value, a stress-strain graph shall be drawn using above values. For plotting the graph, stress-strain relation may be assumed as straight line between above defined points e.g. for Fe 415 grade steel, at $0{\cdot}8\ f_y$ the strain will be $\dfrac{0{\cdot}8 \times 415}{2 \times 10^5} = 0{\cdot}00166$ and at $0{\cdot}85\ f_y$, the strain will be $\dfrac{0{\cdot}85 \times 415}{2 \times 10^5} + 0{\cdot}0001 = 0{\cdot}00186$ and between these two points the stress-strain relation is assumed as straight line. For the given strain, now design stress can be found out from the graph. The stress-strain curves for cold-worked deformed bars are shown in fig. 9-4.

(6) The maximum strain in the tension reinforcement in the section at failure shall not be less than:

$$\frac{f_y}{1{\cdot}15\ E_s} + 0{\cdot}002$$

where

f_y = characteristic strength of steel

E_s = modulus of elasticity of steel.

This assumption restricts the depth of neutral axis. From the strain diagram (refer fig. 9-2)

$$\frac{x_{u,\ max}}{d} = \frac{0{\cdot}0035}{0{\cdot}0035 + \dfrac{0{\cdot}87\ f_y}{E_s} + 0{\cdot}002}$$

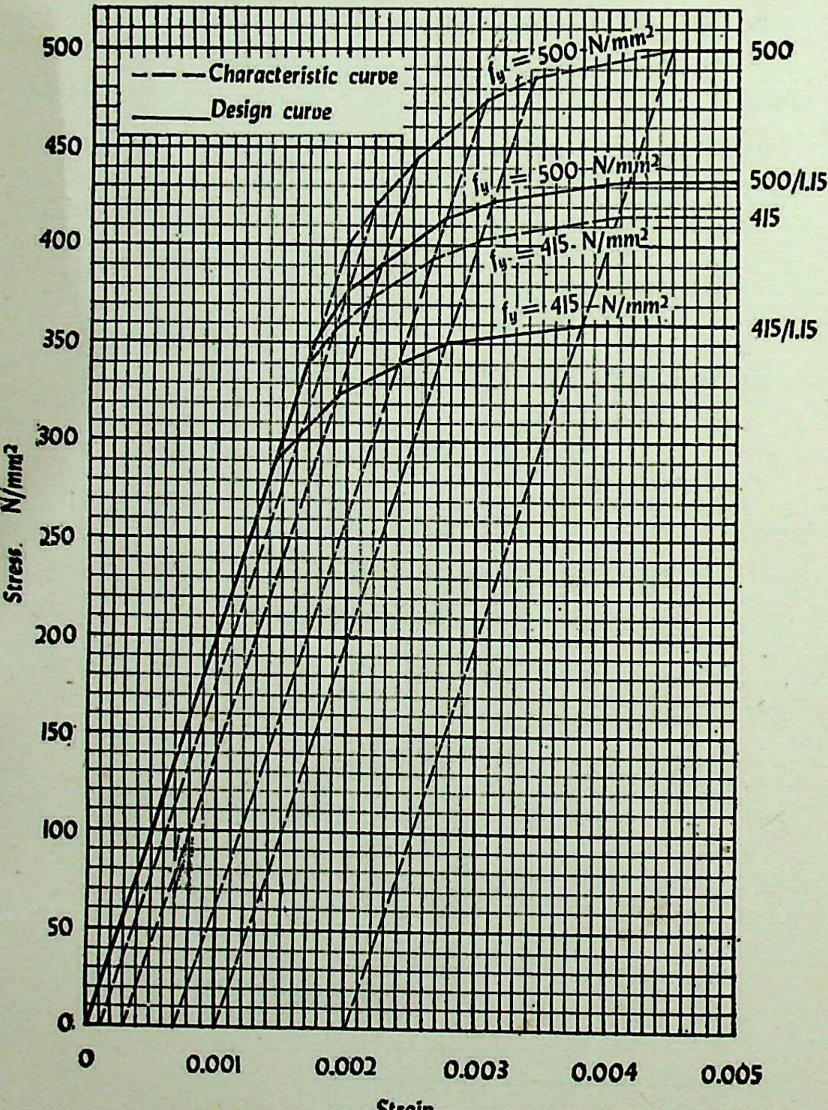
$$= \frac{0{\cdot}0035}{0{\cdot}0055 + \dfrac{0{\cdot}87\ f_y}{E_s}}$$

For mild steel, $f_y = 250\ \text{N/mm}^2$

$$\frac{x_{u,\ max}}{d} = \frac{0{\cdot}0035}{0{\cdot}0055 + \dfrac{0{\cdot}87 \times 250}{2 \times 10^5}}$$

$$= 0{\cdot}53$$

For other steels, $\frac{x_{u, max}}{d}$ may be found out in the same way. The limiting values of depth of neutral axis for



Stress-strain curves for cold-worked steels

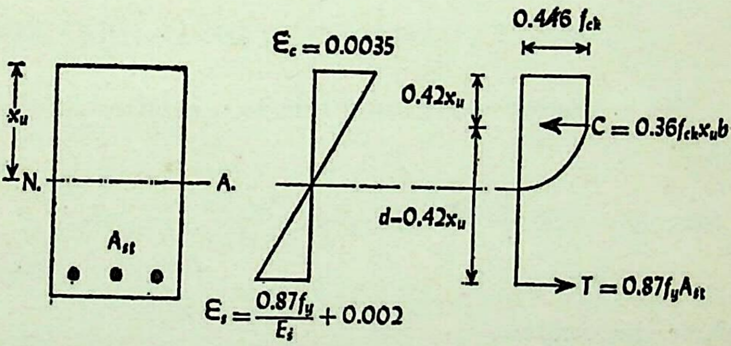
FIG. 9-4

different grades of steel based on the above assumptions are as follows:

f_y	$x_{u,\,max}/d$
250	0·53
415	0·48
500	0·46

SINGLY REINFORCED RECTANGULAR BEAMS

9-6. Derivation of formulae: A singly reinforced rectangular beam section, strain diagram and stress diagram are shown in fig. 9-5. The formulae for balanced section are derived as follow:



(*a*) Section (*b*) Strain diagram (*c*) Stress diagram

Singly reinforced beam

FIG. 9-5

To find neutral axis:

$$\text{Total compression} = \text{total tension}$$

$$0{\cdot}36\, f_{ck}\, x_u\, b = 0{\cdot}87\, f_y\, A_{st}$$

$$\therefore \qquad x_u = \frac{0{\cdot}87\, f_y\, A_{st}}{0{\cdot}36\, f_{ck}\, b} \qquad \text{(9-4)}$$

Note that value of depth of neutral axis as obtained by equation (9-4) should not exceed $x_{u,\,max}$ for a given section. If $x_u > x_{u,\,max}$; the depth of neutral axis shall be taken as $x_{u,\,max}$. This automatically restricts the use of over-reinforced sections.

To find lever arm:

From the stress diagram, the lever arm

$$z = (d - 0{\cdot}42\ x_u).$$

To find moment of resistance:

For a balanced section

M.R. = total compression × lever arm
= total tension × lever arm.

Considering the compressive forces (balanced or limiting value for a given section)

$$M_u = 0{\cdot}36\ f_{ck}\ b\ x_u\ (d - 0{\cdot}42\ x_u)$$

$$= 0{\cdot}36\ \frac{x_u}{d}\ (1 - 0{\cdot}42\ \frac{x_u}{d})\ f_{ck}\ bd^2 \ldots\ldots\ldots\ldots (9\text{-}6a)$$

For a limiting value, substitute $x_{u,\ max}$ for x_u.

Then

$$M_{u,\ lim} = 0{\cdot}36\ \frac{x_{u,\ max}}{d}\ (1 - 0{\cdot}42\ \frac{x_{u,\ max}}{d})\ f_{ck}\ bd^2$$

$$= Qbd^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9\text{-}6b)$$

Where the constant

$$Q = \frac{M_{u,\ lim}}{bd^2} = 0{\cdot}36\ \frac{x_{u,\ max}}{d}\ (1 - 0{\cdot}42\ \frac{x_{u,\ max}}{d}) \ldots\ (9\text{-}7)$$

and is known as *limiting moment of resistance factor for balanced rectangular section.*

Now considering tensile forces (for under-reinforced sections)

$$M_u = 0{\cdot}87\ f_y\ A_{st}\ (d - 0{\cdot}42\ x_u) \ldots\ldots\ldots\ldots\ldots (9\text{-}6c)$$

Substituting $x_u = \dfrac{0{\cdot}87\ f_y\ A_{st}}{0{\cdot}36\ f_{ck}\ b}$

$$M_u = 0{\cdot}87\ f_y\ A_{st}\ (d - \frac{0{\cdot}42 \times 0{\cdot}87\ f_y\ A_{st}}{0{\cdot}36\ f_{ck}\ b})$$

$$= 0{\cdot}87\ f_y\ A_{st}\ d\ (1 - \frac{f_y\ A_{st}}{bd\ f_{ck}}) \ldots\ldots\ldots\ (9\text{-}6d)$$

Limiting moment of resistance index:

From equation (9-6*a*),

$$M_{u, lim} = 0{\cdot}36 \frac{x_{u, max}}{d} \left(1 - 0{\cdot}42 \frac{x_{u, max}}{d}\right) f_{ck}\ bd^2$$

$$\therefore \quad \frac{M_{u, lim}}{f_{ck}\ bd^2} = 0{\cdot}36 \frac{x_{u, max}}{d} \left(1 - 0{\cdot}42 \frac{x_{u, max}}{d}\right) \ldots\ldots\ldots (9\text{-}8)$$

The constant $\frac{M_{u, lim}}{f_{ck}\ bd^2}$ is known as limiting moment of resistance index. This can be obtained by substituting $\frac{x_{u, max}}{d}$ for different grades of steel.

For Fe 250 grade mild steel, $\frac{x_{u, max}}{d} = 0{\cdot}53$

$$\therefore \quad \frac{M_{u, lim}}{f_{ck}\ bd^2} = 0{\cdot}36 \times 0{\cdot}53\ (1 - 0{\cdot}42 \times 0{\cdot}53)$$
$$= 0{\cdot}148.$$

For Fe 415 grade tor steel, $\frac{x_{u, max}}{d} = 0{\cdot}48$

$$\therefore \quad \frac{M_{u, lim}}{f_{ck}\ bd^2} = 0{\cdot}36 \times 0{\cdot}48\ (1 - 0{\cdot}42 \times 0{\cdot}48)$$
$$= 0{\cdot}138.$$

For Fe 500 grade tor steel, $\frac{x_{u, max}}{d} = 0{\cdot}46$

$$\therefore \quad \frac{M_{u, lim}}{f_{ck}\ bd^2} = 0{\cdot}36 \times 0{\cdot}46\ (1 - 0{\cdot}42 \times 0{\cdot}46)$$
$$= 0{\cdot}133.$$

For ready reference, these values are tabulated in table 9-2.

Limiting reinforcement index:

From equation (9-6*c*)

$$M_u = 0{\cdot}87\ f_y\ A_{st}\ d \left(1 - \frac{0{\cdot}42\ x_u}{d}\right).$$

Substituting $A_{st} = \frac{p_t\ bd}{100}$

$$M_u = \frac{0{\cdot}87}{100} f_y\, p_t\, bd^2 \left(1 - \frac{0{\cdot}42\ x_u}{d}\right)$$

$$\therefore \quad p_t f_y = \frac{M_u}{bd^2} \times \frac{100}{0{\cdot}87} \times \frac{1}{\left(1 - \frac{0{\cdot}42\ x_u}{d}\right)}.$$

Dividing both the sides by f_{ck} and writing equation for limiting moment

$$\frac{p_{t,\,lim}\, f_y}{f_{ck}} = \frac{M_{u,\,lim}}{f_{ck}\ bd^2} \times \frac{100}{0{\cdot}87} \times \frac{1}{\left(1 - \frac{0{\cdot}42\ x_{u,\,max}}{d}\right)} \quad \ldots\ldots\ldots(9\text{-}9)$$

The constant $\frac{p_{t,\,lim}\, f_y}{f_{ck}}$ is known as *limiting reinforcement index.*

For mild steel of grade Fe 250

$$\frac{p_{t,\,lim}\, f_y}{f_{ck}} = 0{\cdot}148 \times \frac{100}{0{\cdot}87} \times \frac{1}{(1 - 0{\cdot}42 \times 0{\cdot}53)}$$
$$= 21{\cdot}88.$$

For different grades of steels, these values have been tabulated in table 9-2.

TABLE 9-2

LIMITING MOMENT OF RESISTANCE AND REINFORCEMENT INDEX FOR SINGLY REINFORCED RECTANGULAR SECTIONS

f_y, N/mm^2	250	415	500
$\frac{M_{u,\,lim}}{f_{ck}\ bd^2}$	0·148	0·138	0·133
$\frac{p_{t,\,lim}\, f_y}{f_{ck}}$	21·88	19·87	18·95

Limiting moment of resistance factor:

Limiting moment of resistance factor is defined as

$$Q = \frac{M_{u,\,lim}}{bd^2} = \frac{M_{u,\,lim}}{f_{ck}\ bd^2} \times f_{ck}.$$

For M15 mix and mild steel reinforcement of grade Fe 250

$$Q = 0{\cdot}148 \times 15$$
$$= 2{\cdot}22.$$

For different combinations of materials, the values of Q have been tabulated in table 9-3.

TABLE 9-3

LIMITING MOMENT OF RESISTANCE FACTOR $\frac{M_{u,\,lim}}{ba^2}$, N/mm² FOR SINGLY REINFORCED RECTANGULAR SECTIONS

f_{ck}, N/mm²	f_y, N/mm² 250	415	500
15	2·22	2·07	2·00
20	2·96	2·76	2·66
25	3·7	3·45	3·33
30	4·44	4·14	3·99

Limiting reinforcement:

Limiting percentage of reinforcement can be obtained by using equation (9-9) and values from table 9-2.

For M15 mix and mild steel reinforcement of grade Fe 250,

$$p_{t,\,lim} = 21{\cdot}88\,\frac{f_{ck}}{f_y} \text{ from table 9-2}$$

$$= 21{\cdot}88 \times \frac{15}{250}$$

$$= 1{\cdot}32.$$

For different combinations of materials $p_{t,\,lim}$ have been tabulated in table 9-4.

TABLE 9-4

LIMITING PERCENTAGE OF REINFORCEMENT FOR SINGLY REINFORCED RECTANGULAR SECTIONS

f_{ck}, N/mm²	f_y, N/mm² 250	415	500
15	1·32	0·72	0·57
20	1·75	0·96	0·76
25	2·19	1·20	0·95
30	2·63	1·44	1·14

To find steel area:

(1) For a given ultimate moment (also known as factored moment) and assumed width of section, find out d from equation (9-6a)

$$d = \sqrt{\frac{M_u}{Qb}}.$$

This is a balanced section and steel area may be found out using equation (9-6b). Alternatively, $p_{t,\, lim}$ may be obtained from table 9-4.

(2) For a given factored moment, width and depth of section

Obtain $M_{u,\, lim}$ $= Q\, bd^2$.

If $M_u < M_{u,\, lim}$: design as under-reinforced section as explained below.

If $M_u = M_{u,\, lim}$: design as balanced section as explained in (1) above.

If $M_u > M_{u,\, lim}$: redesign the section either increasing the dimensions of section or design as doubly-reinforced beam.

For under-reinforced section, the steel area can be obtained using equation (9-6c) or (9-6d).

$$M_u = 0{\cdot}87\, f_y\, A_{st}\, d\, \left(1 - \frac{f_y\, A_{st}}{bd\, f_{ck}}\right).$$

Dividing both the sides by $0{\cdot}87\, f_y\, bd^2$

$$\frac{M_u}{0{\cdot}87\, f_y\, bd^2} = \frac{A_{st}}{bd}\left(1 - \frac{f_y\, A_{st}}{f_{ck}\, bd}\right).$$

Substituting $A_{st} = \dfrac{p_t\, bd}{100}$

$$\frac{M_u}{0{\cdot}87\, f_y\, bd^2} = \left(\frac{p_t}{100}\right) - \frac{f_y}{f_{ck}}\left(\frac{p_t}{100}\right)^2$$

which on re-arranging gives

$$\frac{f_y}{f_{ck}}\left(\frac{p_t}{100}\right)^2 - \left(\frac{p_t}{100}\right) + \frac{M_u}{0{\cdot}87\, f_y\, bd^2} = 0$$

which on solving gives

$$\frac{p_t}{100} = \frac{1 - \sqrt{1 - \frac{4 \cdot 6}{f_{ck}} \cdot \frac{M_u}{bd^2}}}{2\,\frac{f_y}{f_{ck}}}$$

$$\therefore \qquad p_t = 50\left[\frac{1 - \sqrt{1 - \frac{4 \cdot 6}{f_{ck}} \times \frac{M_u}{bd^2}}}{f_y / f_{ck}}\right] \quad \ldots\ldots\ldots\ldots\ldots (9\text{-}10)$$

The positive value of root of the equation will give larger area of steel than $p_{t,\,lim}$ and hence is not considered.

From equation (9-10), for given values of f_y and f_{ck} and for different values of $\frac{M_u}{bd^2}$, the reinforcement percentage p_t can be plotted or tabulated. SP : 16 contains charts 1 to 18 and tables 1 to 4 based on above equation. In SP : 16, the exact value of distance of c.g. of compressive force from extreme compression fibre ($0 \cdot 416\, x_u$) is considered which yields the expression

$$p_t = 50\left[\frac{1 \cdot 005 - \sqrt{1 \cdot 005 - \frac{4 \cdot 6}{f_{ck}} \; \frac{M_u}{bd^2}}}{f_y / f_{ck}}\right]$$

9-7. Types of problems: Three different types of problems are possible for singly reinforced rectangular beams:

Type 1: To find out the depth of neutral axis and specifying the type of beam.

For a given section, equate total tension and total compression and thus find out the depth of neutral axis using equation (9-4)

i.e. $\qquad x_u = \frac{0 \cdot 87\, f_y\, A_{st}}{0 \cdot 36\, f_{ck}\, b}$.

Also find out the limiting value of depth of neutral axis $x_{u,\,max}$ using limiting value of $\frac{x_{u,\,max}}{d}$.

Then

if $x_u < x_{u,\,max}$ the section is under-reinforced
if $x_u = x_{u,\,max}$ the section is balanced
if $x_u > x_{u,\,max}$ the section is over-reinforced and adopt $x_u = x_{u,\,max}$.

Type 2: To find out moment of resistance for a given section.

(a) Find out the depth of neutral axis and type of the beam as discussed in type 1.

(b) For over-reinforced and balanced sections, obtain moment of resistance using equation (9-6*a*).

(c) For under-reinforced sections, obtain moment of resistance using equation (9-6*a*), (9-6*c*) or (9·6*d*).

Type 3: To find the steel area for the section for a given factored moment.

This is explained in art. 9-6 under the heading "to find steel area".

Example 9-1.

A rectangular beam 230 mm wide and 520 mm effective depth is reinforced with 4 no. 16 mm diameter bars. Find out the depth of neutral axis and specify the type of beam. The materials are M 15 grade concrete and tor steel reinforcement of grade Fe 415. Also find out the depth of neutral axis if the reinforcement is increased to 5 no. 16 mm diameter bars.

Solution:

Case 1: $A_{st} = 4 \times 201 = 804 \text{ mm}^2$.

$$\begin{aligned}\text{Total compression} &= 0{\cdot}36\, f_{ck}\, b x_u \\ &= 0{\cdot}36 \times 15 \times 230\, x_u \\ &= 1242\, x_u. \\ \text{Total tension} &= 0{\cdot}87\, f_y\, A_{st} \\ &= 0{\cdot}87 \times 415 \times 804 \\ &= 290284 \text{ N.}\end{aligned}$$

Equating

$$1242\, x_u = 290284$$

$$\therefore \quad x_u = 234 \text{ mm.}$$

Limiting value of neutral axis

$$x_{u,\,max} = 0{\cdot}48\ d$$
$$= 0{\cdot}48 \times 520$$
$$= 250 \text{ mm}$$

$x_u < x_{u,\,max}$.

$\therefore$ Section is under-reinforced, and

$$x_u = 234 \text{ mm}.$$

Case 2: $A_{st} = 5 \times 201 = 1005 \text{ mm}^2$.

Total compression $= 0{\cdot}36 \times 15 \times 230\ x_u$
$= 1242\ x_u$.

Total tension $= 0{\cdot}87 \times 415 \times 1005$
$= 362855$ N.

Equating

$$1242\ x_u = 362855$$

$\therefore$ $x_u = 292$ mm.

Here $x_u > x_{u,\,max}$ i.e. over-reinforced section

$\therefore$ $x_u = x_{u,\,max}$
$= 250$ mm.

Example 9-2.

A singly reinforced rectangular beam of width 230 mm and 460 mm effective depth is reinforced with 4 no. 20 mm diameter bars. Find out the ultimate moment of resistance of the section. The materials are M15 grade concrete and mild steel reinforcement. Also find out the ultimate moment of resistance if it is reinforced with 5 no. 20 mm diameter bars.

Solution:

Case 1: $A_{st} = 4 \times 314 = 1256 \text{ mm}^2$.

Total compression $= 0{\cdot}36\ f_{ck}\ b\ x_u$
$= 0{\cdot}36 \times 15 \times 230\ x_u$
$= 1242\ x_u$.

Total tension $= 0{\cdot}87\ f_y\ A_{st}$
$= 0{\cdot}87 \times 250 \times 1256$
$= 273180$ N.

$$\text{Total compression} = \text{total tension}$$
$$1242\ x_u = 273180$$
$$\therefore \quad x_u = 220 \text{ mm}$$
$$x_{u,\,max} = 0{\cdot}53\ d$$
$$= 0{\cdot}53 \times 460$$
$$= 244 \text{ mm}$$
$$x_u < x_{u,\,max}.$$

The section is under-reinforced and $x_u = 220$ mm.

$$M_u = 0{\cdot}87\ f_y\ A_{st}\ (d - 0{\cdot}42\ x_u)$$
$$= 0{\cdot}87 \times 250 \times 1256\ (460 - 0{\cdot}42 \times 220) \times 10^{-6} \text{ kNm}$$
$$= 100{\cdot}42 \text{ kNm.}$$

Alternatively

$$M_u = 0{\cdot}36\ f_{ck}\ b\ x_u\ (d - 0{\cdot}42\ x_u)$$
$$= 0{\cdot}36 \times 15 \times 230 \times 220\ (460 - 0{\cdot}42 \times 220) \times 10^{-6}$$
$$= 100{\cdot}44 \text{ kNm.}$$

Case 2: $A_{st} = 5 \times 314$

$$= 1570 \text{ mm}^2.$$

$$\text{Total compression} = 0{\cdot}36\ f_{ck}\ b\ x_u$$
$$= 0{\cdot}36 \times 15 \times 230\ x_u$$
$$= 1242\ x_u.$$

$$\text{Total tension} = 0{\cdot}87\ f_y\ A_{st}$$
$$= 0{\cdot}87 \times 250 \times 1570$$
$$= 341475 \text{ N.}$$

$$\text{Total compression} = \text{total tension}$$
$$1242\ x_u = 341475$$
$$\therefore \quad x_u = 275 \text{ mm}$$
$$x_{u,\,max} = 244 \text{ mm}$$
$$x_u > x_{u,\,max}.$$

$\therefore$ Section is over-reinforced.

Use $x_u = 244$ mm.

$$\text{Lever arm } z = d - 0{\cdot}42\ x_u$$
$$= 460 - 0{\cdot}42 \times 244$$
$$= 357 \text{ mm.}$$

$$M_u = 0{\cdot}36\ f_{ck}\ b\ x_u\ z$$
$$= 0{\cdot}36 \times 15 \times 230 \times 244 \times 357 \times 10^{-6}\ \text{kNm}$$
$$= 108{\cdot}2\ \text{kNm}.$$

Example 9-3.

A singly reinforced rectangular beam is subjected to a bending moment of 40 kNm at working loads. Design the beam for flexure. The materials are M15 grade concrete and mild steel reinforcement of grade Fe 250.

Solution:

$$\text{Adopt } b = 230\ \text{mm}$$
$$\text{Factored moment} = 1{\cdot}5 \times 40$$
$$= 60\ \text{kNm}.$$

From table 9-3

$$M_{u,\,lim} = 2{\cdot}22\ bd^2.$$

$$\therefore \qquad d = \sqrt{\frac{60 \times 10^6}{2{\cdot}22 \times 230}}$$
$$= 342{\cdot}8\ \text{mm say } 343\ \text{mm}.$$

Steel area can be found out using equation (9-6*c*).

$$M_u = 0{\cdot}87\ f_y\ A_{st}\ d\ \left(1 - 0{\cdot}42\ \frac{x_u}{d}\right)$$

For balanced section

$$\frac{x_u}{d} = \frac{x_{u,\ max}}{d} = 0{\cdot}53.$$

Substituting

$$60 \times 10^6 = 0{\cdot}87 \times 250\ A_{st} \times 343\ (1 - 0{\cdot}42 \times 0{\cdot}53)$$
$$= 57995\ A_{st}$$
$$\therefore \qquad A_{st} = 1035\ \text{mm}^2.$$

Alternatively

This is a balanced section and from table 9-4

$$p_{t,\,lim} = 1{\cdot}32$$
$$A_{st} = \frac{1{\cdot}32}{100} \times 230 \times 343$$
$$= 1040\ \text{mm}^2.$$

Provide 3-20 ϕ + 1-12 ϕ = $3 \times 314 + 113$
$= 1055 \text{ mm}^2.$

$$D = 343 + 10 + 25 = 378 \text{ mm}.$$

Provide overall depth = 380 mm.

Supplementary details: Compare this example with Ex. 2-2 designed using working stress method. While using limit state theory, the saving in concrete per metre length of beam is equal to $0{\cdot}1 \times 0{\cdot}23 = 0{\cdot}023 \text{ m}^3$. At a market rate of 1050 Rs/m³, this saving will be $0{\cdot}023 \times 1050 = 24{\cdot}15$ rupees. The increase in steel area is $1035 - 735 = 300 \text{ mm}^2$. Per metre the additional steel is $300 \times 1000 \times 10^{-9} \times 7850$ kg/m³ = 2·36 kg. Additional cost of this steel = $2{\cdot}36 \times 8$ Rs/kg = 18·88 rupees. Then net saving per metre length of beam is $24{\cdot}15 - 18{\cdot}88 = 5{\cdot}27$ rupees. There may or may not be increase in cost of stirrups while using limit state method. In general, the limit state method provides an economical solution. The other advantages are:

(1) Light sections are used which reduce the dead weight of the structure.

(2) Light sections appear aesthetic.

Example 9-4.

A rectangular beam 230 mm wide × 535 mm effective depth is subjected to a bending moment of 88·5 kNm at working loads. Find the steel area required. The materials are M15 grade concrete and tor steel reinforcement of grade Fe 415.

Solution:

$$\text{Factored B.M. } M_u = 1{\cdot}5 \times 88{\cdot}5 = 132{\cdot}75 \text{ kNm}$$
$$M_{u,\,lim} = 2{\cdot}07\ bd^2$$
$$= 2{\cdot}07 \times 230 \times 535^2 \times 10^{-6} \text{ kNm}$$
$$= 136{\cdot}27 \text{ kNm}$$
$$M_u < M_{u,\,lim}.$$

The beam is under-reinforced.

To find steel area, using equation (9-6c)

$$M_u = 0{\cdot}87\ f_y\ A_{st}\ d\ \left(1 - \frac{f_y\ A_{st}}{bd\ f_{ck}}\right)$$

$$132{\cdot}75 \times 10^6 = 0{\cdot}87 \times 415\ A_{st} \times 535\ \left(1 - \frac{415\ A_{st}}{230 \times 535 \times 15}\right)$$

$$= 193162\ A_{st} - 43{\cdot}43\ A_{st}^2.$$

Dividing both sides by 43·43 and simplifying

$$A_{st}^2 - 4447\ A_{st} + 3056643 = 0$$

which on solving gives

$$A_{st} = 850\ \text{mm}^2.$$

Alternatively

$$p_t = 50 \left[\frac{1 - \sqrt{1 - \dfrac{4{\cdot}6}{f_{ck}} \times \dfrac{M_u}{bd^2}}}{f_y/f_{ck}} \right]$$

$$\frac{M_u}{bd^2} = \frac{132{\cdot}75 \times 10^6}{230 \times 535^2} = 2{\cdot}02$$

$$p_t = 50 \left[\frac{1 - \sqrt{1 - \dfrac{4{\cdot}6}{15} \times 2{\cdot}02}}{415/15} \right]$$

$$= 0{\cdot}692$$

$$A_{st} = \frac{0{\cdot}692}{100} \times 230 \times 535 = 851{\cdot}5\ \text{mm}^2.$$

The value of p_t also can be found out using table 1 from SP : 16.

Note: Compare this example with example 5-8, section at support *B* or *C*.

Example 9-5.

Determine the main reinforcement for a simply supported one-way slab of span 3 m carrying a uniformly distributed load of 5·5 kN/m² inclusive of self weight. Thickness of slab is 100 mm. The materials are M15 grade concrete and mild steel reinforcement.

Solution:

$$M = 5{\cdot}5 \times \frac{3^2}{8} = 6{\cdot}1875\ \text{kNm/m width.}$$

$$M_u = 1{\cdot}5 \times 6{\cdot}1875$$

$$= 9{\cdot}28\ \text{kNm.}$$

Using 10 mm ϕ bars

$$d = 100 - 15 - 5 = 80 \text{ mm}$$

$$b = 1000 \text{ mm}$$

$$\frac{M_u}{bd^2} = \frac{9 \cdot 28 \times 10^6}{1000 \times 80^2} = 1 \cdot 45.$$

From table 1, SP : 16

$$p_t = \frac{100\, A_{st}}{bd} = 0 \cdot 765$$

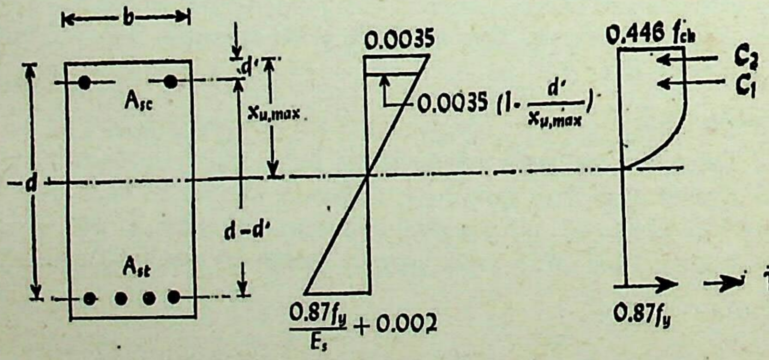
$$A_{st} = \frac{0 \cdot 765}{100} \times 1000 \times 80 = 612 \text{ mm}^2.$$

Use 10 mm ϕ about 125 mm c/c = 628 mm².

DOUBLY REINFORCED BEAMS

9-8. Derivation of formulae: A doubly reinforced beam section, strain diagram and stress diagram are shown in fig. 9-6. A doubly reinforced beam subjected to a moment M_u can be expressed as a rectangular section with tension reinforcement $A_{st,\,lim}$ reinforced for balanced condition giving moment of resistance $M_{u,\,lim}$ + an auxiliary section reinforced with compression reinforcement A_{sc} and tensile reinforcement A_{st2} giving a moment of resistance M_{u2} such that

$$M_u = M_{u,\,lim} + M_{u2}.$$



(*a*) Section (*b*) Strain diagram (*c*) Stress diagram

Doubly reinforced beam

FIG. 9-6

For the moment $M_{u,\,lim}$ the tension steel $A_{st,\,lim}$ is found out as explained for singly reinforced beams. For the additional moment M_{u2}, the additional tension steel and compression steel are provided such that they give a couple of moment M_{u2}.

Let the compression reinforcement be provided at a depth d' from the extreme compression fibre. Then lever arm for additional moment will be $d - d'$.

Considering tension steel

$$M_{u2} = A_{st2} \times 0{\cdot}87\, f_y\, (d - d') \qquad \text{(9-11}a\text{)}$$

Considering compression steel

$$M_{u2} = A_{sc}\,(f_{sc} - f_{cc})\,(d - d') \qquad \text{(9-11}b\text{)}$$

where

A_{st2} = area of additional tensile reinforcement

A_{sc} = area of compression reinforcement

f_{sc} = stress in compression reinforcement

f_{cc} = compressive stress in concrete at the level of compression steel.

Now additional tension = additional compression

$$0{\cdot}87 f_y\, A_{st2} = A_{sc}\,(f_{sc} - f_{cc})$$

$$\therefore \qquad A_{st2} = \frac{A_{sc}\,(f_{sc} - f_{cc})}{0{\cdot}87 f_y} \qquad \text{(9-11}c\text{)}$$

Any two of the above equations may be used to find out the value of A_{sc} and A_{st2}. The total tensile reinforcement

$$A_{st} = A_{st,\,lim} + A_{st2}.$$

To solve the above equations, one needs the value of f_{cc} and f_{sc}. Referring fig. 9-6(b), the strain at the level of compression reinforcement will be $0{\cdot}0035\left(1 - \frac{d'}{x_{u,\,max}}\right)$.

Finding the strain at the level of compression reinforcement, the stress in concrete for this strain (f_{cc}) can be obtained from stress-strain curve of concrete of fig. 9-1. For the values of $\frac{d'}{d}$ upto 0·2, and for all types of steel, f_{cc} is equal to $0{\cdot}446 f_{ck}$. (This corresponds to the straight portion

of curve upto a depth of $\frac{3}{7} x_u$.) Thus, the value of f_{cc} can be found out.

The strain in compression steel is the same as the strain in concrete at the level of compression steel. Using this value of strain, the stress in steel can be found out from the representative stress-strain curves for reinforcement of fig. 9-3 and fig. 9-4.

To simplify the procedure of finding out the stress in compression reinforcement, let us find out the stress for different values of $\frac{d'}{d}$.

For mild steel, $x_{u,\,max} = 0{\cdot}53d$

for $\frac{d'}{d} = 0{\cdot}2$

strain at level of compression reinforcement

$$= 0{\cdot}0035 \left(1 - \frac{0{\cdot}2}{0{\cdot}53}\right) = 0{\cdot}00218.$$

Design stress in mild steel for strain of 0·00109 and above (i.e. upto failure) is 217 N/mm². Therefore, for all values of d'/d from 0 to 0·2, the stress f_{sc} will be taken as 217 N/mm².

For tor steel of grade Fe 415, $x_{u,\,max} = 0{\cdot}48d$.

For $\frac{d'}{d} = 0{\cdot}05$

strain at the level of compression steel

$$= 0{\cdot}0035 \left(1 - \frac{0{\cdot}05}{0{\cdot}48}\right) = 0{\cdot}003135.$$

Stress in compression steel for this value of strain from fig. 9-4, is 355 N/mm².

Similarly for other values of $\frac{d'}{d}$ the stress in compression steel can be worked out. Design stresses in compression reinforcement (f_{sc}) for different values of $\frac{d'}{d}$ and different grades of steel have been tabulated in table 9-5. For inter-

mediate value of $\frac{d'}{d}$, the next higher value may be used for finding out f_{sc} from table 9-5.

TABLE 9-5

STRESS IN COMPRESSION REINFORCEMENT f_{sc}, N/mm^2 IN DOUBLY REINFORCED BEAMS

f_y N/mm^2	$\frac{d'}{d}$			
	0·05	0·1	0·15	0·20
250	217	217	217	217
415	355	353	342	329
500	424	412	395	370

9-9. Types of problems: Two basic types of problems in doubly reinforced beam are discussed below:

Type 1: To find out the moment of resistance of a given section.

Referring fig. (9-6*c*), the equilibrium of forces yields an equation

total compression = total tension

i.e. $C_1 + C_2 = T$

$\therefore \quad 0{\cdot}36\, f_{ck}\, bx_u + A_{sc}\,(f_{sc} - f_{cc}) = 0{\cdot}87\, f_y\, A_{st} \ldots\ldots\ldots (9\text{-}12)$

The stresses f_{sc} and f_{cc} may be found out assuming balanced conditions i.e. for d'/d upto 0·2, $f_{cc} = 0{\cdot}446\, f_{ck}$ and f_{sc} may be found out using table 9-5. Substitute the values in equation (9-12) and find out x_u. Find $x_{u,\,max}$ and type of beam.

The moment of resistance of the section can be found out by taking moments of compressive forces about the centroid of tensile reinforcement. Note that if the section is over-reinforced, the depth of neutral axis obtained will be more than $x_{u,\,max}$. In such cases, $x_u = x_{u,\,max}$ shall be used for calculations. Then

$$M_u = 0{\cdot}36\, f_{ck}\, bx_u\,(d - 0{\cdot}42x_u) + A_{sc}\,(f_{sc} - f_{cc})\,(d - d') \ldots\ldots\ldots\ldots\ldots\ldots (9\text{-}13a)$$

The value of f_{cc} is very small as compared to the value of f_{sc} and can be neglected. Then

$$M_u = 0{\cdot}36\ f_{ck}\ bx_u\ (d - 0{\cdot}42x_u) + A_{sc}\ f_{sc}\ (d - d') \qquad \text{............(9-13}b\text{)}$$

Alternatively, find out the c.g. of compressive forces and calculate lever arm z.

Then

$$M_u = 0{\cdot}87\ f_y\ A_{st}\ z \qquad \text{........................ (9-13}c\text{)}$$

Type 2: To find out reinforcement for flexure for a given section and factored moment.

(1) Find out $M_{u,\,lim}$ and reinforcement $A_{st,\,lim}$ for a given section using the equations

$$M_{u,\,lim} = Qbd^2 = 0{\cdot}36 f_{ck}\ bx_{u,\,max}(d - 0{\cdot}42x_{u,\,max})$$

and $$A_{st,\,lim} = \frac{M_{u,\,lim}}{0{\cdot}87 f_y\ (d - 0{\cdot}42x_{u,\,max})}.$$

(2) Obtain moment $M_{u2} = M_u - M_{u,\,lim}$.

(3) Find compression steel from equation

$$M_{u2} = A_{sc}\ (f_{sc} - f_{cc})\ (d - d').$$

Neglection f_{cc}

$$A_{sc} = \frac{M_{u2}}{f_{sc}\ (d - d')}.$$

(4) Corresponding tension steel A_{st2} may be found out from

$$A_{st2} = \frac{A_{sc}\ f_{sc}}{0{\cdot}87 f_y}$$

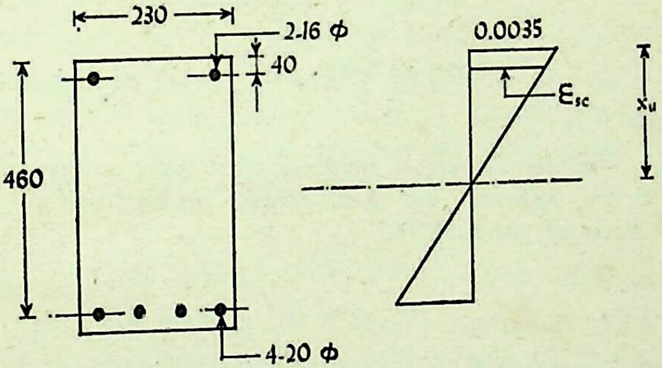
(5) $A_{st} = A_{st,\,lim} + A_{st2}$.

Example 9-6.

Find the moment of resistance of a beam section 230 mm wide × 460 mm effective depth reinforced with 2-16 mm diameter bars as compression reinforcement at an effective cover of 40 mm and 4-20 mm diameter bars as tension reinforcement. The materials are M15 grade concrete and mild steel reinforcement.

Solution :

$$b = 230 \text{ mm}$$
$$d = 460 \text{ mm}$$
$$d' = 40 \text{ mm}$$
$$A_{sc} = 2 \times 201 = 402 \text{ mm}^2$$
$$A_{st} = 4 \times 314 = 1256 \text{ mm}^2.$$



(*a*) Section (*b*) Strain diagram

FIG. 9-7

Equating total forces

$$0{\cdot}36 f_{ck}\, b x_u + A_{sc} f_{sc} = 0{\cdot}87 f_y\, A_{st}$$

$$\frac{d'}{d} = \frac{40}{460}$$

$= 0{\cdot}087$, next higher value $0{\cdot}1$ may be adopted.

$f_{sc} = 217$ N/mm² from table 9-5.

Substituting

$$0{\cdot}36 \times 15 \times 230 x_u + 402 \times 217 = 0{\cdot}87 \times 250 \times 1256$$

which gives $1242 x_u = 273180 - 87234$

$$= 185946$$

$$\therefore \quad x_u = 149{\cdot}7 \text{ mm}$$

$$x_{u,\, max} = 0{\cdot}53 d = 0{\cdot}53 \times 460$$
$$= 243{\cdot}8 \text{ mm}$$
$$x_u < x_{u,\, max}.$$

Hence, section is under-reinforced.

$$x_u = 149{\cdot}7 \text{ mm.}$$

Taking moments of compressive forces about centroid of tensile reinforcement

$$\begin{aligned} M_u &= 0{\cdot}36 f_{ck}\, b x_u\ (d - 0{\cdot}42 x_u) + A_{sc}\, f_{sc}\, (d - d') \\ &= 0{\cdot}36 \times 15 \times 230 \times 149{\cdot}7\ (460 - 0{\cdot}42 \times 149{\cdot}7) \times 10^{-6} + 402 \times 217 \times (460 - 40) \times 10^{-6} \\ &= 73{\cdot}84 + 36{\cdot}64 \\ &= 110{\cdot}48 \text{ kNm.} \end{aligned}$$

Example 9-7.

Find out the moment of resistance of a beam section of Ex. 9-6, if the materials are M15 grade concrete and tor steel reinforcement of grade Fe 415.

Solution:

$$\begin{aligned} b &= 230 \text{ mm} \\ d &= 460 \text{ mm} \\ d' &= 40 \text{ mm} \\ A_{sc} &= 402 \text{ mm}^2 \\ A_{st} &= 1256 \text{ mm}^2. \end{aligned}$$

Equating total forces

total compression = total tension

$$0{\cdot}36 f_{ck}\, b x_u + A_{sc} f_{sc} = 0{\cdot}87 f_y\, A_{st}.$$

For $\dfrac{d'}{d} = 0{\cdot}1$, f_{sc} from table 9-5 = 353 N/mm².

Substituting

$$0{\cdot}36 \times 15 \times 230 x_u + 402 \times 353 = 0{\cdot}87 \times 415 \times 1256$$

$\therefore$ $1242 x_u + 141906 = 453479$

which gives $x_u = 250{\cdot}86$ mm

$x_{u,\,max} = 0{\cdot}48 \times 460 = 220{\cdot}8$ mm.

$x_u > x_{u,\,max}$.

$\therefore$ Section is over-reinforced.

Adopt $x_u = x_{u,\,max} = 220{\cdot}8$ mm.

The moment of resistance can be found out by taking moments of compressive forces about centroid of tensile reinforcement.

$$\therefore \quad M_u = [1242x_u(460 - 0{\cdot}42x_u) + 141906\,(460 - 40)] \times 10^{-6}.$$

Substituting $x_u = 220{\cdot}8$ mm

$$M_u = 100{\cdot}72 + 59{\cdot}60$$
$$= 160{\cdot}32 \text{ kNm.}$$

Example 9-8.

A rectangular beam of size 250 mm wide × 500 mm effective depth is subjected to a factored moment of 175 kNm. Find the reinforcement for flexure. The materials are M15 grade concrete and tor steel reinforcement of grade Fe 415.

Solution:

$$M_{u,\,lim} = 2{\cdot}07 \times 230 \times 500^2 \times 10^{-6} = 119 \text{ kNm.}$$
$$M_{u2} = 175 - 119 = 56 \text{ kNm.}$$

Let the compression reinforcement be provided at an effective cover of 50 mm.

$$\therefore \qquad \frac{d'}{d} = \frac{50}{500} = 0{\cdot}1.$$

Stress in compression steel from table 9-5

$$f_{sc} = 353 \text{ N/mm}^2.$$

Now, $$A_{sc} = \frac{M_{u2}}{f_{sc}\;(d - d')}$$
$$= \frac{56 \times 10^{-6}}{353\;(500 - 50)}$$
$$= 352{\cdot}5 \text{ mm}^2.$$

Corresponding tensile steel

$$A_{st2} = \frac{A_{sc}\,f_{sc}}{0{\cdot}87 f_y}$$
$$= \frac{352{\cdot}5 \times 353}{0{\cdot}87 \times 415}$$
$$= 344{\cdot}6 \text{ mm}^2.$$

$$A_{st,\, lim} = \frac{M_{u,\, lim}}{0{\cdot}87 f_y d\ (1 - 0{\cdot}42\ \frac{x_{u,\, max}}{d})}$$

$$= \frac{119 \times 10^6}{0{\cdot}87 \times 415 \times 500\ (1 - 0{\cdot}42 \times 0{\cdot}48)}$$

$$= 825{\cdot}6 \text{ mm}^2.$$

$A_{sc} = 352{\cdot}5 \text{ mm}^2$, provide 2-16 Φ = 402 mm²

$A_{st} = 344{\cdot}6 + 825{\cdot}6$

$= 1170{\cdot}2 \text{ mm}^2$, provide 4-20 Φ = 1256 mm².

9-10. Use of design aids: While designing, the section is assumed as balanced. The expression for the moment of resistance of doubly reinforced section may be written as follows:

$$M_u = M_{u,\, lim} + A_{st2}\ (0{\cdot}87 f_y)\ (d - d').$$

Substituting $A_{st2} = \dfrac{p_{t2} bd}{100}$

$$M_u = M_{u,\, lim} + \frac{p_{t2}\, bd}{100}\ (0{\cdot}87 f_y)\ (d - d')$$

$$\therefore\quad \frac{M_u}{bd^2} = \frac{M_{u,\, lim}}{bd^2} + \frac{p_{t2}}{100}\ (0{\cdot}87 f_y)\ (1 - \frac{d'}{d})$$

where

p_{t2} is the additional percentage of tensile reinforcement.

$$p_t = p_{t,\, lim} + p_{t2}$$

and $\quad p_c = p_{t2} \left[\dfrac{0{\cdot}87 f_y}{f_{sc} - f_{cc}}\right]$

For different values of $\dfrac{d'}{d}$, the steel percentage p_t and p_c are tabulated for different values of $\dfrac{M_u}{bd^2}$ and various combination of materials in tables 45 to 56 of SP : 16.

Example 9-9.

Design the steel reinforcement for the data given in Ex. 9-8 using design tables.

Solution:

$$M_u = 175 \text{ kNm}$$
$$b = 230 \text{ mm}$$
$$d = 500 \text{ mm}$$
$$\frac{d'}{d} = 0{\cdot}1$$
$$\frac{M_u}{bd^2} = \frac{175 \times 10^6}{230 \times 500^2} = 3{\cdot}04.$$

From table 49 of SP : 16

$$p_t = 1{\cdot}054$$

and $p_c = 0{\cdot}312$

$$A_{st} = \frac{1{\cdot}054}{100} \times 230 \times 500 = 1168 \text{ mm}^2$$

and $A_{sc} = \dfrac{0{\cdot}312}{100} \times 230 \times 500 = 359 \text{ mm}^2.$

FLANGED BEAMS

9-11. Introductory: A tee beam (or ell beam) can be considered as a rectangular beam with dimensions $b_w \times D$ plus a flange of size $(b_f - b_w) \times D_f$. This is indicated in fig. 9-8, where beam (*a*) is equivalent to beam (*b*) + beam (*c*).

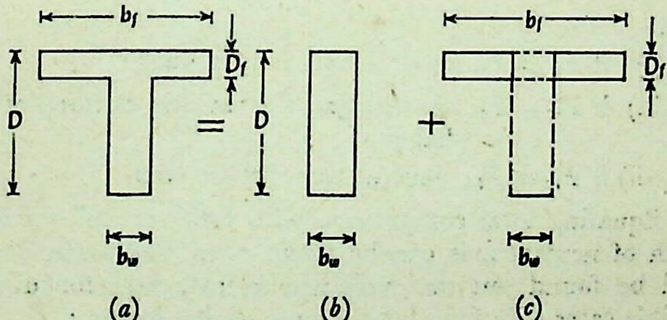


FIG. 9-8

The flanged beam analysis and design are analogous to doubly reinforced rectangular beam. The beam (*a*) of fig. 9-8 can be thought of as a singly reinforced beam (*b*) plus beam (*c*) which provides additional compressive force. In doubly reinforced rectangular beam the compression reinforce-

ment provides additional compressive resistance. A flanged beam also can be doubly reinforced, however, doubly reinforced flanged beams are rare as a large amount of compressive resistance is provided by the slab.

The moment of resistance of a tee beam of fig. 9-8(*a*) is a sum of the moment of resistance of beam (*b*) plus the moment of resistance of beam (*c*).

Similarly the steel area required for beam (*a*) shall be equal to the sum of the steel required for beam (*b*) and the steel area required for beam (*c*).

9-12. Position of neutral axis: For a flanged beam, the neutral axis either (*a*) lies in flange, or (*b*) lies in web. For a given section, to decide whether the neutral axis lies in flange or web, the flange force and the total tension may be compared as explained below.

As a first approximation, let us assume that neutral axis lies at the bottom of flange.

Now, total compression

$F_{tc} = 0{\cdot}36\ f_{ck}\ b_f D_f$ and

total tension

$F_{ts} = 0{\cdot}87\ f_y\ A_{st}$.

Then

(i) if $F_{tc} > F_{ts}$ neutral axis lies in flange

(ii) if $F_{tc} = F_{ts}$ neutral axis lies at the bottom of flange

(iii) if $F_{tc} < F_{ts}$, neutral axis lies in web.

Equating total compression with total tension, the actual depth of neutral axis can be found out. Total compression shall be found out as explained in art. 9-13 for different possible cases. To find out the type of the beam, $x_{u,\,max}$ shall be found and compared with actual value of neutral axis x_u.

Then

(i) if $x_u < x_{u,\,max}$; the section is under-reinforced

(ii) if $x_u = x_{u,\,max}$; the section is balanced

(iii) if $x_u > x_{u,\,max}$; the section is over-reinforced and $x_u = x_{u,\,max}$.

9-13. Derivation of formulae: Depending upon different values of x_u and D_f four different cases are discussed below for a flanged beam.

Case 1: Neutral axis lies in flange $(x_u < D_f)$.

In such a case the beam acts as a rectangular beam of width b_f and the formulae derived for rectangular beams shall be applied. They are summarized below:

For a singly reinforced flanged beam

(a) Equating total compression and total tension

$$x_u = \frac{0{\cdot}87 f_y A_{st}}{0{\cdot}36 f_{ck} b_f} \quad \ldots\ldots \text{(9-14}a\text{)}$$

(b) For under-reinforced section

$$M_u = 0{\cdot}87 f_y A_{st} \quad (d - 0{\cdot}42 x_u) \ldots\ldots \text{(9-14}b\text{)}$$

or $$M_u = 0{\cdot}36 f_{ck}\, b_f x_u\, (d - 0{\cdot}42 x_u) \ldots\ldots \text{(9-14}c\text{)}$$

(c) For balanced or over-reinforced section

$$M_{u,\,lim,T} = 0{\cdot}36 f_{ck}\, b_f x_{u,\,max}\, (d - 0{\cdot}42 x_{u,\,max}) \ldots \text{(9-14}d\text{)}$$

or $$M_{u,\,lim,T} = 0{\cdot}87 f_y A_{st,\,lim} \quad (d - 0{\cdot}42 x_{u,\,max}) \ldots \text{(9-14}e\text{)}$$

For a doubly reinforced flanged beam

(a) $M_u - M_{u,\,lim,T} = f_{sc} A_{sc}\, (d - d') \ldots\ldots$ (9-15a)

(b) $A_{st1} = A_{st,\,lim} \ldots\ldots$ (9-15b)

(c) $A_{st2} = \dfrac{A_s f_{sc}}{0{\cdot}87 f_y} \ldots\ldots$ (9-15c)

(d) $A_{st} = A_{st1} + A_{st2} \ldots\ldots$ (9-15d)

Case 2: Neutral axis lies in web $(x_u > D_f)$, section is balanced (limiting value of the moment of resistance).

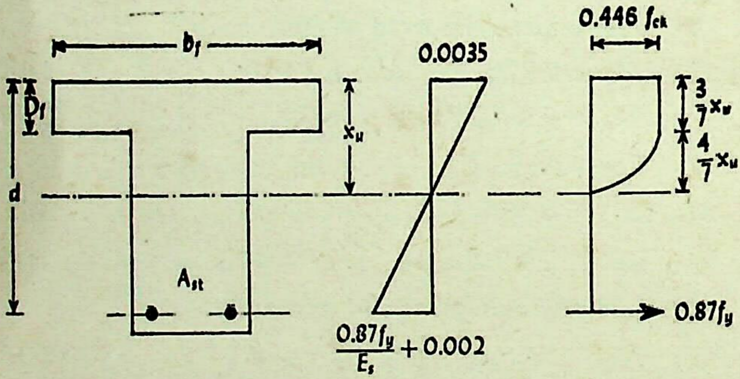
A tee beam section, strain diagram and stress diagram are shown in fig. 9-9. It can be seen that when the thickness of flange is small, that is less than about $0{\cdot}2d$, the stresses in flange are uniform or nearly uniform. (*When mild steel is used, the stresses in concrete are uniform upto* $\frac{3}{7} \times 0{\cdot}53d = 0{\cdot}227d$; *when tor steel of grade Fe 415 is used, the stresses in concrete are uniform*

upto $\frac{3}{7} \times 0{\cdot}48d = 0{\cdot}206d$ *and when tor steel of grade Fe 500 is used, the stresses in concrete are uniform upto* $\frac{3}{7} \times 0{\cdot}46d = 0{\cdot}197d.$)

When the thickness of flange exceeds $0{\cdot}2d$, the stresses in the flange are not uniform. According to expressions given in appendix E-2 of IS : 456, the allowance for non-uniform stresses are made when in the equations considering $\frac{D_f}{d} < 0{\cdot}2d$, D_f is replaced by y_f

where

$$y_f = 0{\cdot}15x_u + 0{\cdot}65D_f \text{ but not greater than } D_f \ldots \text{(9-16)}$$



(*a*) Section (*b*) Strain diagram (*c*) Stress diagram

FIG. 9-9

(i) When $\frac{D_f}{d} \leqslant 0{\cdot}2$

The depth of rectangular portion of stress block is more than flange thickness and thus stresses in flange are uniform having a value of $0{\cdot}446 f_{ck}$.

Total tension $= 0{\cdot}87 f_y A_{st}$ .... (9-17*a*)

Total compression = compression in rectangular beam of size $b_w \times d$ + compression in rectangle of size $(b_f - b_w)\ D_f$

$= 0{\cdot}36 f_{ck}\ b_w x_{u,\,max} + 0{\cdot}446\ f_{ck} \times (b_f - b_w) D_f$ .... (9-17*b*)

Limiting moment of resistance of the section can be found out by taking moment of compressive forces about centroid of tensile reinforcement. Then

$$M_{u,\,lim,\,T} = 0{\cdot}36 f_{ck}\, b_w x_{u,\,max}\, (d - 0{\cdot}42 x_{u,\,max}) + (b_f - b_w) D_f \times 0{\cdot}446 f_{ck}\, \left(d - \frac{D_f}{2}\right) \quad \ldots\ldots\ldots\ldots\ldots \quad (9\text{-}17c)$$

To find out the steel area $A_{s',\,lim}$ for this case, total tension and total compression are equated. Then

$$0{\cdot}87 f_y\, A_{st,\,lim} = 0{\cdot}36 f_{ck}\, b_w x_{u,\,max} + 0{\cdot}446 f_{ck}\, (b_f - b_w) D_f$$

$$\therefore \quad A_{st,\,lim} = \frac{0{\cdot}36 f_{ck}\, b_w x_{u,\,max} + 0{\cdot}446 f_{ck}\, (b_f - b_w) D_f}{0{\cdot}87 f_y} \quad \ldots\ldots\ldots\ldots\ldots\ldots (9\text{-}17d)$$

(ii) When $\frac{D_f}{d} > 0{\cdot}2$

The rectangular portion of stress block in this case is assumed to be equal to y_f.

$$\text{Total tension} = 0{\cdot}87 f_y\, A_{st,\,lim} \quad \ldots\ldots\ldots\ldots\ldots\ldots (9\text{-}18a)$$

$$\text{Total compression} = 0{\cdot}36 f_{ck}\, b_w x_{u,\,max} + 0{\cdot}446 f_{ck}\, (b_f - b_w) y_f \quad \ldots\ldots\ldots\ldots\ldots\ldots (9\text{-}18b)$$

Limiting moment of resistance of the section can be found out by taking moment of compressive forces about centre of tensile reinforcement. Then

$$M_{u,\,lim,\,T} = 0{\cdot}36 f_{ck}\, b_w x_{u,\,max}\, (d - 0{\cdot}42 x_{u,\,max}) + 0{\cdot}446 f_{ck}\, (b_f - b_w) \times y_f \left(d - \frac{y_f}{2}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots (9\text{-}18c)$$

To find out the steel area $A_{st,\,lim}$; total tension and total compression are equated. This gives

$$A_{st,\,lim} = \frac{0{\cdot}36 f_{ck} b_w x_{u,\,max} + 0{\cdot}446 f_{ck}\, (b_f - b_w) y_f}{0{\cdot}87 f_y} \ldots (9\text{-}18d)$$

Note that for a given width of flange, width of web and effective depth, if the thickness of flange is increased, a larger value of moment of resistance is obtained, i.e. case $\frac{D_f}{d} > 0{\cdot}2$ yields larger value of moment of resistance than case $\frac{D_f}{d} \leqslant 0{\cdot}2$.

Case 3: Neutral axis lies in web, section is under-reinforced.

For finding out limiting moment of resistance, the formulae were simplified for $\frac{D_f}{d} \leqslant 0{\cdot}2$ and $\frac{D_f}{d} > 0{\cdot}2$ as discussed in case 2. When the section is under-reinforced, the moment of resistance shall be found out using actual stress block. The stress block for concrete shows a uniform stress upto a depth of $\frac{3}{7}\, x_u$ and then a parabolic shape. These cases are discussed below.

(i) $D_f \leqslant \frac{3}{7} x_u$

In this case, the stresses are uniform in flange.

Total compression

$$= 0{\cdot}36 f_{ck}\, b_w x_u + 0{\cdot}446\, f_{ck}\, (b_f - b_w) D_f \quad \text{(9-19}a\text{)}$$

Total tension

$$= 0{\cdot}87 f_y, A_{st} \quad \text{(9-19}b\text{)}$$

To find out x_u, total compression and total tension are equated.

To find out the moment of resistance of the section, the moment of compressive forces are taken about centroid of tensile reinforcement.

$$M_u = 0{\cdot}36 f_{ck}\, b_w x_u\, (d - 0{\cdot}42 x_u) + 0{\cdot}446 f_{ck}\, (b_f - b_w) \times D_f \left(d - \frac{D_f}{2}\right) \quad \text{(9-19}c\text{)}$$

Referring fig. 9-8, for the given moment M_u, the steel area A_{st} required for beam (*a*) shall be treated as the sum of the steel area A_{st1} required for beam (*b*) resisting moment M_{u1} and the steel area A_{st2} required for beam (*c*) resisting moment M_{u2} such that

$$M_u = M_{u1} + M_{u2} \quad \text{(9-20}a\text{)}$$

and $$A_{st} = A_{st1} + A_{st2} \quad \text{(9-20}b\text{)}$$

For beam (*c*), equating the total compression and total tension

$\therefore \quad 0{\cdot}446 f_{ck}\ (b_f - b_w) D_f = 0{\cdot}87 f_y\ A_{st2}$

$$\therefore \quad A_{st2} = \frac{0{\cdot}446 f_{ck}\ (b_f - b_w) D_f}{0{\cdot}87 f_y} \quad \ldots\ldots\ldots\ldots\ldots \quad (9\text{-}20c)$$

$$M_{u2} = 0{\cdot}446 f_{ck}\ (b_f - b_w) D_f\ \left(d - \frac{D_f}{2}\right) \quad \ldots\ldots\ldots \quad (9\text{-}20d)$$

For beam (b), the approach will be the same as for singly reinforced beam. We have

$$M_{u1} = M_u - M_{u2} \text{ and}$$

$$\frac{100 A_{st1}}{bd} = 50 \left[\frac{1 - \dfrac{\sqrt{1 - 4{\cdot}6}}{f_{ck}} \times \dfrac{M_{u1}}{bd^2}}{f_y / f_{ck}} \right] \quad \ldots\ldots\ldots\ldots \quad (9\text{-}20e)$$

(ii) $D_f > \frac{3}{7}\ x_u$

In this case, the non-uniform stresses in the flange shall be taken into account and the above formulae (9-19) shall be used replacing D_f by y_f.

Total compression

$$= 0{\cdot}36 f_{ck}\ b_w x_u + 0{\cdot}446 f_{ck}\ (b_f - b_w) y_f \quad \ldots\ldots\ldots \quad (9\text{-}21a)$$

Total tension

$$= 0{\cdot}87 f_y\ A_{st} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (9\text{-}21b)$$

where $y_f = 0{\cdot}15 x_u + 0{\cdot}65 D_f$.

Equating total compression and total tension, x_u can be found out.

Moment of resistance

$$M_u = 0{\cdot}36 f_{ck}\ b_w x_u\ (d - 0{\cdot}42 x_u) + 0{\cdot}446 f_{ck}\ (b_f - b_w) \times y_f \left(d - \frac{y_f}{2}\right) \quad \ldots\ldots\ldots \quad (9\text{-}21c)$$

To find out the steel area for a given moment M_u in this case, the approach as described for case (i) cannot be used because the value of x_u is not known and hence y_f cannot be calculated. However, assuming a trial depth of neutral axis, the approach in (i) can be used. A simplified approach is explained below. This approach may be used for design of a flanged beam for any case.

(1) The dimensions of a flanged beam are usually determined using practical concept or a depth of $\frac{1}{12}$ to $\frac{1}{10}$ times the span is assumed. This is explained in art. 2-17 type 4.

(2) Assume the approximate lever arm $d - \frac{D_f}{2}$ and find the steel area from the following formula:

$$A_{st} = \frac{M_u}{0{\cdot}87 f_y \left(d - \frac{D_f}{2}\right)}.$$

(3) Provide the steel as found from (2). Find out the depth of neutral axis and the moment of resistance which shall be greater than or equal to the applied moment.

(4) If moment of resistance is less than the applied moment revise the section.

For case (i) $D_f \leqslant \frac{3}{7} x_u$

i.e. $\frac{7}{3} D_f \leqslant x_u$.

For case (ii) $D_f > \frac{3}{7} x_u$

i.e. $\frac{7}{3} D_f > x_u$.

For a given section $\frac{7}{3} D_f$ is a constant quantity. If the reinforcement is increased, the moment of resistance also increases as long as its value is below the limiting moment of resistance. As the reinforcement increases, x_u also increases. Thus for a given section, the first case will give a larger moment of resistance than the second case. This explanation will be helpful in the analysis of a flanged beam.

Case 4: Neutral axis lies in web and section is over-reinforced.

When the section is over-reinforced, the moment of resistance shall be $M_{u, lim}$ and this can be obtained in a similar way as explained in case 2.

(i) When $\frac{D_f}{d} \leqslant 0{\cdot}2$, use equation (9-17)

(ii) When $\frac{D_f}{d} > 0{\cdot}2$, use equation (9-18).

Example 9-10.

A tee beam of effective flange width 1500 mm, thickness of slab 100 mm, width of rib 300 mm and effective depth of 560 mm is reinforced with 4 no. 25 mm diameter bars. Calculate the ultimate moment of resistance. The materials are M15 grade concrete and tor steel reinforcement of grade Fe 415.

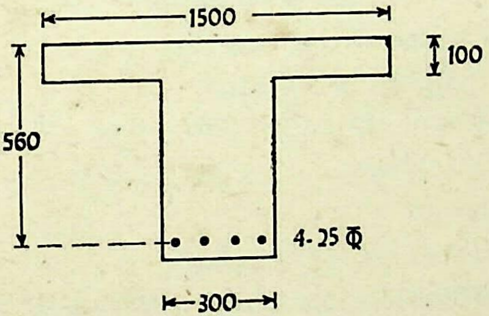


FIG. 9-10

Solution:

$$A_{st} = 4 \times 491 = 1964 \text{ mm}^2.$$

To find whether the neutral axis lies in flange or web, flange compression and tensile force are compared.

$$F_{tc} = 0{\cdot}36 f_{ck}\ b_f D_f$$
$$= 0{\cdot}36 \times 15 \times 1500 \times 100 \times 10^{-3}$$
$$= 810 \text{ kN}$$
$$F_{ts} = 0{\cdot}87 f_y\ A_{st}$$
$$= 0{\cdot}87 \times 415 \times 1964 \times 10^{-3}$$
$$= 709 \text{ kN}$$
$$F_{tc} > F_{ts}$$

$\therefore$ Neutral axis lies in flange.

Equating the forces

$$\text{total compression} = \text{total tension}$$

$$0{\cdot}36 f_{ck}\, b_f x_u = 0{\cdot}87 f_y\, A_{st}$$

$$0{\cdot}36 \times 15 \times 1500 x_u = 0{\cdot}87 \times 415 \times 1964$$

$$8100 x_u = 709102$$

$$\therefore \quad x_u = 87{\cdot}54 \text{ mm} < 100 \text{ mm....(O.K.)}$$

$$x_{u,\,max} = 0{\cdot}48 d$$

$$= 0{\cdot}48 \times 560$$

$$= 268{\cdot}8 \text{ mm.}$$

$$x_u < x_{u,\,max}$$

∴ Section is under-reinforced.

$$M_u = 0{\cdot}87 f_y\, A_{st}\ (d - 0{\cdot}42 x_u)$$

$$= 0{\cdot}87 \times 415 \times 1964\, (560 - 0{\cdot}42 \times 87{\cdot}54) \times 10^{-6}$$

$$= 371 \text{ kNm.}$$

Alternatively

$$M_u = 0{\cdot}36 f_{ck}\, b_f x_u\ (d - 0{\cdot}42 x_u)$$

$$= 0{\cdot}36 \times 15 \times 1500 \times 87{\cdot}54\ (560 - 0{\cdot}42 \times 87{\cdot}54) \times 10^{-6}$$

$$= 371 \text{ kNm.}$$

Example 9-11.

Find the ultimate moment of resistance for the section of Ex. 9-10, if it is reinforced with 5 no. 25 mm diameter bars.

Solution:

$$A_{st} = 5 \times 491 = 2455 \text{ mm}^2.$$

To find neutral axis

$$F_{tc} = 0{\cdot}36 f_{ck}\, b_f D_f$$

$$= 0{\cdot}36 \times 15 \times 1500 \times 100 \times 10^{-3}$$

$$= 810 \text{ kN.}$$

$$F_{ts} = 0{\cdot}87 f_y\, A_{st}$$

$$= 0{\cdot}87 \times 415 \times 2455 \times 10^{-3}$$

$$= 886{\cdot}4 \text{ kN.}$$

$$F_{tc} < F_{ts}$$

$\therefore$ Neutral axis lies in web.

Assume $D_f > \frac{3}{7}\, x_u$

$$y_f = 0{\cdot}15x_u + 0{\cdot}65D_f$$
$$= 0{\cdot}15x_u + 0{\cdot}65 \times 100$$
$$= 0{\cdot}15x_u + 65.$$

Now, total compression

$$= 0{\cdot}36f_{ck}\, b_w x_u + 0{\cdot}446 f_{ck}\, (b_f - b_w) y_f$$
$$= 0{\cdot}36 \times 15 \times 300 x_u + 0{\cdot}446 \times 15 \times 1200\, (0{\cdot}15x_u + 65)$$
$$= 1620x_u + 1204{\cdot}2x_u + 521820$$
$$= 2824{\cdot}2x_u + 521820.$$

Total tension

$$= 0{\cdot}87 f_y\, A_{st}$$
$$= 0{\cdot}87 \times 415 \times 2455$$
$$= 886378.$$

Equating

$$2824{\cdot}2x_u + 521820 = 886378$$

which yields

$$x_u = 129{\cdot}08 \text{ mm}$$

$\frac{3}{7}\, x_u = 55{\cdot}32$ mm $< D_f$. .... (O.K.)

Now, $x_u < x_{u,\,max}$

$\therefore$ Section is under-reinforced.

$$y_f = 0{\cdot}15 \times 129{\cdot}08 + 65 = 84{\cdot}36 \text{ mm}$$

$$M_u = 0{\cdot}36 f_{ck}\, b_w x_u\, (d - 0{\cdot}42x_u) + 0{\cdot}446 f_{ck}\, (b_f - b_w) \times y_f \left(d - \frac{y_f}{2}\right)$$

$$= 0{\cdot}36 \times 15 \times 300 \times 129{\cdot}08\, (560 - 0{\cdot}42 \times 129{\cdot}08) \times 10^{-6} + 0{\cdot}446 \times 15 \times 1200 \times 84{\cdot}36\, (560 - 42{\cdot}18) \times 10^{-6}$$
$$= 105{\cdot}76 + 350{\cdot}69$$
$$= 456{\cdot}45 \text{ kNm}.$$

Example 9-12.

Find the ultimate moment of resistance for the section of Ex. 9-10 if it is reinforced with 3 no. 28 mm diameter bars plus 3 no. 25 mm diameter bars. Assume that effective depth remains the same.

Solution:

$$A_{st} = 3(616 + 491) = 3321 \text{ mm}^2.$$

From Example 9-11, it can be observed that N.A. lies in web. Assume $D_f < \frac{3}{7} x_u$.

Total compression

$$= 0{\cdot}36 f_{ck}\, b_w x_u + 0{\cdot}446 f_{ck}\, (b_f - b_w) D_f$$
$$= 0{\cdot}36 \times 15 \times 300 x_u + 0{\cdot}446 \times 15 \times 1200 \times 100$$
$$= 1620 x_u + 802800.$$

Total tension

$$= 0{\cdot}87 f_y\, A_{st}$$
$$= 0{\cdot}87 \times 415 \times 3321$$
$$= 1199047.$$

Equating

$$1620 x_u + 802800 = 1199047$$

$$\therefore \quad x_u = 244{\cdot}6 \text{ mm}$$

$$\frac{3}{7} x_u = 104{\cdot}8 \text{ mm} > D_f \ldots\ldots\ldots\ldots \text{(O.K.)}$$

$$x_{u,\, max} = 0{\cdot}48 \times 560$$
$$= 268{\cdot}8 \text{ mm}$$

$$x_u < x_{u,\, max}$$

$\therefore$ Section is under-reinforced.

$$M_u = 0{\cdot}36 f_{ck}\, b_w x_u (d - 0{\cdot}42 x_u) + 0{\cdot}446 f_{ck}\, (b_f - b_w) D_f \left(d - \frac{D_f}{2}\right)$$
$$= 0{\cdot}36 \times 15 \times 300 \times 244{\cdot}6\, (560 - 0{\cdot}42 \times 244{\cdot}6) \times 10^{-6}$$
$$+ 0{\cdot}446 \times 15 \times 1200 \times 100\, (560 - 50) \times 10^{-6}$$
$$= 181{\cdot}20 + 409{\cdot}43$$
$$= 590{\cdot}63 \text{ kNm}.$$

Example 9-13.

A tee beam as shown in fig. 9-11 is subjected to a factored moment of 400 kNm. Design the steel reinforcement for flexure. The materials are M15 grade concrete and tor steel reinforcement of grade Fe 415.

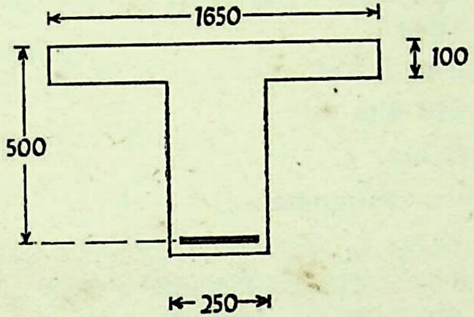


FIG. 9-11

Solution:

To start with, assume lever arm

$$z = d - \frac{D_f}{2}$$
$$= 500 - 50$$
$$= 450 \text{ mm.}$$

$$\text{Approximate} \quad A_{st} = \frac{400 \times 10^6}{0{\cdot}87 \times 415 \times 450}$$
$$= 2462 \text{ mm}^2.$$

Provide 5-25 Φ ; $A_{st} = 5 \times 491 = 2455 \text{ mm}^2$.

The approximate design is now checked.

To find lever arm

$$F_{tc} = 0{\cdot}36 f_{ck}\, b_f D_f$$
$$= 0{\cdot}36 \times 15 \times 1650 \times 100 \times 10^{-3}$$
$$= 891 \text{ kN.}$$
$$F_{ts} = 0{\cdot}87 f_y\, A_{st}$$
$$= 0{\cdot}87 \times 415 \times 2455 \times 10^{-3}$$
$$= 886{\cdot}4 \text{ kN.}$$
$$F_{tc} > F_{ts}$$

$\therefore$ Neutral axis lies in flange.

Equating the forces

$$0{\cdot}36 f_{ck}\ b_f x_u = 0{\cdot}87 f_y\ A_{st}$$
$$0{\cdot}36 \times 15 \times 1650 x_u = 0{\cdot}87 \times 415 \times 2455$$
$$\therefore \quad x_u = 99{\cdot}5 \text{ mm} < 100 \text{ mm} \ldots\ldots\ldots\ldots\ldots\ldots (\text{O.K.})$$
$$x_{u,\,max} = 0{\cdot}48d$$
$$= 0{\cdot}48 \times 500$$
$$= 240 \text{ mm}$$
$$x_u < x_{u,\,max}.$$

$\therefore$ Section is under-reinforced.

$$M_u = 0{\cdot}87 f_y\ A_{st}\ (d - 0{\cdot}42 x_u)$$
$$= 0{\cdot}87 \times 415 \times 2455\ (500 - 0{\cdot}42 \times 99{\cdot}5) \times 10^{-6}$$
$$= 406{\cdot}2 \text{ kNm} > 400 \text{ kNm} \ldots\ldots\ldots\ldots\ldots (\text{O.K.})$$

The design is satisfactory.

Alternatively

$$M_u = 0{\cdot}36 f_{ck}\ b_f x_u\ (d - 0{\cdot}42 x_u)$$
$$= 0{\cdot}36 \times 15 \times 1650 \times 99{\cdot}5\ (500 - 0{\cdot}42 \times 99{\cdot}5) \times 10^{-6}$$
$$= 406{\cdot}2 \text{ kNm}.$$

Example 9-14.

In Example 9-13, if now the factored moment is increased to 470 kNm, design the reinforcement for flexure.

Solution:

Approximate lever arm

$$= d - \frac{D_f}{2}$$
$$= 500 - 50$$
$$= 450 \text{ mm}.$$

Approximate

$$A_{st} = \frac{470 \times 10^6}{0{\cdot}87 \times 415 \times 450}$$
$$= 2893 \text{ mm}^2.$$

Provide 6-25 Φ ; $A_{st} = 6 \times 491 = 2946$ mm².

To find neutral axis

$$F_{tc} = 0{\cdot}36 f_{ck}\ b_f D_f$$
$$= 0{\cdot}36 \times 15 \times 1650 \times 100 \times 10^{-3}$$
$$= 891\ \text{kN}.$$
$$F_t = 0{\cdot}87 f_y\ A_{st}$$
$$= 0{\cdot}87 \times 415 \times 2946 \times 10^{-3}$$
$$= 1063{\cdot}6\ \text{kN}.$$

$F_{tc} < F_{ts}$

$\therefore$ Neutral axis lies in web.

Assume $D_f > \dfrac{3}{7}\ x_u$

$$y_f = 0{\cdot}15 x_u + 0{\cdot}65 D_f$$
$$= 0{\cdot}15 x_u + 0{\cdot}65 \times 100$$
$$= 0{\cdot}15 x_u + 65.$$

Now total compression

$$= 0{\cdot}36 f_{ck}\ b_w x_u + 0{\cdot}446 f_{ck}\ (b_f - b_w) y_f$$
$$= 0{\cdot}36 \times 15 \times 250 x_u + 0{\cdot}446 \times 15 (1650 - 250)(0{\cdot}15 x_u + 65)$$
$$= 1350 x_u + 1405 x_u + 608790$$
$$= 2755 x_u + 608790.$$

Total tension

$$= 0{\cdot}87 f_y\ A_{st}$$
$$= 0{\cdot}87 \times 415 \times 2946$$
$$= 1063653.$$

Equating

$$2755 x_u + 608790 = 1063653$$

$\therefore$ $x_u = 165$ mm

$\dfrac{3}{7} x_u = 70{\cdot}7$ mm $< D_f$.......................... (O.K.)

Now $x_u < x_{u,\ max}$.

$\therefore$ Section is under-reinforced.

$$y_f = 0{\cdot}15 \times 165 + 65$$
$$= 89{\cdot}75\ \text{mm}.$$

$$M_u = 0{\cdot}36 f_{ck}\, b_w x_u\, (d - 0{\cdot}42 x_u) + 0{\cdot}446 f_{ck}\, (b_f - b_w)\, y_f\, (d - \frac{y_f}{2})$$

$$= 0{\cdot}36 \times 15 \times 250 \times 165 (500 - 0{\cdot}42 \times 165) \times 10^{-6} +$$

$$0{\cdot}446 \times 15\, (1650 - 250) \times 89{\cdot}75\, (500 - \frac{89{\cdot}75}{2}) \times 10^{-6}$$

$$= 95{\cdot}94 + 382{\cdot}58$$

$$= 478{\cdot}52 \text{ kNm} > 470 \text{ kNm} \ldots\ldots\ldots\ldots\ldots\ldots \text{(O.K.)}$$

Example 9-15.

In Ex. 9-14, now the factored moment is increased to 540 kNm. Design the reinforcement for flexure. Also find out the limiting moment of resistance of the section.

Solution:

Approximate lever arm

$$= d - \frac{D_f}{2}$$

$$= 500 - 50$$

$$= 450 \text{ mm.}$$

Approximate

$$A_{st} = \frac{540 \times 10^6}{0{\cdot}87 \times 415 \times 450}$$

$$= 3323 \text{ mm}^2.$$

Provide 7-25 Φ = 3437 mm².

It can be seen from above examples that neutral axis lies in web.

Assume $D_f > \frac{3}{7}\, x_u$.

From Ex. 9-14

total compression $= 2755 x_u + 608790$

total tension $= 0{\cdot}87 \times 415 \times 3437$

$= 1240929$ N.

Equating

$$2755x_u + 608790 = 1240929$$

$$x_u = 229{\cdot}45 \text{ mm}$$

$$x_{u,\,max} = 240 \text{ mm.}$$

$\therefore$ Section is under-reinforced.

Also $\frac{3}{7}x_u = \frac{3}{7} \times 229{\cdot}45 = 98{\cdot}34$ mm $< D_f$..........(O.K.)

Now

$$M_u = 0{\cdot}36 f_{ck}\, b_w x_u\, (d - 0{\cdot}42 x_u) + 0{\cdot}446 f_{ck}\, (b_f - b_w) \times y_f\, (d - \frac{y_f}{2})$$

$$y_f = 0{\cdot}15 x_u + 65$$
$$= 0{\cdot}15 \times 229{\cdot}45 + 65$$
$$= 99{\cdot}42 \text{ mm.}$$

$$M_u = 0{\cdot}36 \times 15 \times 250 \times 229{\cdot}45\,(500 - 0{\cdot}42 \times 229{\cdot}45) \times 10^{-6} + 0{\cdot}446 \times 15\ (1650 - 250) \times 99{\cdot}42 \times (500 - \frac{99{\cdot}42}{2}) \times 10^{-6}$$

$$= 125{\cdot}02 + 419{\cdot}3$$

$= 544{\cdot}32$ kNm > 540 kNm...............(O.K.)

Limiting moment of resistance:

$$\frac{D_f}{d} = \frac{100}{500} = 0{\cdot}2.$$

Using equation (9-17*c*)

$$M_{u,\,lim,T} = 0{\cdot}36 f_{ck}\, b_w x_{u,\,max}\, (d - 0{\cdot}42 x_{u,\,max}) + (b_f - b_w) \times D_f \times 0{\cdot}446 f_{ck}\ (d - \frac{D_f}{2})$$

$$= 0{\cdot}36 \times 15 \times 250 \times 240\,(500 - 0{\cdot}42 \times 240) \times 10^{-6} + 1400 \times 100 \times 0{\cdot}446 \times 15\ (500 - 50) \times 10^{-6}$$

$$= 129{\cdot}34 + 421{\cdot}47$$

$$= 550{\cdot}81 \text{ kNm.}$$

SHEAR

9-14. Introductory: The behaviour of a concrete beam in shear and design for shear using elastic theory was discussed in chapter 3, where the limit state behaviour of concrete in shear was used for design. In this chapter we shall design the beam in shear using limit state method. The procedure of design is similar to that using elastic theory, though the shear strength of concrete, maximum shear stress etc. will be different.

9-15. Nominal shear stress: The nominal shear stress τ_v in beams of uniform depth shall be obtained by the following equation:

$$\tau_v = \frac{V_u}{bd} \qquad \text{(9-22)}$$

where

V_u = factored shear force due to design loads

b = breadth of the member, which for flanged sections shall be taken as the breadth of the web, b_w

d = effective depth.

9-16. Design shear strength of concrete: The design shear strength of concrete shall be taken as follows:

(a) *Without shear reinforcement:* The design shear strength of concrete in beams without shear reinforcement is given in table 9-6.

For solid slabs, the design shear strength for concrete shall be $k\ \tau_c$ where k has the values given below:

Overall depth of slab, mm	300 or more	275	250	225	200	175	150 or less
k	1·00	1·05	1·10	1·15	1·20	1·25	1·30

(b) *With shear reinforcement:* Under no circumstances, even with shear reinforcement shall the nominal shear stress in beams τ_v exceed $\tau_{c\ max}$ given in table 9-7. For solid slabs, the nominal shear stress shall not exceed half the appropriate values given in table 9-7.

TABLE 9-6

DESIGN SHEAR STRENGTH OF CONCRETE τ_c, N/mm²

$\frac{100 A_s}{bd}$	Concrete grade					
	M15	M20	M25	M30	M35	M40
(1)	(2)	(3)	(4)	(5)	(6)	(7)
0·25	0·35	0·36	0·36	0·37	0·37	0·38
0·50	0·46	0·48	0·49	0·50	0·50	0·51
0·75	0·54	0·56	0·57	0·59	0·59	0·60
1·00	0·60	0·62	0·64	0·66	0·67	0·68
1·25	0·64	0·67	0·70	0·71	0·73	0·74
1·50	0·68	0·72	0·74	0·76	0·78	0·79
1·75	0·71	0·75	0·78	0·80	0.82	0·84
2·00	0·71	0·79	0·82	0·84	0·86	0·88
2·25	0·71	0·81	0·85	0·88	0·90	0·92
2·50	0·71	0·82	0·88	0·91	0·93	0·95
2·75	0·71	0·82	0·90	0·94	0·96	0·98
3.00	0·71	0·82	0·92	0·96	0·99	1·01

Note: The term A_s is the area of longitudinal tension reinforcement which continues at least one effective depth beyond the section being considered except at supports where the full area of tension reinforcement may be used provided the detailing conforms code requirements.

TABLE 9-7

MAXIMUM SHEAR STRESS, $\tau_{c\ max}$, N/mm²

Concrete grade	M15	M20	M25	M30	M35	M40
$\tau_{c\ max}$, N/mm²	2·5	2·8	3·1	3·5	3·7	4·0

9-17. Design of shear reinforcement: When τ_v exceeds τ_c given in table 9-6, shear reinforcement shall be provided in any of the following forms:

(a) Vertical stirrups
(b) Bent-up bars along with stirrups
(c) Inclined stirrups.

Where bent-up bars are provided, their contribution towards shear resistance shall not be more than half that of the total shear reinforcement.

Shear reinforcement shall be provided to carry a shear equal to $V_u - \tau_c\, bd$. The strength of shear reinforcement V_{us} shall be calculated as below:

(a) For vertical stirrups:

$$V_{us} = \frac{0{\cdot}87 f_y \, A_{sv} \, d}{s_v} \quad \text{.....................} \quad (9\text{-}23a)$$

(b) For inclined stirrups or a series of bars bent-up at different cross-sections:

$$V_{us} = \frac{0{\cdot}87 f_y \, A_{sv} \, d}{s_v} (\sin \alpha + \cos \alpha) \quad \text{.............} \quad (9\text{-}23b)$$

(c) For single bar or single group of parallel bars, all bent-up at the same cross-section

$$V_{us} = 0{\cdot}87 \, f_y \, A_{sv} \sin\alpha \quad \text{.....................} \quad (9\text{-}23c)$$

where

A_{sv} = total cross-section area of stirrup legs or bent-up bars within a distance s_v

s_v = spacing of the stirrups or bent-up bars along the length of the member

τ_v = nominal shear stress

τ_c = design shear strength of the concrete

b = breadth of the member which for flanged beams shall be taken as the breadth of the web b_w

f_y = characteristic strength of the stirrup or bent-up reinforcement which shall not be taken greater than 415 N/mm²

α = angle between the inclined stirrup or bent-up bar and the axis of the member, not less than 45°

d = effective depth.

Note 1: Where more than one type of shear reinforcement is used to reinforce the same portion of the beam, the total shear resistance shall be computed as the sum of the resistances for the various types separately.

Note 2: While designing the shear reinforcement, the practical considerations as discussed in art. 3-7 shall be observed.

Example 9-16.

A simply supported tee beam 230 mm wide × 460 mm effective depth is reinforced with 5 no. 16 mm diameter bars as tension reinforcement. The beam is subjected to a factored shear of 52·5 kN at support. Check the shear stresses and design the shear reinforcement at support. Assume that ends of reinforcement are not confined with compressive reaction. The materials are M15 grade concrete and mild steel reinforcement.

Solution :

At support

$$V_u = 52{\cdot}5 \text{ kN}$$

$$A_{st} = 5 \times 201 = 1005 \text{ mm}^2,\ b = 230 \text{ mm},\ d = 460 \text{ mm}.$$

Nominal shear stress

$$\tau_v = \frac{52{\cdot}5 \times 10^3}{230 \times 460} = 0{\cdot}496 \text{ N/mm}^2.$$

$$\frac{100\ A_s}{bd} = \frac{100 \times 1005}{230 \times 460} = 0{\cdot}95.$$

From table 9-6, $\tau_c = 0{\cdot}588$ N/mm².

Now, $\tau_v < \tau_c$, therefore only nominal shear reinforcement is required.

Select 6 mm diameter M.S. bars for stirrups.

$A_{sv} = 2 \times 28 = 56$ mm² for two-legged stirrups.

For minimum stirrups

$$s_v \leqslant \frac{A_{sv}\, f_y}{0{\cdot}4 b}$$

$$\leqslant \frac{56 \times 250}{0{\cdot}4 \times 230}$$

$$\leqslant 152 \text{ mm}.$$

The spacing shall be lesser of

(a) $0{\cdot}75d = 0{\cdot}75 \times 460 = 345$ mm

(b) 450 mm

(c) 152 mm as calculated above.

Provide 6 mm ϕ two-legged stirrups about 150 mm c/c.

Example 9-17.

If the shear of above section is increased to 90 kN, check the shear stresses and find the spacing of 6 mm diameter stirrups at the support.

Solution:

$$V_u = 90 \text{ kN}$$
$$b = 230 \text{ mm}$$
$$d = 460 \text{ mm}$$

Shear stress $\tau_v = \dfrac{90 \times 10^3}{230 \times 460}$

$= 0{\cdot}85 \text{ N/mm}^2 < 2{\cdot}5 \text{ N/mm}^2$ where $2{\cdot}5 \text{ N/mm}^2$ is the maximum permissible shear stress for M15 mix.

$\tau_c = 0{\cdot}588 \text{ N/mm}^2$ as calculated in Ex. 9-16.

Now, $\tau_v > \tau_c$, therefore shear reinforcement shall be designed.

Shear resistance of concrete

$$\tau_c\, bd = 0{\cdot}588 \times 230 \times 460 \times 10^{-3}$$
$$= 62{\cdot}2 \text{ kN}.$$

Shear to be resisted by stirrups

$$V_{us} = 90 - 62{\cdot}2 = 27{\cdot}8 \text{ kN}.$$

Using 6 mm dia. two-legged stirrups

$$s_v = \frac{0{\cdot}87 f_y\, A_{sv}\, d}{V_{us}}$$
$$= \frac{0{\cdot}87 \times 250 \times 56 \times 460}{27{\cdot}8 \times 10^3}$$
$$= 201{\cdot}5 \text{ mm}.$$

The spacing shall be lesser of

(a) $0{\cdot}75d = 0{\cdot}75 \times 460 = 345$ mm

(b) 450 mm

(c) minimum shear reinforcement required i.e. 152 mm as shown in Ex. 9-16.

(d) required spacing = 201·5 mm.

Provide 6 mm ϕ two-legged stirrups about 150 mm c/c.

Example 9-18.

In Ex. 9-17, if 2 no. 16 mm diameter bars are bent up at 45° and shear is increased to 120 kN, find the spacing of 6 mm diameter stirrups at support.

Solution:

2 bars bent up $A_{sv} = 2 \times 201 = 402 \text{ mm}^2$.

Shear resistance $= 402 \times 0{\cdot}87 \times 250 \times \sin 45° \times 10^{-3}$
$= 61{\cdot}82$ kN.

For the remaining 3 bars

$$\frac{100\ A_s}{bd} = \frac{100 \times 603}{230 \times 460} = 0{\cdot}57$$

$$\tau_c = 0{\cdot}482 \text{ N/mm}^2.$$

Shear resistance of concrete

$$\tau_c bd = 0{\cdot}482 \times 230 \times 460 \times 10^{-3} = 51 \text{ kN}.$$

Shear resistance to be provided by shear reinforcement

$$V_{us} = 120 - 51 = 69 \text{ kN}.$$

Bent bars provide 50% = 34·5 kN< 61·82 kN .....(O.K.)

Stirrups provide 50% = 34·5 kN.

Using 6 mm ϕ two-legged stirrups

$$s_v = \frac{56 \times 0{\cdot}87 \times 250 \times 460}{34{\cdot}5 \times 10^3}$$

$$= 162{\cdot}4 \text{ mm}.$$

The spacing shall be lesser of

(a) $0{\cdot}75d = 0{\cdot}75 \times 460 = 345$ mm

(b) 450 mm

(c) 152 mm (minimum shear reinforcement)

(d) 162·4 mm (required).

Provide 6 mm ϕ two-legged stirrups about 150 mm c/c.

Note: Compare examples 9-16, 9-17 and 9-18 with examples 3-1, 3-2 and 3-3. Also observe the comments made in examples 3-1, 3-2 and 3-3.

BOND AND DEVELOPMENT LENGTH

9-18. Development of stress in reinforcement: IS : 456 states, "The calculated tension or compression in any bar at any section shall be developed on each side of the section by an appropriate development length or end anchorage or by a combination thereof".

Sufficient discussion on development length of bar is given in chapter 3, based on working stress method. In limit state method, the design bond stress will be different and the comments made in chapter 3 are applicable in limit state method also.

The development length L_d is given by

$$L_d = \frac{\phi\, \sigma_s}{4\, \tau_{bd}} \qquad \text{(9-24)}$$

where

ϕ = nominal diameter of the bar

σ_s = stress in bar at the section considered at design load

τ_{bd} = design bond stress as given below.

Note 1: The development length includes anchorage values of hooks in tension reinforcement.

Note 2: For the bars of sections other than circular, the development length should be sufficient to develop the stress in the bar by bond.

Design bond stress in limit state method for plain bars in tension shall be as below:

Grade of concrete	M15	M20	M25	M30	M35	M40
Design bond stress, τ_{bd}, N/mm²	1·0	1·2	1·4	1·5	1·7	1·9

For deformed bars confirming to IS : 1786-1979 or IS : 1139-1966, these values shall be increased by 60 per cent.

For bars in compression, the values of bond stress for bars in tension shall be increased by 25 per cent.

Example 9-19.

Calculate the anchorage length in tension and compression for a single mild steel bar of diameter ϕ in concrete grade of M15.

Solution:

(1) Tension:

$$\text{Design stress for M.S. } \sigma_s = 0{\cdot}87 f_y = 0{\cdot}87 \times 250 = 217{\cdot}5 \text{ N/mm}^2$$

$$\tau_{bd} = 1{\cdot}0 \text{ N/mm}^2.$$

$$\text{Anchorage length} = \text{development length} = \frac{\phi \times 0{\cdot}87 f_y}{4\tau_{bd}} = \frac{\phi \times 217{\cdot}5}{4 \times 1} = 54{\cdot}4\,\phi.$$

(2) Compression:

$$\text{Design stress for M.S. } \sigma_s = 0{\cdot}87 f_y = 217{\cdot}5 \text{ N/mm}^2$$

$$\tau_{bd} = 1{\cdot}0 \times 1{\cdot}25 \text{ (for compression)} = 1{\cdot}25 \text{ N/mm}^2$$

$$L_d = \frac{\phi \times 217{\cdot}5}{4 \times 1{\cdot}25} = 43{\cdot}5\,\phi.$$

Development length for different types of bars in different grades of concrete is tabulated in table 9-8.

TABLE 9-8

DEVELOPMENT LENGTH FOR SINGLE BARS

f_y N/mm²	Tension bars M15	Tension bars M20	Compression bars M15	Compression bars M20
250	55 ϕ	46 ϕ	44 ϕ	37 ϕ
415	56 ϕ	47 ϕ	45 ϕ	38 ϕ
500	69 ϕ	58 ϕ	54 ϕ	46 ϕ

DEFLECTION AND CRACKING

9-19. General: In chapter 4, deflection and cracking of reinforced concrete member has been discussed. In limit state theory also, these criteria remain unchanged. Thus, while designing a beam or slab using limit state method, the rules discussed in chapter 4 shall be observed. The, calculations of deflection and crack width is outside the scope of this book and specialist literature may be referred.

TORSION

9-20. Code provisions:

General: According to IS : 456, using limit state method, the design for torsion shall be made as follows:

In general, where the torsional resistance or stiffness of members has not been taken into account in the analysis of structure, no specific calculations for torsion will be necessary. Adequate control of any torsional cracking being provided by the required nominal shear reinforcement. Where the torsional resistance or stiffness of members is taken into account in the analysis, the members shall be designed for torsion.

The approach to design for torsion is as follows:

Torsional reinforcement is not calculated separately from that required for bending and shear. Instead the total longitudinal reinforcement is determined for a fictitious bending moment which is a function of actual bending moment and torsion. Similarly web reinforcement is determined for a fictitious shear which is a function of actual sheaı and torsion.

The following design rules shall apply to beams of solid rectangular cross-section. However, these may also be applied to flanged beams by substituting b_w for b in which case they are generally conservative; therefore specialist literature may be referred.

For design of torsion, sections located less than a distance d, from the face of the support may be designed for the same torsion as computed at a distance d, where d is the effective depth.

Design rules: The design rules for torsion as indicatde above are based on equivalent shear and equivalent moment and are explained below:

(*a*) *Shear and torsion:*

Equivalent shear: Equivalent shear, V_e shall be calculated from the formula:

$$V_e = V_u + 1{\cdot}6\ \frac{T_u}{b} \qquad \text{(9-25)}$$

where

V_e = equivalent shear

V_u = shear

T_u = torsional moment

b = breadth of beam.

The equivalent nominal shear stress τ_{ve} is given by

$$\tau_{ve} = \frac{V_e}{bd} \qquad \text{(9-26)}$$

The equivalent nominal shear stress τ_{ve} shall not exceed the values of $\tau_{c\,max}$ as given in table 9-7.

If the equivalent nominal shear stress, τ_{ve} does not exceed τ_c, given in table 9-6, minimum shear reinforcement shall be provided as explained in art. 3-7. However, if τ_{ve} exceeds τ_c given in table 9-6, both longitudinal and transverse reinforcement shall be provided as explained below.

(*b*) *Longitudinal reinforcement:*

The longitudinal reinforcement shall be designed to resist an equivalent bending moment, M_{e1}, given by

$$M_{e1} = M_u + M_t$$

where

M_u = bending moment at the cross-secticn

and
$$M_t = T_u\left(\frac{1 + D/b}{1{\cdot}7}\right) \qquad \text{(9-27)}$$

where

T_u = torsional moment

D = overall depth of the beam

b = breadth of the beam.

If the numerical value of M_t as defined above exceeds the numerical value of the moment M_u, longitudinal reinforcement shall be provided on the flexural compression face, such that $M_{e2} = M_t - M_u$, the moment M_{e2} being taken as acting in the opposite sense to the moment M_u.

(c) *Transverse reinforcement:*

Two-legged closed hoops enclosing the corner longitudinal bars shall have an area of cross-section A_{sv}, given by

$$A_{sv} = \frac{T_u\, s_v}{b_1 d_1\ (0{\cdot}87 f_y)} + \frac{V_u\, s_v}{2{\cdot}5 d_1\ (0{\cdot}87 f_y)} \qquad \text{(9-28)}$$

But the total transverse reinforcement shall not be less than

$$\frac{(\tau_{ve} - \tau_c)\, b s_v}{0{\cdot}87 f_y}$$

where

T_u = torsional moment

V_u = shear force

s_v = spacing of the stirrup reinforcement

b_1 = centre to centre distance between corner bars in the direction of the width

d_1 = centre to centre distance between corner bars in the direction of the depth

b = breadth of the member

f_y = characteristic strength of the stirrup reinforcement

τ_{ve} = equivalent shear stress as calculated with equation 9-26

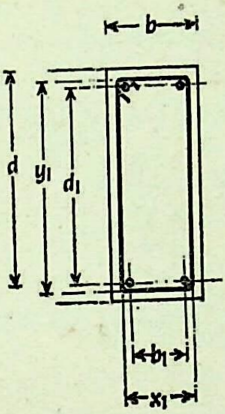
τ_c = shear strength of the concrete as per table 9-6.

(d) *Distribution of torsion reinforcement:*

When a member is designed for torsion, torsion reinforcement shall be provided as below:

(1) The transverse reinforcement for torsion shall be rectangular closed stirrups placed perpendicular to the axis of the member. The spacing of stirrups shall not exceed the least of x_1, $\dfrac{x_1 + y_1}{4}$ and 300 mm, where x_1 and y_1 are

respectively the short and long dimensions of the stirrup. Refer fig. 9-12.



Details of torsion reinforcement
FIG. 9-12

(2) Longitudinal reinforcement shall be placed as close as practicable to the corners of the cross-section and in all cases, there shall be at least one longitudinal bar in each corner of the ties. When the cross-sectional dimension of the member exceeds 450 mm, additional longitudinal bars shall be provided along the two faces. The total area of such reinforcement shall be not less than 0·1 per cent of the web area and shall be distributed equally on two faces at a spacing not exceeding 300 mm or web thickness whichever is less.

Example 9-20.

A rectangular beam of size 230 mm wide × 600 mm overall depth is subjected to a factored sagging bending moment of 48 kNm; factored shear force of 48 kN and a factored torsional moment of 18 kNm. Design the reinforcement at the section. The materials are M15 grade concrete and mild steel reinforcement.

Solution:

$$M_u = 48 \text{ kNm}$$
$$V_u = 48 \text{ kN}$$
$$T_u = 18 \text{ kNm}$$
$$b = 230 \text{ mm}$$
$$d = 600 \text{ mm}$$

Assuming 25 mm cover and 20 mm diameter bars in one layer, $d = 600 - 25 - 10 = 565$ mm.

Equivalent shear $V_e = V_u + 1{\cdot}6 \dfrac{T_u}{b}$

$$= 48 + 1{\cdot}6 \times \frac{18}{0{\cdot}23}$$

$$= 48 + 125{\cdot}2$$

$$= 173{\cdot}2 \text{ kN.}$$

Equivalent shear stress

$$\tau_{ve} = \frac{V_e}{bd} = \frac{173{\cdot}2 \times 10^3}{230 \times 565} = 1{\cdot}33 \text{ N/mm}^2.$$

For M15 mix from table 9-7

$$\tau_{c\,max} = 2{\cdot}5 \text{ N/mm}^2$$

$\tau_{ve} < \tau_{c\,max}$ ....(O.K.)

Assuming tension reinforcement $= 0{\cdot}5\%$

$$\tau_c = 0{\cdot}46 \text{ N/mm}^2 < \tau_{ve}.$$

Thus, design of torsion is necessary.

Longitudinal reinforcements:

Equivalent bending moment

$$M_{e1} = M_u + M_t$$

$$= M_u + T_u \left(\frac{1 + D/b}{1{\cdot}7}\right)$$

$$= 48 + 18 \left(\frac{1 + 60/23}{1{\cdot}7}\right)$$

$$= 48 + 38{\cdot}2$$

$$= 86{\cdot}2 \text{ kNm.}$$

Since $M_u > M_t$, no reversal of moment is considered and therefore steel on compression side is not required.

Now

$$M_{e1} = 86{\cdot}2 \text{ kNm}$$

$$d_{required} = \sqrt{\frac{86{\cdot}2 \times 10^6}{2{\cdot}22 \times 230}}$$

$= 410$ mm < 565 mm.................(O.K.)

To find steel area

$$p_t = 50\left[\frac{1-\sqrt{1-\frac{4\cdot6}{f_{ck}}\times\frac{M_{e1}}{bd^2}}}{f_y/f_{ck}}\right]$$

$$\frac{M_{e1}}{bd^2} = \frac{86\cdot2\times10^6}{230\times565\times565}$$

$$= 1\cdot17$$

$$\therefore \quad p_t = 50\left[\frac{1-\sqrt{1-\frac{4\cdot6}{15}\times1\cdot17}}{250/15}\right]$$

$$= 0\cdot6$$

$$A_{st} = \frac{0\cdot6}{100}\times230\times565$$

$$= 780 \text{ mm}^2.$$

Provide 4 no. 16 mm diameter bars = 804 mm².

At top provide 2 no. 12 mm diameter anchor bars.

As the depth of beam is more than 450 mm, side reinforcement has to be provided.

Minimum area on each face

$$= \frac{1}{2}\times\frac{0\cdot1}{100}\times230\times600$$

$$= 69 \text{ mm}^2.$$

However, use 1-12 ϕ on each face at centre of web.

Spacing of bars $= \frac{530}{2} = 265$ mm.

Spacing shall not exceed

(1) 300 mm

(2) web thickness = 230 mm.

Second criteria is not satisfied. Therefore, provide 4-12 ϕ side face reinforcement as shown in fig. 9-13.

Now spacing $= \frac{530}{3} = 176\cdot6$ mm $<$ 230 mm...... (O.K.)

Transverse reinforcements:

Assuming 10 mm ϕ two-legged stirrups

$A_{sv} = 2 \times 78{\cdot}5 = 157\ \text{mm}^2.$

$$A_{sv} = \frac{T_u\ s_v}{b_1 d_1 (0{\cdot}87 f_y)} + \frac{V_u\ s_v}{2{\cdot}5 d_1 (0{\cdot}87 f_y)}$$

Substituting, referring fig. 9-13

$$157 = \frac{18 \times 10^6\ s_v}{168 \times 534 \times 0{\cdot}87 \times 250} + \frac{48 \times 10^3\ s_v}{2{\cdot}5 \times 534 \times 0{\cdot}87 \times 250}$$

$$= (0{\cdot}922 + 0{\cdot}165)\ s_v$$

$$= 1{\cdot}087\ s_v$$

$\therefore \quad s_v = 144{\cdot}4$ mm. . . . . (1)

Also $\quad A_{sv} \nless \dfrac{(\tau_{ve} - \tau_c)\ b.s_v}{0{\cdot}87 f_y}$

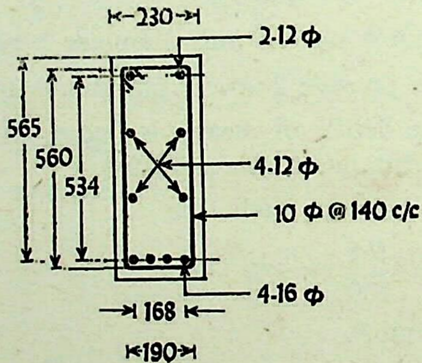


Fig. 9-13

$$A_s = A_{st} = 804\ \text{mm}^2$$

$$\frac{100 A_s}{bd} = \frac{100 \times 804}{230 \times 565}$$

$$= 0{\cdot}62$$

$$\tau_c = 0{\cdot}5\ \text{N/mm}^2.$$

Substituting

$$157 \nless \frac{(1{\cdot}33 - 0{\cdot}62) \times 230 s_v}{0{\cdot}87 \times 250}$$

which gives $s_v < 209$ mm.......................... (2)

Now spacing should not exceed

(a) $x_1 = 190$ mm

(b) $\dfrac{x_1 + y_1}{4} = \dfrac{190 + 560}{4}$

$= 187 \cdot 5$ mm

(c) 300 mm.

i.e. $s_v \not> 187 \cdot 5$ mm.......................... (3)

From (1), (2) and (3), provide 10 mm ϕ two-legged stirrups about 140 mm c/c. The designed section is shown in fig. 9-13.

Note: Compare this example with Ex. 4-3.

AXIALLY LOADED COLUMNS

9-21. Assumptions: In addition to the assumptions given in art. 9-5 for flexure, the following shall be assumed:

(1) The maximum compressive strain in concrete in axial compression is taken as 0·002.

(2) The maximum compressive strain at the highly compressed extreme fibre in concrete subjected to axial compression and bending and when there is no tension on the section shall be 0·0035 minus 0·75 times the strain at the least compressed extreme fibre.

9-22. Minimum eccentricity: All the members in compression shall be designed for the minimum eccentricity equal to the unsupported length of column/500 plus lateral dimension/30, subject to a minimum of 20 mm. Where the calculated eccentricity is larger, the minimum eccentricity should be ignored.

9-23. Short axially loaded columns: Axially loaded short columns shall be designed considering the above assumptions and minimum eccentricity. When the minimum eccentricity does not exceed 0·05 times the lateral dimension, the members may be designed by the following equation:

$$P_u = 0{\cdot}4 f_{ck} \,.\, A_c + 0{\cdot}67 f_y \,.\, A_{sc} \qquad \text{(9-29)}$$

where

P_u = axial load on the member

f_{ck} = characteristic compressive strength of the concrete

A_c = area of concrete

f_y = characteristic strength of the compression reinforcement

A_{sc} = area of longitudinal reinforcement for columns.

In the design of columns subjected to axial load and moment, IS : 456 gives a formula for the capacity of section for pure axial load P_{uz} in clause 38.6 of IS: 456

where

$$P_{uz} = 0{\cdot}45 f_{ck} \,.\, A_c + 0{\cdot}75 f_y \, A_{sc} \qquad \text{(9-30)}$$

The comparison of equations 9-29 and 9-30 shows that equation 9-29 is simplified for the eccentric load with eccentricity less than 0·05 times the lateral dimension by reducing the load carrying capacity of column by about 10%.

Example 9-21.

A short column 230 mm × 230 mm is reinforced with 4 no. 16 mm diameter bars. Find the ultimate load carrying capacity of the column, if the minimum eccentricity is less than 0·05 times the lateral dimension. The materials are M15 grade concrete and tor steel reinforcement of grade Fe 415.

Solution:

$$\begin{aligned}
A_{sc} &= 4 \times 201 = 804 \text{ mm}^2 \\
A_c &= 230 \times 230 - 804 = 52096 \text{ mm}^2 \\
P_u &= 0{\cdot}4 f_{ck} \,.\, A_c + 0{\cdot}67 f_y \, A_{sc} \\
&= 0{\cdot}4 \times 15 \times 52096 \times 10^{-3} + 0{\cdot}67 \times 415 \times 804 \times 10^{-3} \\
&= 312{\cdot}58 + 223{\cdot}55 \\
&= 536{\cdot}13 \text{ kN.}
\end{aligned}$$

Example 9-22.

A 230 mm × 350 mm size column has to carry a factored load of 1000 kN. The column is short and the minimum eccentricity

is less than 0·05 times the lateral dimension. Design the reinforcement. The materials are M20 grade concrete and tor steel reinforcement of grade Fe 415.

Solution:

$$A_c = 230 \times 350 - A_{sc}$$
$$= 80500 - A_{sc}.$$
$$P_u = 0{\cdot}4 f_{ck}\, A_c + 0{\cdot}67 f_y\, A_{sc}$$
$$1000 \times 10^3 = 0{\cdot}4 \times 20(80500 - A_{sc}) + 0{\cdot}67 \times 415 A_{sc}$$
$$= 644000 - 8A_{sc} + 278{\cdot}05 A_{sc}$$

which gives

$$270{\cdot}05\, A_{sc} = 356000$$
$$\therefore \quad A_{sc} = 1318{\cdot}2 \text{ mm}^2.$$

Provide 4-20 Φ + 2-16 Φ = 1658 mm^2.

Ties: Diameter shall not be less than 20/4 = 5. Use 6 mm ϕ M.S. ties.

Spacing should not exceed

(1) least lateral dimension = 230 mm

(2) 16 × 16 = 256 mm

(3) 48 × 6 = 288 mm.

Use 6 mm ϕ M.S. ties about 230 mm c/c.

EXAMPLES IX

Note: In data of the following examples, all values of moment shear etc. are factored one. Solve all the examples using limit state method.

(1) A rectangular beam 200 mm wide and 450 mm effective depth is reinforced with 3 no. 16 mm diameter bars. Find out the depth of neutral axis and specify the type of beam. The materials are M15 grade concrete and tor steel reinforcement of grade Fe 415. Also find out the depth of neutral axis if the reinforcement is increased to 4 no. 16 mm diameter bars.

(2) A singly reinforced rectangular beam 230 mm wide $\times$ 450 mm effective depth is reinforced with 3 no. 25 mm diameter bars. Find out the ultimate moment of resistance of the section. The materials are M15 grade concrete and mild steel reinforcement. Also find out the moment of resistance if the materials are M15 grade concrete and tor steel reinforcement of grade Fe 415.

(3) A singly reinforced rectangular beam is subjected to a factored bending moment of 75 kNm. Design the beam for flexure. The materials are M15 grade concrete and mild steel reinforcement. With the same size of beam, design for flexure if the tor steel reinforcement of grade Fe 415 is used.

(4) Find the moment of resistance of a beam section 230 mm wide $\times$ 500 mm effective depth reinforced with 2 no. 20 mm diameter bars as compression reinforcement at an effective cover of 40 mm and 4 no. 25 mm diameter bars as tension reinforcement. The materials are M15 grade concrete and mild steel reinforcement. Also find out the moment of resistance of the section if the tor steel reinforcement of grade Fe 415 is used.

(5) A rectangular beam of size 230 mm wide $\times$ 565 mm effective depth is subjected to a factored moment of 215 kNm. Find the reinforcement for flexure. The materials are M15 grade concrete and mild steel. Also find out the reinforcement for flexure, if the tor steel reinforcement of grade Fe 415 is used. Compare the economy for designed sections.

(6) A tee beam of effective flange width of 1800 mm, thickness of slab 120 mm, width of rib 230 mm and effective depth of 500 mm is reinforced with 4 no. 25 mm diameter bars. Calculate the ultimate moment of resistance. The materials are M15 grade concrete and tor steel reinforcement of grade Fe 415.

(7) Find out the ultimate moment of resistance for a section of Ex. (6), if the reinforcement is increased to (i) 5 no. 25 mm diameter bars (ii) 6 no. 25 mm diameter bars.

(8) A tee beam of effective flange width of 1800 mm, thickness of slab 120 mm, width of rib 230 mm and effective depth of 500 mm is subjected to a factored moment of 450 kNm. Design the reinforcement for flexure. The materials are M15 grade concrete and tor steel reinforcement of grade Fe 415. Also design the reinforcement for flexure if the moment is increased to 500 kNm. Find out the limiting value of moment of resistance of beam.

(9) A simply supported rectangular beam 230 mm wide × 500 mm effective depth is reinforced with 4 no. 20 mm diameter bars as tension reinforcement. The beam is subjected to a factored shear of 135 kN at support. Design the shear reinforcement at support. Assume that ends of reinforcement are not confined with compressive reaction. The materials are M15 grade concrete and mild steel reinforcement.

(10) A cantilever beam of 1·35 m span requires 2360 mm^2 area to carry the negative moment. If 4 no. 28 mm dia. bars are used as reinforcement, (a) is sufficient anchoragement available in the cantilever to anchor the bars properly? (b) if the bars are not properly anchored, determine the largest size bar that can be used as reinforcement. The materials are M15 grade concrete and tor steel reinforcement of grade Fe 415.

(11) A rectangular beam of size 300 mm wide × 600 mm effective depth is subjected to a factored hogging moment of 50 kNm, factored shear force of 50 kN and a factored torsional moment of 22 kNm. Design the reinforcement at the section. The materials are M15 grade concrete and tor steel reinforcement of grade Fe 415.

(12) A short column of size 230 mm × 300 mm is reinforced with 6 no. 16 mm diameter bars. Determine the safe factored load on column. The materials are M15 grade concrete and mild steel reinforcement.

(13) A short column of size 300 mm × 300 mm has to carry an axial load of 975 kN. Design the column using M15 grade concrete and tor steel reinforcement of grade Fe 415.

(14) Design a short circular column to carry a load of 1200 kN. The materials are grade M20 concrete and mild steel reinforcement.

APPENDIX A

Formwork for R.C.C. Members

A-1. General: The formwork shall be designed and constructed to the shapes, lines and dimensions shown on the drawings within the tolerances given below:

(a)	Deviation from specified dimensions of cross-section of columns and beams	− 6 mm + 12 mm
(b)	Deviation from dimensions of footings (see note):	
(1)	Dimensions in plan	− 12 mm + 50 mm
(2)	Eccentricity	0·02 times the width of the footing in the direction of deviation but not more than 50 mm
(3)	Thickness	± 0·05 times the specified thickness

Note: Tolerances apply to concrete dimensions only, not to positioning of vertical reinforcing steel or dowels.

A-2. Cleaning and Treatment of Forms: All rubbish, particularly chippings, shavings and sawdust, shall be removed from the interior of the forms before the concrete is placed and the formwork in contact with the concrete shall be cleaned and thoroughly wetted or treated with an approved composition. Care shall be taken that such approved composition is kept out of contact with the reinforcement.

A-3. Stripping time: Forms shall not be struck until the concrete has reached a strength at least twice the stress to which the concrete may be subjected at the time of removal of formwork. The strength referred to shall be that of concrete using the same cement and aggregates, with the same proportions and cured under conditions of temperature and moisture similar to those existing on the work. Where possible, the formwork shall be left longer as it would assist the curing.

Note 1: In normal circumstances and where ordinary Portland cement is used, forms may generally be removed after the expiry of the following periods:

(a)	Walls, columns and vertical faces of all structural members	24 to 48 hours as may be decided by the engineer-in-charge
(b)	Slabs (props left under)	3 days
(c)	Beam soffits (props left under)	7 days
(d)	Removal of props under slabs:	
	(1) Spanning upto 4·5 m	7 days
	(2) Spanning over 4·5 m	14 days
(e)	Removal of props under beams and arches:	
	(1) Spanning upto 6 m	14 days
	(2) Spanning over 6 m	21 days

For other cements, the stripping time recommended for ordinary Portland cement may be suitably modified.

Note 2: The number of props left under, their sizes and disposition shall be such as to be able to safely carry the full dead load of the slab, beam or arch as the case may be together with any live load likely to occur during curing or further construction.

Where the shape of the element is such that the formwork has re-entrant angles, the formwork shall be removed as soon as possible after the concrete has set, to avoid shrinkage cracking occurring due to the restraint imposed.

APPENDIX B

Useful Tables

TABLE 1

PROPERTIES OF ROUND BARS USED AS REINFORCEMENT

Size	Area mm^2	Perimeter mm	Mass per metre kg	Remark
5	20	15·7	0·157	
6	28·2	18·8	0·222	
8	50	25·1	0·395	
10	78·5	31·4	0·617	Unit mass
12	113	37·7	0·888	of steel =
14	154	44	1·208	7850 kg/m^3
16	201	50·3	1·578	
18	254	56·5	2·000	
20	314	62·8	2·466	
22	380	69·1	2·980	
25	491	78·5	3·854	
28	616	88	4·830	
32	804	100·5	6·313	
36	1018	113	7·990	
40	1257	125·7	9·864	

91·0

804

TABLE 2
LIVE LOADS ON FLOORS

Loading class No. (1)	Types of floors (2)	Minimum live loads in N/m² of floor area (3)	Alternative minimum live load (4)
2000	Floors in dwelling houses, tenements, hospital wards, bed rooms and private sitting rooms in hostels, and dormitories	2000	Subject to a minimum total load of 2·5 times the values in column 3 for any given slab panel and 6 times the values in column 3 for any given beam This total load shall be assumed uniformly distributed on the entire area of the slab panel or the entire length of the beam
2500	Office floors other than entrance halls, floors of light workrooms	*2500-4000	
3000	Floors of banking halls, office entrance halls, and reading rooms	3000	
4000	Shop floors used for the display and sale of merchandise; floors of workrooms generally; floors of class-rooms in schools, floors or places of assembly with fixed seating, restaurants; circulation space in machinery halls, power stations, etc., where not occupied by plant or equipment	4000	
5000	Floors of warehouses, workshops, factories and other buildings or parts of buildings of similar category for light weight loads; office floors for storage and filing purposes; floors of places of assembly without fixed seating public rooms in hotels, dance halls, waiting halls, etc.	5000	
7500	Floors of warehouses, workshops, factories and other buildings or parts of buildings of similar category for medium-weight loads	7500	—
10000	Floors of warehouses, workshops, factories and other buildings or parts of buildings of similar category for heavy-weight loads, floors of book stores and libraries, roofs and pavement lignts over basements projecting under the public footpath	10000	—

TABLE 2 (Continued)

Garage, light	Floors used for garages for vehicles not exceeding 25 kN gross weight:		
	Slabs	4000	or the worst combination of actual wheel loads, whichever is greater
	Beams	2500	or the worst combination of actual wheel loads, whichever is greater
Garage, heavy	Floors used for garages for vehicles not exceeding 40 kN gross weight	7500	Subject to a minimum of one-and-a half times maximum wheel load but not less than 9000 N considered to be distributed over 75 cm square
Stairs	Stairs, landings and corridors for class 2000 loading but not liable to overcrowding	3000	Subject to a minimum of 1300 N concentrated load at the unsupported end of each step for stairs constructed out of structurally independent cantilever steps
	Stairs, landings and corridors for class 2000 loading but liable to overcrowding, and for all other classes	5000	
Balcony	Balconies not liable to overcrowding:		
	For class 2000 loading	3000	—
	For all other classes	5000	—
	Balconies liable to overcrowding	5000	—

*The lower value of 2500 N/m² should be taken where separate storage facilities are provided and the higher value of 4000 N/m² should be taken where such provisions are lacking.

Table 2 (Continued)

Note 1 : In the above table a reference to a 'floor' includes a reference to any part of that floor, and a reference to 'slabs' includes boarding and beams or ribs spaced not further apart than one metre between centres, and a reference to 'beams' means all other beams and ribs.

Note 2 : Under loading class No. 2500, the reference to 'light workrooms' envisages rooms in which some light machines (for example, sewing machines used by milliners or tailors) are operated without a central power-driven unit, that is, the machines are independently operated, either by hand or by small motors. Under loading class No. 4000, the reference to 'workrooms' generally envisages the installation of machines operated with a central power-driven unit, with the individual machines being belt driven.

Note 3 : 'Fixed seating' implies that the removal of the seating and the use of the space for other purposes is improbable. The maximum likely load in this case, is, therefore, closely controlled.

Note 4 : The loading in workshops, warehouses and factories varies considerably and so three loadings under the terms 'light', 'medium' and 'heavy' are introduced in order to allow for more economical designs but the terms have no special meaning in themselves other than the live load for which the relevant floor is designed. It is, however, important particularly in the case of heavy weight loads, to assess the actual loads to ensure that they are not in excess of 10000 N/m^2; in cases where they are in excess, the design shall be based on the actual loading.

Note 5 : The load classification for stairs, corridors, balconies and landings, provide for the fact that these often serve several occupancies and are used for transporting the furniture and goods.

TABLE 3

LIVE LOADS ON ROOFS

Sr. No. (1)	Type of roof (2)	Live load measured on plan (3)	Minimum live load measured on plan (4)
(i)	Flat, sloping or curved roof with slopes upto and including 10 degrees:		
	(a) Access provided	1500 N/m^2	3750 N uniformly distributed over any span of one metre width of the roof slab and 9000 N uniformly distributed over the span in the case of all beams
	(b) Access not provided, except for maintenance	750 N/m^2	1900 N uniformly distributed over any span of one metre width of the roof slab and 4500 N uniformly distributed over the span in the case of beams.
(ii)	Sloping roof with slope greater than 10 degrees	(a) For roof membrane, sheets or purlins — 750 N/m^2 less 20 N/m^2 for every degree increase in slope over 10 degrees	Subject to a minimum of 400 N/m^2
		(b) For members supporting the roof membrane and roof purlins, such as trusses, beams, girders, etc. — 2/3 of load in (a)	—
		(c) Loads in (a) and (b) do not include loads due to snow, rain, dust collection, etc. and the affects of such loads shall be appropriately considered.	—

TABLE 3 (Continued)

(iii)	Curved roofs with slope at springing greater than 10 degrees	$(750 - 3450\,\gamma^2)$ N/m^2 where $\gamma = \frac{h}{l}$ h = the height of the highest point of the structure measured from its springing; and l = chord width of the roof if singly curved and shorter of two sides, if doubly curved. Subject to a minimum of 400 N/m^2

Note : For special types of roofs with highly permeable and absorbent material, the contingency of roof material increasing in weight due to absorption of moisture shall be provided for.

TABLE 4

AREAS OF BARS IN SLABS (in mm²/m)

Spacing	Bar diameter in millimetres							
mm	6	8	10	12	14	16	18	20
50	565	1005	1571	2262	3079	4021	5089	6283
60	471	838	1309	1885	2566	3351	4241	5236
70	404	718	1122	1616	2199	2872	3635	4448
80	353	628	982	1414	1924	2513	3181	3927
90	314	558	873	1257	1710	2234	2827	3491
100	283	503	785	1131	1539	2011	2545	3142
110	257	457	714	1028	1399	1828	2313	2856
120	236	419	654	942	1283	1675	2121	2618
130	217	387	604	870	1184	1547	1957	2417
140	202	359	561	808	1100	1436	1818	2244
150	188	335	524	754	1026	1340	1696	2094
160	177	314	491	707	962	1257	1590	1963
170	166	296	462	665	905	1183	1497	1848
180	157	279	436	628	855	1117	1444	1745
190	149	265	413	595	810	1058	1339	1653
200	141	251	393	565	770	1005	1272	1571
210	135	239	374	539	733	957	1212	1496
220	128	228	357	514	700	914	1157	1428
230	123	218	341	492	669	874	1106	1366
240	118	209	327	471	641	838	1060	1309
250	113	201	314	452	616	804	1018	1257
260	109	193	302	435	592	773	979	1208
270	105	186	291	419	570	745	942	1164
280	101	179	280	404	550	718	909	1122
290	97	173	271	390	531	693	877	1083
300	94	168	262	377	513	670	848	1047
320	88	157	245	353	481	628	795	982
340	83	148	231	333	453	591	748	924
360	78	140	218	314	428	558	707	873
380	74	132	207	298	405	529	670	827
400	71	126	196	283	385	503	636	785

APPENDIX C

Objective Questions

Q-1. Define the characteristic strength of materials.

The characteristic strength of materials is defined as the strength of material below which not more than 5 per cent of the test results are expected to fall.

Q-2. What do you mean by M20 mix ?

M20 is the designation of concrete mix. Letter M refers to the mix and number 20 refers to the characteristic strength of 15 cm cube after 28 days equal to 20 N/mm^2.

Q-3. What is the increase in strength of concrete after six months ?

15 per cent.

Q-4. Why is the concrete cover to reinforcement required ?

The concrete cover to reinforcement is required (a) to protect the reinforcement from weather and fire, and (b) to ensure the grip of concrete over reinforcement so that they act as one and resist the loads.

Q-5. Why is the maximum and minimum limits on the spacing of bars specified ?

Maximum spacing of bars is specified to limit the width of crack in concrete while minimum spacing of bars is specified to allow the concrete to enter when poured or a vibrator can be immersed.

Q-6. How is the modular ratio defined ?

The modular ratio m is defined as

$$m = \frac{\text{modulus of elasticity of steel}}{\text{modulus of elasticity of concrete}} = \frac{E_s}{E_c}.$$

According to IS : 456, the modular ratio m has the value $\frac{280}{3\,\sigma_{cbc}}$ where σ_{cbc} is the permissible compressive stress due to bending in N/mm².

Q-7. What is a balanced design ?

In balanced design the section is so proportioned that the steel and concrete both reach their maximum permissible value of stresses at the same time. Thus at some value of loads, both the materials will fail at the same time.

Q-8. Define (a) effective depth (b) depth of neutral axis and (c) lever arm.

The *effective depth* is defined as the distance from extreme compression fibre to the centre of tensile reinforcement.

The *depth of neutral axis* is defined as the distance of neutral axis from extreme compressive fibre.

The *lever arm* is defined as the distance between centroid of compressive force to the centroid of tensile force.

Q-9. What is a transformed section ?

A transformed section is a section in which the steel area is replaced by the equivalent concrete area.

Q-10. Why is the over-reinforced design not preferred?

For the over-reinforced section, concrete fails first. Prior to failure concrete does not give notice as it does not yield. Thus over-reinforced structure may collapse without giving a notice when over-loaded. Therefore, over-reinforced design is not preferred.

Q-11. How does a concrete beam resist shear?

The shear in concrete beam is resisted by:

(1) Above neutral axis the shear resistance is provided by the uniform shear stress in uncracked concrete.

(2) Along the crack, the shear resistance is provided by the vertical component of force due to the inter-locking of aggregates.

(3) At the tensile reinforcements, shear is resisted by dowel action of the longitudinal bars.

Q-12. How does an increase in tension steel improve shear capacity of a concrete beam?

(1) When amount of tension steel increases, the depth of neutral axis increases and thus the depth of uncracked concrete increases. This increases the capacity of concrete in shear.

(2) When amount of tension steel increases, the cracks formed are smaller which improves the aggregate interlock. Also because of larger steel area, the dowel action is improved. This improves the capacity of section in shear.

Q-13. How is nominal shear stress defined?

The nominal shear stress τ_v in beams of uniform depth shall be obtained from the equation

$$\tau_v = \frac{V}{bd}$$

where

V = shear force due to design loads

b = width of member in case of rectangular beam and width of rib in case of flanged beam

d = effective depth.

Q-14. Why is the upper limit to shear strength (even with shear reinforcement) of concrete specified?

The upper limit (tables 3-2 and 9-7) is specified to prevent the failure of beam by diagonal compression.

Q-15. Why is the maximum spacing of vertical stirrups limited to $0{\cdot}75d$?

The horizontal distance between two successive cracks is approximately equal to effective depth d. The spacing of stirrup shall be such that it crosses the crack and also no crack shall remain unreinforced. To ensure this, the spacing of vertical stirrups is limited to $0{\cdot}75d$.

Q-16. Are the bent bars alone satisfactory as shear reinforcement ?

No. They should design to carry a maximum of 50% design shear.

Q-17. For the same percentage of steel, the shear resistance in slabs is more than the beam. Why?

The increased shear strength in slab is based on test results which show that thin plates would fail at loads corresponding to a higher nominal shear stress.

Q-18. What is a development length ?

A length of reinforcement embedded in concrete so that it can develop the stress by bond is termed as development length and is denoted by L_d.

Q-19. How is the development length calculated for single bar and bundled bars?

The development length for single bar is obtained from the formula

$$L_d = \frac{\phi \sigma_s}{4 \tau_{bd}}$$

where

ϕ = nominal diameter of the bar

σ_s = stress in bar at the section considered at design load

τ_{bd} = design bond stress.

For bundled bars, the development length of each bar of bundled bars shall be that for the individual bar, increased by 10% for two bars in contact, 20% for three bars in contact and 33% for four bars in contact.

Q-20. What are the limits of deflection of a structure or a structural member?

The deflection of a structure or a structural member due to dead loads only should not exceed span/350 or 20 mm whichever is less. For $DL + LL$ and other total effects such as temperature, shrinkage, creep etc. it shall not exceed span/250.

Q-21. Larger the tension reinforcement area, lesser is the modification factor while calculating permissible span/depth ratio. Why?

This is because (1) when the area of steel reinforcement increases, the neutral axis shifts towards the tension steel. Thus the area of concrete in compression zone increases which leads to a larger deflection due to creep, and (2) the smaller area of concrete in tension zone reduces the stiffness of the beam.

Q-22. What is the function of distribution bars in slabs?

The function of distribution reinforcement in slabs is:

(1) To distribute uniformly the concentrated loads on the slab.

(2) To resist stresses due to shrinkage and temperature.

Q-23. What is a camber?

Camber is a term applied to the slight upward curve of a beam made in construction such that on loading (usually dead load) it will straighten out and attain its correct shape.

Q-24. What is a kicker?

To provide an exact alignment to the upper column, starter or kicker is casted above the footing or above the slab. Thickness of kicker is usually 8 to 10 cm and casted in a rich mix than lower column or footing. The bars should be lapped above the kicker.

Q-25. What is called a Limit state?

The acceptable limit for the safety and serviceability requirement of a structure or structural element before failure occurs is called a 'Limit state'.

Q-26. What is a characteristic load?

The value of load which has a 95 per cent probability of not being exceeded during the life of the structure is known as characteristic load and is denoted by F.

Q-27. What are the partial safety factors?

The partial safety factors when applied to loads and materials give the design values. The partial safety factors take into account the possible overloads, the limit state considered and inaccurate accessment of the effects of loading.

Q-28. Select the answers.

(1) Shear design in IS : 456-1978 refers to
— elastic analysis
— plastic analysis
— limit state analysis

(2) Side face reinforcement in the beam designed for flexure shall be provided when the depth of web in a beam exceeds
— 450 mm
— 750 mm
— 1050 mm

(3) Unit weight of reinforced concrete is
— 23 kN/m^3
— 24 kN/m^3
— 25 kN/m^3

(4) Maximum spacing of vertical stirrups permitted is
— $0{\cdot}75d$
— $\sqrt{2}\, d$
— $\frac{1}{\sqrt{2}}\, d$

(5) Basic span to effective depth ratio of a simply supported beam is
— 20
— 26
— 30

(6) To find out modification factor for span-depth ratio with respect to percentage of tension reinforcement for a tee beam, the width is taken as
— effective flange width
— effective rib width
— average of above two

(7) In a slab of 100 mm thickness, the maximum size of reinforcing bar is
— 10 mm
— 12 mm
— 16 mm

(8) The minimum tor steel reinforcement in slab is
— 0·12%
— 0·15%
— 0·20%

(9) In elastic design, percentage of redistribution of moment in a continuous beam is limited to
— 10%
— 15%
— 30%

(10) The limiting value of the depth of neutral axis in a beam when tor steel reinforcement of grade Fe 415 is used, is
— 0·53 *d*
— 0·48 *d*
— 0·46 *d*

Answers:

(1) Limit state analysis
(2) 750 mm
(3) 25 kN/m^3
(4) 0·75 *d*
(5) 20
(6) effective flange width
(7) 12 mm
(8) 0·12%
(9) 15%
(10) 0·48 *d*.

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